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Extreme Linkages in Financial Markets: Macro Shocks and Systemic Risk

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Abstract

The recent IMF World Economic Outlook (2013) investigates how real and financial shocks can cause a sharp increase in cross country output co-movements. This paper looks at the reverse issue by asking how macro regimes of extreme low and high inflation or productivity growth are conducive to spillover of financial market shocks between major open economies. Using a non-parametric measure we study the largest movements in the US and German equity index returns conditional on a specific macro regime in one or both of the countries. It is known that the unconditional probability of different stock markets crashing jointly is non-negligible, see e.g. Hartmann et al. (2004) and Poon et al. (2004). The results suggest that the factor related to real economy, i.e. industrial production growth, is a major driver behind the extreme loss linkage, but inflation is not. One explanation is that monetary policy shocks are absorbed by the exchange rate, whereas technology shocks do spillover.

Keywords: Spillover, Systemic risk, Macro shock, Extreme Value Theory

JEL classification: G1, F3, C49, E44

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1 Introduction

Stock markets move together for various reasons. The recent credit crisis depressed stock markets worldwide. Since then, much heed has been paid to studying rare events and their co-movements in financial markets. It is known that stock returns and stock index returns are asymptotically dependent, meaning that even in the limit the interdependency does not vanish, like in the case of a multivariate Student-t distribution. Per contrast the interdependency between correlated normal distributions eventually vanishes deep in the tail area, see De Vries (2005).

The conventional correlation measure is frequently used to characterise such interdependency. For example, Lin, Engle and Ito (1994) investigate correlations and spillovers between the Tokyo and New York equity markets using a GARCH-based model. Correlation analysis puts a strong assumption of normality on the underlying distribution which is prone to the underestimation of tail risks (Poon et al., 2004). To model the rare events, extreme statistics (i.e. extreme value theory - EVT) may be preferable since only extreme data are considered and hence the estimation will not be contaminated by the data from the central. Some literature regarding the use of EVT approach is Hartmann, Straetmans, and de Vries (2008) in which they apply EVT to examine heavy tails and currency crises. Hartmaan et al. (2004) also measure asset market linkages between and across equity and bond markets in crisis periods using a non-parametric EVT method.

To our knowledge, EVT has not been employed much in macro study. One main reason is due to the nature of its low frequency data. Nevertheless, this paper exploits a way to make use of it. In this paper we ask which macro regimes are conducive to the observed asymptotic dependency between stock markets of major open economies. We mainly consider the largest movements in the US and German equity index returns conditional on specific regimes of inflation and total factor productivity growth, representing demand side and supply side regimes respectively.

The monetary model of the exchange rate holds that exchange rate returns Δs absorb the changes in (country-) relative money supply Δm , relative real income shocks Δy and shocks to the interest differential Δi . Specifically, in its crudest form the monetary model reads

$$\Delta s = \Delta m - \gamma \Delta y + \lambda \Delta i$$

where λ is the semi-interest elasticity of money demand and γ is the income elasticity of money demand. As presented, monetary shocks and hence regimes of high or low inflation (deflation) are fully absorbed in the exchange rate changes. Typical regressions do reject monetary neutrality and hence monetary shocks in one country may be transmitted to the other country and be partly reflected in Δy , at least in the short run. This would be a case of international demand side externalities that may spillover into foreign stock market indices.

On the real side, standard macro models nowadays start from a representative consumer who faces choices between consuming a competitive good Z, differentiated goods Q_i and supplying labor L. The archetypical utility function reads

$$U = Z^{1-\theta} \left[\frac{1}{n} \sum_{i=1}^{n} Q_i^{\rho} \right]^{\theta/\rho} - \frac{1}{1+\gamma} L^{1+\gamma},$$

where ρ is constrained to $\rho \in (0, 1)$.

On the supply side Ricardian production functions

$$Z = BL_z$$
 and $Q_i = AL_i$

generate output, where A and B are the labor productivity coefficients. Solving for the profit of the differentiated goods sector, one finds that

$$\Pi(Q) = c(\theta, \rho) wL(A, B; \theta, \gamma, \rho)$$

and where w is the nominal wage rate, which might capture monetary spillover (see full derivation in Appendix A). Factor productivity coefficients A and B drive the profit levels and presumably the stock indices. In the case that productivity growth is more or less entirely a domestic development, like extreme weather conditions influencing crop size, supply side factors would not generate the tail interdependency between stock markets. In the case of productivity spillover, however, stock markets are more susceptible to joint crashes or jubilation. Novel computer technology typically spreads rapidly around the globe.

We test for the presence or absence of spillover by conditioning the extreme downward movements in the US S&P index and the German based DAX index returns on macro regimes of high and low inflation and productivity growth.

The remainder of the paper is organised as follows. Section 2 introduces the concept of heavy tails and the measure of extreme dependency. The equity returns and macro data are described in Section 3. In Section 4, the empirical results of estimating extreme loss linkages are discussed. Section 5 concludes. Appendix A provides the theoretical investigation for the fat-tailed distribution of macro factors using a standard closed economy macro model.

2 Estimation

In this section, the concept of heavy tails is first presented. Then, we introduce the tail index estimator and extreme dependency measure.

2.1 Fundamentals of Heavy Tails

Suppose that F(x) is a distribution function of a random variable x. F(x) exhibits heavy tails if it varies regularly at infinity. Specifically, for the upper tail, we have

$$\lim_{t \to \infty} = \frac{1 - F(tx)}{1 - F(t)} = x^{-\alpha}, x > 0, \alpha > 0,$$
(1)

where α is a tail index.

Furthermore, random variables whose tails are regularly varying also have an additivity property, i.e. Feller's Convolution Theorem (1971, VIII.8). More precisely, assume that

$$P\{X > x\} = 1 - F(x) = Ax^{-\alpha} + o(x^{-\alpha}), \text{ as } x \to \infty.$$
 (2)

Then, if X_1 and X_2 are i.i.d. with c.d.f. F(x) in (2),

$$P\{X_1 + X_2 > s\} \sim 2As^{-\alpha}, \text{ as } s \to \infty.$$
(3)

If X and Y are two random variables such that $P\{X > x\} \sim A_1 x^{-\alpha}$ and $P\{Y > x\} \sim A_2 x^{-\gamma}$, where $\gamma > \alpha$, it can be shown that

$$P\{X+Y>s\} \sim As^{-\alpha}, \text{ as } s \to \infty.$$
(4)

In other words, the convolution is dominated by the heavier tail.

2.2 Tail Index Estimator

To estimate the tail index α , Hill (1975) estimator is frequently proposed. Suppose that $X_1 \leq X_2 \leq \ldots \leq X_n$ is the order statistics. Thus, the Hill estimator is the inverse tail index

$$\widehat{\gamma} = \frac{\widehat{1}}{\alpha} = \frac{1}{k} \sum_{i=1}^{k} \log \frac{X_{n+1-i}}{X_{n-k}},\tag{5}$$

where X_{n-k} is an appropriate threshold. In this case, there will be k observations above the threshold. For $k(n) \to \infty$ and $k(n)/n \to 0$, $\hat{\gamma}$ is asymptotically normally distributed with zero mean and variance $1/\alpha^2$.

A suitable threshold or cutoff point can be chosen using the eye-balling technique (Embrechts et al., 1997). Specifically, the cutoff point is decided where the plot of the estimated tail indices against descending threshold values (the so-called Hill plot) is first relatively stable.

Alternatively, the Dekkers-Einmahl-de Haan (1989) (DEdH) estimator may also be used to obtain the tail index. The DEdH tail index is defined as

$$\widehat{\gamma} = \frac{\widehat{1}}{\alpha} = 1 + H + \frac{1}{2} \frac{K/H}{H - K/H},\tag{6}$$

where

$$H = \frac{1}{k} \sum_{i=1}^{k} \log \frac{X_{n+1-i}}{X_{n-k}}, \quad K = \frac{1}{k} \sum_{i=1}^{k} (\log \frac{X_{n+1-i}}{X_{n-k}})^2.$$

Note that H is the Hill estimator and if the distribution varies regularly at infinity, K/2H is an alternative to the Hill estimator.

2.3 Extreme Dependence Measure

One may use correlation analysis to measure the conditional dependence. The correlation concept is, however, closely connected to the multivariate normal distribution and hence it may not be a reliable dependency measure for data from the tails, see Ang and Chen (2002). More importantly, the amount of correlation, for example, is not very informative regarding the probability of tail spillover. The pitfalls of using correlation based measure are discussed in Embrechts et al. (1999).

Given the shortcomings of correlation measure, we directly quantify the extreme dependence in terms of probability by employing a non-parametric count estimator that does not require any distributional assumption regarding the data. Suppose that Xand Y are two random variables whose failure regions are defined by X > s and Y > s, where s is the loss level, say. Then the probability of markets crashing jointly given that at least one market crashes is

$$\frac{P(X > s, Y > s)}{1 - P(X \le s, Y \le s)} = \frac{P(X > s) + P(Y > s)}{1 - P(X \le s, Y \le s)} - 1$$
$$= \frac{P(\min[X, Y] > s)}{P(\max[X, Y] > s)}$$
$$\approx \frac{\#\min[X, Y] > s}{\#\max[X, Y] > s}.$$
 (7)

Note that the conditional probability of market crashes in (7) is in fact the summation of the marginal probabilities divided by their joint probability. The joint distribution typically can be estimated by a number of approaches such as Copula (a parametric approach), Stable Tail Dependence Function (a non-parametric multivariate approach) and so on. We here utilise the count estimator in which the probability of a joint crash given a crash in at least one of the markets is estimated by simply counting the joint minima and maxima over a high threshold s; see Slijkerman et al. (2013). Subsequently we condition the probabilities in the numerator and denominator on certain macro regimes.

Consider the following simplified example. Suppose that the losses X and Y are driven by the shocks ε , u and w as follows

$$X = \varepsilon + u$$
 and $Y = \varepsilon + w$

and where the three shocks are independently Pareto distributed. Then one shows that

$$\lim_{s \to \infty} \frac{P(X > s, Y > s)}{1 - P(X \le s, Y \le s)} = \frac{1}{3}$$

Per contrast, if the shocks are normally distributed, this limit is zero! Conditioning on the joint factor ε gives

$$\lim_{s \to \infty} \frac{P(X > s, Y > s | \varepsilon > s)}{1 - P(X \le s, Y \le s | \varepsilon > s)} = 1,$$

while if one conditions one of the other two factors, say u,

$$\lim_{s \to \infty} \frac{P(X > s, Y > s | u > s)}{1 - P(X \le s, Y \le s | u > s)} = 0$$

Conditioning on the common factor raises the asymptotic dependence.

2.3.1 The Impact of Marginal Distributions

To measure the dependency of random variables, it is conventional to eliminate the impact of individual marginal distributions using data transformation. One may transform the raw data to common unit Pareto marginals (Hartmann et al., 2006). More precisely, suppose that we have the random variables X_i for i = 1, ..., M. Then, X_i can be transformed to \tilde{X}_i such that

$$\tilde{X}_i = \frac{1}{1 - F_X(X_i)}, \text{ for } i = 1, \cdots, M,$$

where $F_{X_i}(X_i)$ denotes the marginal cumulative distribution function for X_i . Once transformed, each \tilde{X}_i will obtain the common marginal distribution in which the dependence structure remains the same as before. Nonetheless, since the marginal distributions are unknown, it is suggested to use their empirical counterparts. Eventually, we achieve

$$\tilde{X}_i = \frac{n+1}{n+1-R_{X_i}},$$

where $R_{X_i} = \operatorname{rank}(x_{ik}, k = 1, \dots, n)$. Alternatively, other transformation methods such as unit Frétchet marginals (see Poon et al., 2004) may be employed.

3 Data

This section describes the data of equity returns and macro fundamentals that will be used for the estimation.

3.1 Equity Index Returns

Figure 1 illustrates the scatter plot of S&P500 and DAX daily index returns from January 1973 to June 2012 (10,304 days). From the plot, it can be observed that several extremes occurred over the period. For instance, the pair of most extreme returns in the left below quadrant represents the well-known Black Monday co-crash in October 1987. During the credit crisis in 2008, there exists a dramatic slump in both markets simultaneously, following a sharp rebound after a while. Moreover, there was also the event that the German market realised the biggest loss in 40 years, whereas the US market slightly moved. Such event is pertaining to the German unification. In the next section, we will particularly consider the dependency of large losses in the left below quadrant.



Figure 1: Scatter plot of S&P500 and DAX daily index returns from January 1977 to June 2012 (10,304 days).

3.2 Macro Fundamentals

Regarding macro data, we consider the following four main macro factors:

- 1. Inflation (Consumer Price Index; CPI)
- 2. Industrial production growth
- 3. Unemployment rate
- 4. Money supply (M2) growth.

The data were obtained from DataStream inc. at a monthly frequency ranging from January 1970 to December 2012.¹ All are seasonally adjusted. In Figure 2, we can notice that US inflation is relatively higher for the first 15 years while German inflation appears relatively less volatile over the period considered. An immediate question arises how we determine stressed levels for inflation regimes.

3.2.1 Stressed Thresholds

It is natural to specify stressed thresholds for the regimes of macro factors as follows.

• First and the simplest way is to compute constant lower and upper (i.e. 5% and 95%) quantiles. The plots of US and German inflations with the constant thresholds are demonstrated in Figure 3 (top row). Typically, the policy related to inflation tends to change over time. Those extremely high levels, like in the 1970s, are unlikely to

¹the data of all four macro factors except unemployment rate are in year-on-year percentage change.



Figure 2: US inflation with S&P500 daily index returns (left) and German inflation with daily DAX index returns (right) from January 1970 to December 2011. Inflations are seasonally adjusted.

be repeated since the central banks of both countries are now adhering to inflation targets. In that sense, using the constant thresholds may be not suitable. It would be reasonable if the thresholds are allowed to vary over time.

- Secondly, the stressed levels may be determined by employing N-year moving average 5% and 95% quantiles. Figure 3 (bottom row) presents the US and German inflations with 10-year moving average thresholds. With the moving average, the thresholds turn to be more realistic comparing to the constant thresholds.
- Figures 4, 5 and 6 plot the remaining three macroeconomic fundamentals (industrial production growth, unemployment rate and money supply growth respectively) with constant and 10-year moving average 5% and 95% quantile thresholds.

To this end, two observations are made as below. We would emphasise that public announcement dates of macro data should not have a significant effect to our study since we are not investigating the impact of an announcement date to extreme events but merely employing thresholds to specify the regimes of macro factors. Note that the official macro data of the US and Germany are generally released around two weeks after the end of every month and the date is not predetermined.



Figure 3: US (left) and German (right) seasonally adjusted inflation (year-on-year) from January 1970 to December 2011 with constant (top) and 10-year moving average (bottom) 5% and 95% quantile thresholds.



Figure 4: US (left) and German (right) monthly seasonally adjusted industrial production growth (year-on-year) from January 1970 to December 2011 with constant (top) and 10-year moving average (bottom) 5% and 95% quantile thresholds.



Figure 5: US (left) and German (right) monthly seasonally adjusted unemployment rate from January 1970 to December 2011 with constant (top) and 10-year moving average (bottom) 5% and 95% quantile thresholds.



Figure 6: US (left) and German (right) monthly seasonally adjusted money supply (M2) growth (year-on-year) from January 1970 to December 2011 with constant (top) and 10-year moving average (bottom) 5% and 95% quantile thresholds.

4 Empirical Results

This section discusses main empirical results. To begin with, we carry out the estimates of tail index for macro fundamentals. Consequently, we the count estimator is employed to measure the extreme loss linkages between the US and German equity markets unconditional and conditional on the regimes of stressed macro fundamentals.

4.1 Tail Index of Macro Fundamentals

We investigate the right-tailed heaviness for macro fundamentals by computing the tail index $\hat{\alpha}$ of monthly US and German macro data (inflation, industrial production growth, unemployment rate and money supply growth) from January 1973 to June 2012 (474 months). The results are illustrated in Figures 7 and 8 respectively. An appropriate threshold is selected where the plot first stabilises. (eye-balling technique).

For the US, it can be found that inflation has the heaviest tail with $\hat{\alpha} \approx 2.0$, whereas industrial production growth achieves the thinnest tail (with a short stable region in the Hill plot) $\hat{\alpha} \approx 7.5$. For the German indicators, the results are slightly different, particularly for the money supply growth which clearly has a fatter tail than that of the US.



Figure 7: Hill plots for the right tails of four US macro fundamentals: inflation, industrial production growth, unemployment rate and money supply growth. The monthly data ranges from January 1973 to June 2012 (474 months).

Figure 8: Hill plots for the right tails of four German macro fundamentals: inflation, industrial production growth, unemployment rate and money supply growth. The monthly data ranges from January 1973 to June 2012 (474 months).

4.2 Extreme Linkages: Equity Index Returns

We measure the conditional extreme loss linkages between S&P500 and DAX daily index returns from January 1973 to June 2012 (10,304 days) using the count measure. To eliminate the effect of individual marginal distributions, we transform the raw data to common unit Pareto marginals as discussed in the earlier section. After this transformation, each series exhibits the common marginal distribution in which the dependence structure is still embedded and unaltered. Two cases of the extreme dependency are investigated: unconditional and conditional on the stressed macro regimes. The macro fundamentals are monthly inflation, industrial production growth, unemployment rate and money supply (M2) growth. The stress levels for these data are said to occur if these are outside the 5% - 95% quantile ranges of their 10-year moving average of the historical data.

4.2.1 Unconditional on the Stressed Macro Regimes

Figure 9 gives the extreme loss linkage estimates for the full sample, i.e. without conditioning on the stressed macro regimes. Eventually, if s is sufficiently low, all data are included and the plot reaches 1 (this would be on the right hand side and outside the frame of Figure 9). At high threshold levels s, however, the plot either lingers in the neighborhood of zero in the case that there is no asymptotic dependence (as in the case of correlated normals), or almost immediately jumps to the level of asymptotic dependence that is in the data. From the Figure 9, it can be seen that the linkage probability quickly stabilizes (reading from the left to the right) around 0.2. This means that "approximately in one per five market crashes, there is one co-crash" between the two indices.

Figure 9: Co-crash probability between S&P500 and DAX daily index returns from January 1973 to June 2012.

4.2.2 Conditional on the Stressed Macro Regimes

To estimate the extreme loss linkages conditional on a stressed macro fundamental, we extract the return data from the full sample using four cases of severe macroeconomic regimes: a) above the 95% quantile threshold in both countries, b) above the 95% quantile threshold in at least one country, c) below the 5% quantile threshold in both countries, and d) below the 5% quantile threshold in at least one country.

In other words, we investigate the extreme linkages between the two markets during periods of high and low levels of macro fundamentals, either when the severe macro situation occurs in both countries simultaneously, or when it happens in at least one country. Due to the nature of the macro variables, the macro data are lagged by one month. For instance, if the value of a macro fundamental of the current month exceeds the specified threshold value, daily returns of the whole next month are used for estimating the extreme linkage probabilities.

Figures 11 presents the estimates of the extreme linkage probabilities conditional on stressed inflation levels. The number of observations of the four regimes are 152, 1,861, 523 and 782 respectively (as shown in Figure 10). Surprisingly perhaps, the results indicate that in almost all cases the extreme linkages between US and German equity markets during severe low and high inflations are close to that of the full sample data. The linkage probability when inflation is extremely low in at least in one country turns out to be slightly higher than in the other three cases. It appears that the monetary or demand side factor does not contribute towards explaining the tail spillover between the two stock markets.

Figures 13 gives the estimated extreme linkages conditional on stressed industrial production growth, based on respectively 236, 1,178, 303 and 782 observations for the four cases (see Figure 12). The two graphs on the right stand out as the estimated limit probability is around 0.4. During periods of dramatic contraction of the industrial production the co-crash probabilities are clearly above the unconditional full sample estimate. When the industrial production growth is extremely high in both countries, the extreme linkage estimate is also elevated above the unconditional estimate. There appears to be real or supply side spillovers that drive the joint dowturns of the two markets.

The estimated extreme linkages conditional on stressed levels of two additional macro fundamentals (unemployment rate and money supply growth) are illustrated in Figures 15 and 17 respectively. During periods of dramatic unemployment rate, we can observe that the most impact on the extreme linkage is when at least one country encounters very high unemployment. For money supply growth, it can be found that we achieve a very low extreme linkage probability when conditioning the returns on money supply growth below 5% quantile in both countries,.

Table 1 ranks the impact of stressed macro regimes on the extreme loss linkages between S&P500 and DAX daily index returns. We here have not reported the results numerically since it appears to have high variation in the plots due to few conditional data obtained. Nevertheless, the results still can be differentiated which cases are more influential. From the table, the results are generally mixed up but it is obvious that industrial production growth has the most impact in almost all cases.

10-year Moving Average Thresholds		Ranking of Stressed Macro Regimes by the Impact on Extreme Loss Linkages		
Above 95%	In Both Countries	 Industrial Production Growth Inflation, Unconditional* Unemployment Rate 		
Quantile Threshold	In at Least One Country	 Unemployment Rate, Money Supply Growth Industrial Production Growth Inflation, Unconditional* 		
Below 5% Quantile Threshold	In Both Countries	 Industrial Production Growth Inflation Unconditional* Money Supply Growth 		
	In at Least One Country	 Industrial Production Growth Unemployment Rate, Money Supply Growth Inflation, Unconditional* 		

*Unconditional on stressed macro regimes, i.e. using full sample data.

Table 1: Ranking of stressed macro regimes by the impact on extreme loss linkages between US and German daily index returns from from January 1973 to June 2012.

Figure 10: S&P500 and DAX daily index returns from January 1973 to June 2012 conditional on four cases of stressed inflation.

Figure 11: Co-crash probabilities between S&P500 and DAX daily index returns from January 1973 to June 2012 conditional on four cases of stressed inflation.

Figure 12: S&P500 and DAX daily index returns from January 1973 to June 2012 conditional on four cases of stressed industrial production growth.

Figure 13: Co-crash probabilities between S&P500 and DAX daily index returns from January 1973 to June 2012 conditional on four cases of stressed industrial production growth.

Figure 14: S&P500 and DAX daily index returns from January 1973 to June 2012 conditional on four cases of stressed unemployment rate.

Figure 15: Co-crash probabilities between S&P500 and DAX daily index returns from January 1973 to June 2012 conditional on four cases of stressed unemployment rate.

Figure 16: S&P500 and DAX daily index returns from January 1973 to June 2012 conditional on four cases of stressed money supply growth.

Figure 17: Co-crash probabilities between S&P500 and DAX daily index returns from January 1973 to June 2012 conditional on four cases of stressed money supply growth.

4.3 Extreme Linkages: Sectorial Index Returns

Instead of the index, we also examine the extreme loss linkage of US and German equity markets in the sectorial level. Eight sectors considered are financials, industrials, materials, consumer goods, consumer services, utilities, health care and telecom. The results are concluded as follows.

• Comparing the extreme linkages unconditional on stressed macro regimes among all sectors, we can categorise the results as three main groups as demonstrated in Table 2. From the table, we can observe that three sectors in the first group (financials, industrial and materials) achieve the highest co-crash probabilities at around 0.20 which is close to that of the index returns. The second and third groups obtain the linkage probability lower respectively.

	Sector	Co-Crash Probability	
1.	Financials, Industrials, Materials	≈ 0.20	
2.	Consumer Services, Consumer Goods, Telecom	≈ 0.15	
3.	Utility, Health Care	≈ 0.12	

Table 2: Co-crash probabilities for the US and German sectorial index returns from January 1973 to June 2012.

- Once the sectorial index returns are conditioned on the regimes of stressed macro fundamentals, it reveals that industrial production growth is still the most influential macro factor on the extreme linkage for almost all sectors, except for the health care in which the impact by all four stressed macro fundamentals are rather similar¹. Furthermore, we also notice that given the regime of high stressed inflation, the co-crash probabilities between US and German index returns in materials, industrials, telecom and health care sectors are higher than that of the index.
- Table 3 summarises which sectors are affected most and least for each stressed macro regime. In general, we can see that three sectors: industrials, financials and materials (the first group in Table 2) are most affected during severe macro conditions. The least affected sectors are mixed up but all belong to the second and third groups.

5 Conclusion

This paper attempts to associate multivariate extreme value theory with macro study. The main contribution is the analysis and investigation of the extreme loss linkages in the financial markets conditional on stressed macro regimes. The count measure is deliberately opted for the estimation as it is convenient to implement and does not rely on any strong assumption of the underlying distribution.

¹To save the space, the figures are not shown here but available upon request.

Magra Factors	Above 95% Quantile Thresholds		Below 5% Quantile Thresholds	
Macio Factors	Quantile Thresholds		Quantile Thresholds	
	Most Impact	Least Impact	Most Impact	Least Impact
Inflation	Industrials	Consumer Services	Industrials	Health Care
	Materials	Consumer Goods	Materials	Telecom
				Consumer Services
IP growth	Industrials	Telecom	Industrials	Telecom
	Financials	Health Care	Financials	Health Care
	Materials	Consumer Services	Materials	Utilities
	Industrials	Health Care	Industrials	Consumer Services
Unemployment rate	Materials	Telecom	Financials	Telecom
	Financials	Consumer Services	Materials	Utilities
Money supply growth	Materials	Telecom	Industrials	Utilities
	Industrials	Utilities	Materials	Health Care
	Financials	Health Care	Consumer Services	Consumer Services

Table 3: The impacts of stressed macro regimes on the extreme loss linkages between US and German sectorial index returns (from January 1973 to June 2012).

We investigate the co-crash probabilities between US and German equity markets both unconditional and conditional on four regimes of stressed macro fundamentals. Productivity growth represents the supply side, while inflation plays the part on the demand side. Interestingly, the results suggest that real macro factor is more influential than the monetary factor. Monetary policy shocks between the two economies are not transmitted to the stock indices if these are absorbed by the exchange rate, for example. Supply side based technology spillovers may drive both stock markets. Apparently these are not purely domestic innovations (or retardations), but affect both markets simultaneously.

There is a plenty of room for further research. Other macro factors might be considered and the analysis is not limited to the linkages between major economies. The spillovers from developed to emerging market countries would be worth investigating. Perhaps that demand side factors play a larger role in this case.

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A Macroeconomics with Shocks

In this appendix, we use a standard closed economy macro model to study how shocks propel through the macro economy in equilibrium. Both supply and demand shocks are studied. It is shown how standard assumptions on the shock distribution lead to a power law distribution explaining the bouts of severe changes and asymptotic dependency between the various macro factors.

A.1 Demand Side

Current macro models typically entertain a two sector model. One sector is competitive and the other sector produces differentiated products. The pricing power in the latter sector determines price setting behavior.

The macro literature has focussed almost exclusively on the Dixit and Stiglitz (1977) specification for the differentiated goods demand, see e.g. Walsh (2010, ch.8). The familiar Dixit-Stiglitz (DS) specification with endogenous labor supply is derived from the following utility function.

$$U = Z^{1-\theta} \left[\frac{1}{n} \sum_{i=1}^{n} Q_i^{\rho} \right]^{\theta/\rho} - \frac{1}{1+\gamma} L^{1+\gamma},$$
(8)

where Z is the composite good, the Q_i are the differentiated goods and L is labor. To guarantee concavity and allow for zero demand, the parameter ρ is constrained to $\rho\epsilon(0, 1)$. Macro literature, see e.g. Walsh (2010, ch.8), mostly uses a continuum of differentiated goods, here we use a specification with a discrete number n, to facilitate the computation of the distribution of the various macro factors in equilibrium. We envision the Z good to be a staple good like agricultural produce, while the Q_i goods capture the production of other goods and services.

The budget constraint reads

$$wL + \Pi(Q) = qZ + \frac{1}{n} \sum_{i=1}^{n} p_i Q_i,$$
(9)

where w is the wage rate and q, p_i are the goods prices, while $\Pi(Q)$ are the profits received from the differentiated goods sector.

The first order conditions for optimality conditions entail

$$(1-\theta) Z^{-\theta} n^{-\theta/\rho} \left[\sum_{i=1}^{n} Q_i^{\rho} \right]^{\theta/\rho} - \lambda q = 0,$$

$$\theta \left(\frac{Z}{\left[\sum_{i=1}^{n} Q_i^{\rho}\right]^{1/\rho}} \right)^{1-\theta} n^{-\theta/\rho} \left[\sum_{i=1}^{n} Q_i^{\rho} \right]^{\frac{1}{\rho}-1} Q_i^{\rho-1} - \lambda \frac{1}{n} p_i = 0,$$

$$-L^{\gamma} + \lambda w = 0$$

and

$$wL + \Pi(Q) = qZ + \frac{1}{n} \sum_{i=1}^{n} p_i Q_i.$$

The first order conditions imply the familiar price and wage ratios

$$\frac{p_i}{p_j} = \frac{Q_i^{\rho-1}}{Q_j^{\rho-1}},$$

$$\frac{p_i}{q} = \frac{\theta}{1-\theta} \frac{Z}{Q_j} \frac{p_j^{\rho/(\rho-1)}}{\frac{1}{n} \sum_{i=1}^n p_i^{\rho/(\rho-1)}},$$

and

$$\frac{w}{q} = \left(q^{-1}P\right)^{\theta} \frac{L^{\gamma}}{\left(1-\theta\right)^{1-\theta}\theta^{\theta}},$$

where the price index for differentiated goods is defined as

$$P = \left(\frac{1}{n}\sum_{i=1}^{n} p_i^{\rho/(\rho-1)}\right)^{\frac{\rho-1}{\rho}}$$

Then the labor supply can be written as

$$L = \left((1-\theta)^{1-\theta} \,\theta^{\theta} \frac{w}{q^{1-\theta} P^{\theta}} \right)^{1/\gamma}.$$
(10)

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The goods demanded can be expressed as

$$Z = (1 - \theta) \frac{wL + \Pi(Q)}{q} \tag{11}$$

the goods demanded and

$$Q_i = \theta \frac{wL + \Pi(Q)}{p_i} \left(\frac{p_i}{P}\right)^{\rho/(\rho-1)}.$$
(12)

A.2 Supply Side

Assume Ricardian technologies for all the goods, where

$$Z = BN$$

and

$$Q_i = AN_i.$$

Here A and B are the productivity coefficients while N and N_i are the respective labor inputs. Both A and B are random variables. These TFP shocks are the familiar supply side total factor productivity shocks due to innovation and nature.

Suppose that the market for Z is perfectly competitive

$$\Pi(Z) = qZ - wN = \left(q - \frac{w}{B}\right)Z = 0,$$

so that

$$q = w/B. \tag{13}$$

A.2.1 DS Differentiated Goods

In the DS specification the differentiated goods profit function reads

$$\Pi(Q_i) = p_i Q_i - w N_i = \left(p_i - \frac{w}{A}\right) Q_i$$
$$= \left(p_i - \frac{w}{A}\right) \theta \frac{wL + \Pi(Q)}{p_i} \left(\frac{p_i}{P}\right)^{\rho/(\rho-1)}$$

The producer exploits his pricing power, but ignores his pricing effect on the price index P of the differentiated goods and the consumer income $wL + \Pi(Q)$.² Differentiation gives

$$\frac{\partial \Pi(Q_i)}{\partial p_i} = \frac{1}{\rho - 1} Q_i \left\{ \rho - \frac{1}{A} \frac{w}{p_i} \right\}.$$

Exploiting the pricing power therefore implies setting prices

$$p_i = \frac{w}{\rho A}.\tag{14}$$

Hence, $P = w/\rho A$ as all prices are identical. Total profits in the differentiated goods sector are

$$\Pi(Q) = \frac{1}{n} \sum_{i=1}^{n} \Pi(Q_i) = \frac{1}{n} \sum_{i=1}^{n} \left(1 - \frac{w/p_i}{A}\right) \theta \left[wL + \Pi(Q)\right] \left(\frac{p_i}{P}\right)^{\rho/(\rho-1)}$$
$$= (1-\rho) \theta \left[wL + \Pi(Q)\right].$$

Solve for the total sectorial profits as

$$\Pi(Q) = \frac{(1-\rho)\,\theta}{1-(1-\rho)\,\theta} wL.$$
(15)

A.3 Macro Equilibrium

It follows that in equilibrium after substituting the price levels into the labor supply equation (10)

$$L = \left(\theta^{\theta} \left(1-\theta\right)^{1-\theta} A^{\theta} B^{1-\theta}\right)^{1/\gamma} \rho^{\theta/\gamma} = \varphi \rho^{\theta/\gamma}, \tag{16}$$

say, and where

$$\varphi = \left(\theta^{\theta} \left(1-\theta\right)^{1-\theta} A^{\theta} B^{1-\theta}\right)^{1/\gamma}$$

Furthermore, from (11), (15) and (16)

$$Z = (1 - \theta) \frac{B}{1 - (1 - \rho)\theta} \varphi \rho^{\theta/\gamma}.$$
(17)

 $^{^{2}}$ One can easily incorporate this effect as well if desired, see Heijdra and Yang (1993). But for two reasons we do not follow this route. One may doubt that producers take this macro effect of their pricing behavior into account. Moreover, it adds little to the insights derived form specifying the differentiated goods sector.

Similarly, using (12), (15) and (16)

Hence

$$Q_j = \theta \frac{A}{1 - (1 - \rho) \theta} \rho \varphi \rho^{\theta/\gamma}.$$

$$\frac{1}{n} \sum_{j=1}^n Q_j = \theta \frac{A}{1 - (1 - \rho) \theta} \varphi \rho^{\theta/\gamma+1}.$$
(18)

To determine the price level, we also assume a simple quantity type relation for the money supply process

$$M = wL. (19)$$

A.4 Equilibrium Price Distribution

With the above preparations, we can now derive the implications for the equilibrium prices, quantities and macro factors. Most macro models consider supply shocks originating from total factor productivity A and B and demand shocks originating from the money supply process M or from the markup elasticity ρ , see Smets and Wouters (2003). We will first look at the implication of such shocks for the prices p_i of industrial production.

From the above (13) combined with (19) and (16), we get that

$$q = \frac{w}{B} = \frac{M}{B} \frac{1}{L} = M \frac{1/\rho^{\theta/\gamma}}{B\left(\theta^{\theta} \left(1-\theta\right)^{1-\theta} A^{\theta} B^{1-\theta}\right)^{1/\gamma}}.$$

Similarly, using (14) combined with (19) and (16) yields

$$p_{i} = p = \frac{w}{\rho A} = \frac{M}{\rho A} \frac{1}{L} = M \frac{1/\rho^{\theta/\gamma+1}}{A \left(\theta^{\theta} \left(1-\theta\right)^{1-\theta} A^{\theta} B^{1-\theta}\right)^{1/\gamma}}.$$

Consider a supply shock A such that A follows a beta distribution:

$$\Pr\left\{A \le t\right\} = t^{\beta}$$

on [0, 1] and $\beta > 0$. Consider the implication for the distribution of the differentiated goods Q_i . Some calculation reveals

$$\Pr \left\{ p_i \le s \right\} = \Pr \left\{ M \frac{1/\rho^{\theta/\gamma+1}}{A \left(\theta^{\theta} \left(1-\theta\right)^{1-\theta} A^{\theta} B^{1-\theta} \right)^{1/\gamma}} \le s \right\}$$
$$= \Pr \left\{ \frac{M/\rho^{\theta/\gamma+1}}{\left(\theta^{\theta} \left(1-\theta\right)^{1-\theta} B^{1-\theta} \right)^{1/\gamma}} A^{-(1+\theta/\gamma)} \le s \right\}$$
$$= \Pr \left\{ c A^{-(1+\theta/\gamma)} \le s \right\}$$

say, where
$$c = M / \left[\rho^{\theta/\gamma+1} \left(\theta^{\theta} \left(1-\theta\right)^{1-\theta} B^{1-\theta} \right)^{1/\gamma} \right]$$
. So that

$$\Pr \left\{ p_i \le s \right\} = \Pr \left\{ cA^{-(1+\theta/\gamma)} \le s \right\}$$

$$= \Pr \left\{ c/s \le A^{(1+\theta/\gamma)} \right\}$$

$$= \Pr \left\{ (c/s)^{1/(1+\theta/\gamma)} \le A \right\}$$

$$= 1 - \Pr \left\{ A \le (c/s)^{1/(1+\theta/\gamma)} \right\}$$

$$= 1 - c^{\beta/(1+\theta/\gamma)} s^{-\beta/(1+\theta/\gamma)}$$

with support on $[c, \infty)$. The distribution of equilibrium prices is heavy tailed.

Also note that one can then easily obtain that the price changes are also fat tailed distributed as ratios of random variables that have fat tails are also fat tailed distributed. Interestingly, if we look at the implication for nominal output of the sector or profits, we do not get the fat tail implication since

$$p_i Q_i = M \frac{1/\rho^{\theta/\gamma+1}}{A\varphi} * \theta \frac{A}{1 - (1 - \rho)\theta} \rho \varphi \rho^{\theta/\gamma}$$
$$= M \frac{\theta}{1 - (1 - \rho)\theta}$$

and

$$\Pi(Q) = \frac{(1-\rho)\,\theta}{1-(1-\rho)\,\theta} wL = \frac{(1-\rho)\,\theta}{1-(1-\rho)\,\theta} M.$$

But if we have demand shocks of the sorts discussed in Smets and Wouters (2003) regarding ρ , assuming that (recalling that by assumption $\rho\epsilon(0,1)$)

$$\Pr\left\{1 - \rho \le t\right\} = t^{\beta}$$

then

$$\Pr \left\{ \Pi(Q) \le s \right\} = \Pr \left\{ \frac{(1-\rho)\,\theta}{1-(1-\rho)\,\theta} M \le s \right\}$$
$$= \Pr \left\{ (1-\rho)\,\theta \le \frac{s}{M} - \frac{s}{M}\,(1-\rho)\,\theta \right\}$$
$$= \Pr \left\{ \left[1 + \frac{s}{M} \right] (1-\rho)\,\theta \le \frac{s}{M} \right\}$$
$$= \Pr \left\{ (1-\rho) \le \frac{s}{\theta} \frac{1}{M+s} \right\}$$
$$= \left(\frac{s}{\theta} \frac{1}{M+s} \right)^{\beta}$$
$$= \frac{1}{\theta^{\beta}} \left(1 - \frac{M}{M+s} \right)^{\beta}$$

with support $[0, \frac{\theta M}{1-\theta}]$. If $\theta = 1$, only differentiated goods, we have again a heavy upper tail. The ratio of profits in the change of profits, though, is certainly heavy tailed. To see this,

note that

$$\Pr\left\{\frac{1}{\Pi(Q)} \le x\right\} = \Pr\left\{\frac{1}{x} \le \Pi(Q)\right\} = 1 - \Pr\left\{\Pi(Q) \le \frac{1}{x}\right\}$$
$$= 1 - \frac{1}{\theta^{\beta}} \left(1 - \frac{M}{M + 1/x}\right)^{\beta}$$
$$= 1 - \frac{1}{\theta^{\beta}} \left(\frac{1}{Mx + 1}\right)^{\beta}$$

on $\left[\frac{1-\theta}{\theta M},\infty\right)$. The inverses has tail index β .

Looking at nominal GDP, we get

$$qZ + \frac{1}{n} \sum_{i=1}^{n} p_i Q_i = wL + \Pi(Q)$$
$$= \left[1 + \frac{(1-\rho)\theta}{1 - (1-\rho)\theta} \right] wL$$
$$= \frac{1}{1 - (1-\rho)\theta} M.$$

So if we assume again that $\Pr\{1 - \rho \le t\} = t^{\beta}$, then due to θ in the denominator $1 - (1 - \rho) \theta \epsilon [1 - \theta, 1]$ and there are no fat tails. But if we assume that for example that M is exponentially distributed, then the ratio of the money supply and time t divided by the time t-1 supply is fat tailed, since if

$$\Pr\{M \le t\} = 1 - e^{-t},$$

then

$$\Pr\{\frac{1}{M} \le s\} = \Pr\{\frac{1}{s} \le M\} = \exp(-1/s),$$

which is a Frechet extreme value distribution with a tail index of one. Note that we can obtain the distribution of the change as follows (use the last result in the third step and the exponential distribution for the numerator in the fourth step)

$$\Pr\left\{\frac{M(t)}{M(t-1)} - 1 \le x\right\} = E_{M(t)} \left[\Pr\left\{\frac{m(t)}{M(t-1)} - 1 \le x \middle| M(t) = m(t)\right\}\right]$$
$$= E_{M(t)} \left[\Pr\left\{\frac{1}{M(t-1)} \le \frac{x+1}{m(t)}\middle| M(t) = m(t)\right\}\right]$$
$$= E_{M(t)} \left[e^{-\frac{m(t)}{1+x}}\right]$$
$$= \int_{0}^{\infty} e^{-\frac{m}{1+x}} e^{-m} dm$$
$$= \int_{0}^{\infty} e^{-\frac{2+x}{1+x}m} dm$$
$$= \frac{1+x}{2+x}$$
$$= 1 - \frac{1}{2+x}$$

which is a Burr distribution with tail index 1 and support $[-1, \infty)$.

A.5 Implication for Systemic Risk

As for a start, consider the asymptotic dependence between the GDP measure

$$qZ + \frac{1}{n}\sum_{i=1}^{n} p_i Q_i = wL + \Pi(Q) = \frac{1}{1 - (1 - \rho)\theta}M$$

and industrial output in nominal terms

$$p_i Q_i = M \frac{\theta}{1 - (1 - \rho)\,\theta,}$$

where ${\cal M}$ itself follows a Pareto law

$$\Pr\{M \le t\} = 1 - t^{-\alpha}.$$

This example may be less interesting as M is directly assumed to be fat tailed but we can use the above ideas to derive the fat tail property endogenously. Given the assumption on M, we get immediately that

$$\Pr\{\frac{1}{1-(1-\rho)\theta}M > t\} = \Pr\{M > [1-(1-\rho)\theta]t\} = [1-(1-\rho)\theta]^{-\alpha}t^{-\alpha}$$

and

$$\Pr\{\frac{\theta}{1 - (1 - \rho)\,\theta}M > t\} = \theta^{\alpha} \left[1 - (1 - \rho)\,\theta\right]^{-\alpha} t^{-\alpha}.$$

Since $\theta \epsilon(0,1)$ we find that

$$\begin{split} \Pr\{\frac{1}{1-(1-\rho)\,\theta}M>t, \frac{\theta}{1-(1-\rho)\,\theta}M>t\} &= & \Pr\{\frac{\theta}{1-(1-\rho)\,\theta}M>t\}\\ &= & \theta^{\alpha}\left[1-(1-\rho)\,\theta\right]^{-\alpha}\,t^{-\alpha}, \end{split}$$

while

$$1 - \Pr\{\frac{1}{1 - (1 - \rho)\theta}M \le t, \frac{\theta}{1 - (1 - \rho)\theta}M \le t\} = \Pr\{\frac{1}{1 - (1 - \rho)\theta}M > t\}$$
$$= [1 - (1 - \rho)\theta]^{-\alpha}t^{-\alpha}.$$

Hence, the measure for asymptotic dependence gives

$$1 + \frac{\Pr\{\frac{1}{1-(1-\rho)\theta}M > t, \frac{\theta}{1-(1-\rho)\theta}M > t\}}{1-\Pr\{\frac{1}{1-(1-\rho)\theta}M \le t, \frac{\theta}{1-(1-\rho)\theta}M \le t\}} = 1 + \theta^{\alpha} > 1.$$