Extracting Market Inflation Expectations: A Semi-structural Macro-finance Term Structure Model

by

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Abstract

This paper estimates the term structure of inflation expectations using a semi-structural macro-finance term structure model based on new Keynesian macroeconomic framework and the arbitrage-free affine term structure model which defines bond prices as an affine function of state variables. Key economic variables and Thai government bond yield curve data are used to filter out for unobserved components. While letting the inflation target adapts over time, the results suggest that the inflation target has trended down under inflation targeting regime. The long-term inflation expectation is well anchored while the inflation risk premium has dropped substantially over the past five years. The real interest rate is considerably volatile and is a major contributor to movements in the 10-year government bond yield.

Keywords: inflation expectations, inflations risk premium, affine term structure models

JEL classification: E43, E44, G12.

1. **Introduction**

Measuring inflation expectations is an important task for central banks targeting inflation. With that goal, economists prefer getting information from financial markets over surveys since the market decisions are driven by investors' return thus reflected what they really believe. Many central banks may gauge this unobserved variable, inflation expectation, via the difference between nominal bond yield and inflation-linked bond yields or the so-called "the break-even inflation". Unfortunately, this method is infeasible when inflation-linked market is absent or illiquid, which is in Thailand's case, and even with the break-even inflation, one can still suffer from the noisy inflation risk premium embedded in this measure. The objective of this study is to extract the time-series of inflation expectations, net of the risk premium, and other associated components given only nominal yield curve information is provided.

There have been several researches made on extracting inflation expectations and risk premia from the bond yields. One standard approach is to use the canonical finance model which purely describes the dynamics of yields and risk premia based on latent factors Beechey (2008) estimates using 3-factor arbitrage-free affine model and finds that UK's long-term inflation expectations decrease slowly since its adoption of inflation targeting in 1992 while inflation risk premia were quite maintained except a sudden drop in 1997 after the Bank of England was granted operational independence. D'Amico et al. (2010) use information from Treasury Inflation-Protected Securities (TIPS) market and survey data for the U.S. and find that model-based inflation expectations can differ significantly from TIPS break-even inflations due to relatively large liquidity risk premium. Joyce et al. (2010) add an extra factor to a joint model of the UK nominal and real yield curves for flexibility and find similar results. They also use long-term inflation expectations from surveys to help pinning down the inflation expectations. Finlay and Wende (2012) develop a new technique to estimate likewise from the prices of coupon inflation-linked bonds and applied for Australia where there is a limited number of inflation-linked bonds. Their results suggest that Australia's long-term inflation expectations are well anchored within the target range and are less volatile than inflation risk premium.

Several studies also employ the mix of macroeconomic and latent variables as pioneered by Ang and Piazzesi (2003). Ang et al. (2008) combine inflation and other two unobserved factors and estimate with the U.S. data. They conclude that changes in inflation expectations are the key driver of the nominal term spreads. Chernov and Mueller (2012) develop a model that allows for heterogeneous expectations in surveys and find that the U.S. monetary policy becomes more effective over time as the inflation expectations are anchored at about 2%. The number of state variables can overreach to eight variablesas in Dewachter et al. (2014) where macroeconomic, finance and latent variables are used together.

Among others, instead of unobserved latent factors, Gimeno and Marqués (2009) and Christensen et al. (2010) utilize the principal components or well-known Nelson-Siegel factors (Nelson and Siegel, 1987) as observed state variables. These studies extend a simple version of dynamic Nelson-Siegel created by Diebold and Li (2006) by imposing no-arbitrage

condition and add inflation or other additional factors to the state variables. Although we seemingly see advantages from flexibility and capability to fit in the data featured by the above classes of models, this approach gives a very little insight about how bond yields and risk premia react to the changes in economic fundamentals.

Alternatively, though less popular, the joint model of bond yields and macroeconomic variations is used. Instead of simple affine function, this class of models determines the dynamics of the short rate through a policy reaction function which characterizes central bank's decision on monetary policy. Based on a reduced-form of New-Keynesian macro model[1], Hördahl and Tristani (2010, 2012) and Hördahl (2008) impose an affine term structure formulation onto the solution of the macro model. Their results show that model-implied inflation expectations are close to the observed break-even inflations because of relatively small inflation risk premia. Rudebusch and Wu (2008) and Rudebusch (2010) use a similar model and point that shocks to the inflation target is the primary determinant of long-term yields. Extra feature such as Natural rate of interest can be added to the macroeconomic structure as in Lemke (2008).

Although the development of structural macro-finance model and its estimation are vastly complex, the model benefit from the ability to explain linkages between economic fundamentals and term structure in a consistent way. This study, which aims to extract for inflation expectations without any supplements of inflation-linked bonds or surveys, will follow the latter strand of researches to fill the needs of explicit and concrete explanations for the behavior of the yield curve and its components.

The remains of this paper organize as follows. The next section outlines the model. Section 3 describes the data, estimation method, and how to decompose the yield curve into associated components. Section 4 discusses the results, and Section 5 concludes.

2. The Model

The model[2] lies on the idea of how investors form their views about the future real activity (output), inflation, and risks to make decisions on investments based on the perceived current economic situation. Hence, the prices of the assets (or implied bond yields) are the mirrors of investors' expectations; one can infer those underlying expectations from the observed bond prices (yields) and macroeconomic variables. This model is built up from two parts: the macroeconomic module and the finance module (bond pricing formulation).

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[1] See Bekaert et al. (2010) for an integration of full-fledged New-Keynesian DSGE model and affine term structure model which will not be discussed in this study.

[2] This model is much inspired by those of Lemke (2008) and Hördahl (2008).

2.1 The Macroeconomic Module

The macroeconomic part is driven mainly by a semi-structural forward-looking[3] New-Keynesian model with three key equations[4] which describe the joint dynamics of the output gap (z_t), inflation (π_t), and the short-term interest rate (i_t) where ϵ_t's denote shocks to their three main variables.

$$z_t = \alpha_1 z_{t-1} + \alpha_2 z_{t+1} - \alpha_3 (r_t - \bar{r}_t) + \tilde{z}_t + \epsilon_t^z \tag{1}$$

$$\pi_t = \beta_1 \pi_{t-1} + (1 - \beta_1)\pi_{t+1} + \beta_2 z_t + \tilde{\pi}_t + \epsilon_t^\pi \tag{2}$$

$$i_t = \rho_i i_{t-1} + (1 - \rho_i)[\bar{r}_t + \bar{\pi}_t + \Phi_z z_t + \Phi_\pi(\pi_{t+1} - \bar{\pi}_t)] + \epsilon_t^i \tag{3}$$

The output gap, which is defined by the percentage deviation of the actual output from the potential output, is described by the Euler equation (1). The current output (equals to consumption in this setup) is determined by (i) the lagged output and expected output; these two terms are used to capture consumption habits formation and consumption smoothing, (ii) the real interest rate gap or the difference between ex-ante real interest rate (r_t) and the natural real rate of interest ($\bar{r}_t$). The current period real interest rate is defined as the nominal interest rate deflated with one-period ahead expected inflation:

$$r_t = i_t - \pi_{t+1}. \tag{4}$$

The minus sign prior to α_3 gives a straightforward interpretation: consumption is stimulated when real interest rate is below its natural level. The last term is a demand shock.

The Phillips curve equation (2) is specified in a similar fashion; current inflation depends on lead and lagged inflations and the output gap. The output gap enters this equation so that inflationary pressure is induced when the economy operates above its potential level. The last term indicates a supply shock or so-called cost-push shock.

The central bank sets the short-term nominal interest rate following a modified Taylor (1993) rule (3). Typically, the policy rate tracks the natural level of the nominal rate which is the sum of natural real rate of interest ($\bar{r}_t$) and inflation target ($\bar{\pi}_t$); but when the output is above potential level or the expected inflation is above the target, the central bank raise policy rate beyond the natural level depends on weights given to the output gap (Φ_z) and the deviation of expected inflation from target (Φ_π). A gradualism behavior of the central bank is characterized by the lagged interest rate and the last term denotes monetary policy shock.

The standard shocks to the Euler and Phillips curve, however, are short-lived. This study specifies two additional shocks, $\tilde{z}_t$ and $\tilde{\pi}_t$, which are used to capture more persistent shocks that might be expected by investors. The persistent shocks are assumed to follow first-order autoregressive processes and are triggered by transitory shocks: $\epsilon_t^{\tilde{z}}$ and $\epsilon_t^{\tilde{\pi}}$.

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[3] For backward-looking versions, see Lemke (2008) and Laubach and Williams (2003).

[4] Expectation operators (E_t) are omitted here for simplicity.

$$\tilde{z}_t = \rho_{\tilde{z}}\tilde{z}_{t-1} + \epsilon_t^{\tilde{z}} \tag{5}$$

$$\tilde{\pi}_t = \rho_{\tilde{\pi}}\tilde{\pi}_{t-1} + \epsilon_t^{\tilde{\pi}} \tag{6}$$

Equation (7) relates the output gap to the log level of output (y_t) and potential output ($\bar{y}_t$) where g_t in equation (8) denotes the potential growth rate.

$$y_t = \bar{y}_t + z_t \tag{7}$$

$$g_t = \bar{y}_t - \bar{y}_{t-1} \tag{8}$$

The potential growth rate and the natural rate of interest are assumed to evolve around their pre-determined steady-state values with some degree of inertia as specified in equation (9) and (10). These trends share a common cycle through a trend growth rate (a_t) which follows a process described by the equation (11). Note that potential growth rate can still deviate from the common trend since a specific shock to potential growth (ϵ_t^g) is defined herein.

$$g_t = \rho_g g_{t-1} + (1 - \rho_g)g_{ss} + \theta_g a_t + \epsilon_t^g \tag{9}$$

$$\bar{r}_t = \rho_{\bar{r}}\bar{r}_{t-1} + (1 - \rho_{\bar{r}})r_{ss} + \theta_r a_t \tag{10}$$

$$a_t = \rho_a a_{t-1} + \epsilon_t^a \tag{11}$$

Equation (12) describes the behavior of the inflation target that fluctuates around the long-run inflation target (π_{ss}). This long-run variable, which in fact is set by the central bank, can be viewed as investors' perception of inflation target; so we can infer about central bank's credibility from the value of $\rho_{\bar{\pi}}$. When $\rho_{\bar{\pi}}$ approaches to 1, shock to inflation target will last longer; and it takes long time for inflation target to be anchored at the long-run level again. This indicates that the central bank might not gain a full credibility. On the other hand, $\rho_{\bar{\pi}}$ approaches to zero means that inflation target will remain close to the long-run target for most of the time, and this could be happened in the context of full credibility. This study, however, will model inflation target in a conservative way and try to avoid the "inherent credibility." The parameter $\rho_{\bar{\pi}}$ will be fixed at 0.999 to make inflation target a near random walk process and implicitly let this variable varies freely over time. Therefore, only the data can eventually determine how volatile the inflation target is.

$$\bar{\pi}_t = \rho_{\bar{\pi}}\bar{\pi}_{t-1} + (1 - \rho_{\bar{\pi}})\pi_{ss} + \epsilon_t^{\bar{\pi}} \tag{12}$$

Finally, all shocks are assumed to be normal and uncorrelated (i.e. $\epsilon_t \sim IID\ N(0, \Omega)$) with variance-covariance matrix Ω where $\epsilon_t = diag(\epsilon_t^z, \epsilon_t^\pi, \epsilon_t^i, \epsilon_t^{\tilde{z}}, \epsilon_t^{\tilde{\pi}}, \epsilon_t^a, \epsilon_t^g, \epsilon_t^{\bar{\pi}})$ and $\Omega = diag(\sigma_z^2, \sigma_\pi^2, \sigma_i^2, \sigma_{\tilde{z}}^2, \sigma_{\tilde{\pi}}^2, \sigma_a^2, \sigma_g^2, \sigma_{\bar{\pi}}^2)$. The system of equations (1) - (12) together with variance-covariance matrix can be represented in the form of state-space model:

$$X_t = \mu + \Phi X_{t-1} + R\epsilon_t \qquad \text{(State Equation)}$$

$$Y_t = d + ZX_t + H\epsilon_t \qquad \text{(Measurement Equation)}$$

where $X_t = [y_t, r_t, \pi_t, i_t, z_t, a_t, \tilde{z}_t, \tilde{\pi}_t \bar{r}_t, \bar{y}_t, g_t, \bar{\pi}_t]'$. Given the variance-covariance matrix, we can rewrite the state equation to follow a first-order vector autoregressive (VAR(1)) with normalized shocks which is common in related works:

$$X_t = \mu + \Phi X_{t-1} + \Sigma \epsilon_t \qquad \epsilon_t \sim IID(0, I_8) \qquad (13)$$

where $\Sigma = R\Omega^{\frac{1}{2}}$ and I_8 is 8 x 8 identity matrix.

2.2 Bond Pricing Formulations

The finance part of the model is based on an affine term structure model which is constructed under the arbitrage-free condition. The bonds of all maturities are priced in a way that nobody can make any zero-cost profit. Let P_t^n denotes the price at time t of one unit of bond with n period of maturity left. The bond price under arbitrage-free condition is given by

$$P_t^n = E_t(M_{t+1} P_{t+1}^{n-1}) \qquad (14)$$

where M_t is the pricing kernel or stochastic discount factor (SDF). A matured bond pays one unit of account with no uncertainty ($P_{t+n}^0 = 1$). The pricing kernel follows the log-normal process:

$$M_{t+1} = \exp\left(-i_t - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\epsilon_{t+1}\right) \qquad (15)$$

where λ is the 8 x 1 vector of the market price of risk which is defined by an affine function of the state variables:

$$\lambda_t = \lambda_0 + \lambda_1 X_t \qquad (16)$$

The short rate (i_t) in the equation (15) plays a central role of interconnection between the macro module and the finance model; the investors price bonds using expected interest rate path perceived in macroeconomic views. On the other hand, i_t is an affine function of the state vector obtained from the state-space solution of the model:

$$i_t = \delta_i' X_t \qquad (17)$$

where δ_i is a 12 x 1 vector containing zeros in all elements except a one in the fourth position which is the order number of i_t in the state vector. Solving (14) recursively[5], the prices of bonds can be expressed in the form of exponential affine function:

$$P_t^n = \exp(A_n + B_n' X_t) \qquad (18)$$

where A_n and B_n satisfy the difference equations[6]:

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[5] See appendix for derivation steps.

[6] These equations are quite typical in related literature, see e.g. Ang and Piazzesi (2003).

$$A_{n+1} = A_n + B_n'(\mu - \Sigma\lambda_0) + \frac{1}{2}B_n'\Sigma\Sigma'B_n$$

$$B_{n+1}' = -\delta_i + B_n'(\Phi - \Sigma\lambda_1) \tag{19}$$

with initial conditions $A_1 = 0$ and $B_1 = -\delta_i$.

Finally, bond yields are related to bond prices in the following manner:

$$y_t^n = -\frac{1}{n}\ln P_t^n \tag{20}$$

where y_t^n is a continuously compounded n-period yield at time t and one-period yield equals the short rate (i.e. $y_t^1 = i_t$).

2.3 Hypothetical Real Bonds

Although the real bonds are not observed in this study, it still worth to consider the model-implied real yield curve and make use of it in decomposition for other components. The model setup allows us to price the real bonds consistently with the nominal bonds. One can derive the prices of real bonds using exactly the same technique as nominal bonds except that real pricing kernel ($\bar{M}_{t+1}$) is used to discount the prices of bonds.

$$\bar{M}_{t+1} = \exp\left(-r_t - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\epsilon_{t+1}\right) \tag{21}$$

Note the difference between real and nominal pricing kernel is determined by the type of short rate used as a discount factor. Instead of i_t, the short real rate r_t can be extracted from the state vector X_t using:

$$r_t = \delta_r'X_t \tag{22}$$

where δ_r' is a vector containing only zeros except for one in the second element which associates with the order of r_t in the state vector. Using the same derivation steps as nominal bonds, the real bond pricing equations will be:

$$\bar{P}_t^n = \exp(\bar{A}_n + \bar{B}_n'X_t) \tag{23}$$

and the coefficients $\bar{A}_n$ and $\bar{B}_n$ are subject to:

$$\bar{A}_{n+1} = \bar{A}_n + \bar{B}_n'(\mu - \Sigma\lambda_0) + \frac{1}{2}\bar{B}_n'\Sigma\Sigma'\bar{B}_n$$

$$\bar{B}_{n+1}' = -\delta_r + \bar{B}_n'(\Phi - \Sigma\lambda_1) \tag{24}$$

with initial values $\bar{A}_1 = 0$ and $\bar{B}_1 = -\delta_r$.

2.4 Decomposing Yield Curve

The other components in term structure can be computed as the followings. First, simply calculate nominal rates (NOM) and real rates (REAL) using pricing formulas discussed previously. Second, by setting all lambdas to be zeros, the formulas give us risk-free nominal rates (RFN) and risk-free real rates (RFR) respectively. Third, the inflation expectation (PIE) and the real rate expectation (RRE) in term structure are derived from the averages of the consecutive one-period inflation expectations and the real rate expectations respectively:

$$\pi_t^{e,n} = E_t[\pi_{t+1} + \pi_{t+2} + \cdots + \pi_{t+n}]/n$$

$$r_t^{e,n} = E_t[r_t + r_{t+1} + \cdots + \pi_{t+n-1}]/n$$

where $\pi_t^{e,n}$ is the average annualized n-quarter expected inflation and $r_t^{e,n}$ is the average annualized n-quarter expected real rate. Note that $\pi_t^{e,1} = E_t(\pi_{t+1})$ while $r_t^{e,1} = r_t$. The other components can be derived from the above ones by:

Real Convexity (RCV) = [RFR] - [RRE],

Real Risk Premium (RRP) = [REAL] - [RFR],

Inflation Convexity (PICV) = ([RFN] - [RFR]) - [PIE], and

Inflation Risk Premium (IRP) = ([NOM]-[REAL]) – ([RFN] - [RFR]).

With relations above, one can decompose the nominal rate into six components:

[NOM] = [RRE] + [RCV] + [RRP] + [PIE] + [PICV] + [IRP].

Moreover, given that no liquidity risk involves, one can also define the term premium as:

Term Premium (TP) = [RRP] + [IRP].

3. Data and Estimation

3.1 Data

The data used in this study are end-of-quarter zero-coupon yield curves of Thai government bonds provided by Thai Bond Market Association (TBMA)[7]. The sample dates back to 2001Q3. Thailand's real GDP (seasonally adjusted) and CPI inflation are taken from the National Economic and Society Development Board (NESDB) and the Ministry of Commerce (MOC) respectively. Note that the real GDP is log-transformed, and the CPI is seasonally adjusted and takes the form of quarter-on-quarter annualized growth rate.

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[7] www.thaibma.or.th

3.2 Estimation Approach

When combining macro module with bond pricing component, the whole system involves a vast set of parameters, and it is almost technically infeasible to estimate all parameters simultaneously. To overcome such difficulty in estimation, two-step estimation approach is employed here.

Step 1: Macro Module Calibration and Estimation

In this step, a set of parameters in the macro module (Ψ_{mac}) will be estimated with a specific set of measurement variables (Θ_{mac}). Before going to the estimation, it will help reduce the dimensions of the problem if some parameters can be determined to particular values. The following parameters in the macro module are fixed: (i) π_{ss}, r_{ss}, g_{ss} are set to 0.0063, 0, and 0.0015 in order to match the sample averages which are approximately equivalent to 2.5, 0, and 4 percent per annum respectively, (ii) θ_r is set to 1 since θ_g is related to this parameter, (iii) $\rho_{\bar{\pi}}$ is set to 0.999 as mentioned previously, (iv) ρ_i is set to 0.65 to match the past behavior of the MPC[8], and (v) σ_i is set to 0.0013 (0.5 percent per annum) using the standard error from simple AR(1) short rate model as a benchmark. Thus, Ψ_{mac} is

$$\Psi_{mac} = \left(\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \rho_{\tilde{z}}, \rho_{\tilde{\pi}}, \Phi_z, \Phi_\pi, \rho_a, \rho_g, \rho_{\bar{r}}, \theta_g, \sigma_z, \sigma_\pi, \sigma_{\tilde{z}}, \sigma_{\tilde{\pi}}, \sigma_a, \sigma_g, \sigma_{\bar{\pi}}\right)',$$

and Θ_{mac} is

$$\Theta_{mac} = (y_t, \pi_t, i_t)'.$$

The Kalman filter[9] is applied here to recover shocks and unobserved components in the model and later to compute the likelihood. Those unobserved components are; the potential output ($\bar{y}$), the inflation target ($\bar{\pi}$), and the natural rate of interest ($\bar{r}$). This study estimates parameters using the Bayesian technique, see appendix for details of priors and posteriors.

Step 2: Price of Risk Estimation

In this step, all parameters in Ψ_{mac} are taken from step 1. The solution to the macro part gives the state-space matrices (i.e. μ, Φ, and Σ) which are key inputs for the finance part. This study aims to estimate the yield curve without measurement errors, and the model consists of eight shocks; this means we can have up to eight measurement variables without measurement errors. However, two shocks are reserved for the extra degree of freedom to calibrate for sensible unobserved trends in the model. Three of the rest are paired with the observed macro variables (output, inflation, and short rate). Then, three maturities of the yield curve can be picked up as observed variables: 4-, 20-, and 40-quarter yields which, in fact, the most liquid maturities in the market. One can also see that all other maturities are estimated with measurement errors.

Henceforth, what is left for estimating is the set of price of risk (Λ)

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[8] During sample period, the MPC meeting ended with the decisions of policy rate unchanged 77 times out of 122.

[9] The model is solved and estimated using IRIS toolbox for MATLAB. See http://iristoolbox.codeplex.com/.

$$\Lambda = (\lambda_0, \lambda_1)'$$

with the full set of measurement variables (Θ_{full})

$$\Theta_{full} = (y_t, \pi_t, i_t, y_t^4, y_t^{20}, y_t^{40})'.$$

Again, some parameters can be ruled out to reduce estimating burden in this step. As can be seen in the macro module, shock to potential growth (ϵ_g) effects only potential output which has nothing to do with the interest rate, so this paper sets all lambdas in seventh row which is the order number of ϵ_g in the shock vector to zeros. Likewise, lambdas corresponding with potential output variables, $\bar{y}$ and g (i.e. 10th and 11th column) in λ_1 are set to zeros as well. Also the lambdas corresponding to y and r are set to zeros since they are just identities of other variables. It can be shown that the following lambdas are included in estimating.

$$\lambda_0 = [\lambda_{0,1} \quad \lambda_{0,2} \quad \lambda_{0,3} \quad \lambda_{0,4} \quad \lambda_{0,5} \quad \lambda_{0,6} \quad 0 \quad \lambda_{0,8}]'$$

$$\lambda_1 = \begin{bmatrix} 0 & 0 & \lambda_{1,13} & \lambda_{1,14} & \lambda_{1,15} & \lambda_{1,16} & \lambda_{1,17} & \lambda_{1,18} & \lambda_{1,19} & 0 & 0 & \lambda_{1,112} \\ 0 & 0 & \lambda_{1,23} & \lambda_{1,24} & \lambda_{1,25} & \lambda_{1,26} & \lambda_{1,27} & \lambda_{1,28} & \lambda_{1,29} & 0 & 0 & \lambda_{1,212} \\ 0 & 0 & \lambda_{1,33} & \lambda_{1,34} & \lambda_{1,35} & \lambda_{1,36} & \lambda_{1,37} & \lambda_{1,38} & \lambda_{1,39} & 0 & 0 & \lambda_{1,312} \\ 0 & 0 & \lambda_{1,43} & \lambda_{1,44} & \lambda_{1,45} & \lambda_{1,46} & \lambda_{1,47} & \lambda_{1,48} & \lambda_{1,49} & 0 & 0 & \lambda_{1,412} \\ 0 & 0 & \lambda_{1,53} & \lambda_{1,54} & \lambda_{1,55} & \lambda_{1,56} & \lambda_{1,57} & \lambda_{1,58} & \lambda_{1,59} & 0 & 0 & \lambda_{1,512} \\ 0 & 0 & \lambda_{1,63} & \lambda_{1,64} & \lambda_{1,65} & \lambda_{1,66} & \lambda_{1,67} & \lambda_{1,68} & \lambda_{1,69} & 0 & 0 & \lambda_{1,612} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_{1,83} & \lambda_{1,84} & \lambda_{1,85} & \lambda_{1,86} & \lambda_{1,87} & \lambda_{1,88} & \lambda_{1,89} & 0 & 0 & \lambda_{1,812} \end{bmatrix}$$

However, there are still numerous free parameters and are more than enough to fit only three points of the yield curve. To avoid over-fitting situation that could lead to insensible results, one might add two further restrictions to the estimating process. First, all lambdas are to have minus signs which represent that investors would require some compensation when facing uncertainty and require more if the state they are in correlate with higher interest rate in the future, i.e. higher values of the state variables.

Another restriction is to put two additional dummy measurement variables, 20- and 40- period yields (y_{ss}^{20} and y_{ss}^{40}):

$$y_{ss}^{20} = -\frac{1}{n} ln P_{ss}^{20} = -\frac{1}{n}(A_{20} + B_{20}' X_{ss})$$

$$y_{ss}^{40} = -\frac{1}{n} ln P_{ss}^{40} = -\frac{1}{n}(A_{40} + B_{40}' X_{ss})$$

where X_{ss} is the state vector in steady state and fix them to sample averages of yields at corresponding maturities. This will guide the optimizer towards parameterization that gives a close match between steady state yield curve and sample average yield curve.

4. Results

4.1 Analysis of Impulse Response Functions

For macro-finance model, it is useful to look at the behaviors of key economic variables by examining how they respond to shocks in the model before considering the results. These actions are summarized in Figure 1 where responses of the output gap (z), the inflation (π), the short rate (i), and the real interest rate (r) are shown. The size of each shock is given by its standard deviation; so the responses at the same time implicitly represent their relative sizes of influences to the macroeconomic variables.

Figure 1: Impulse responses of key macroeconomic variables
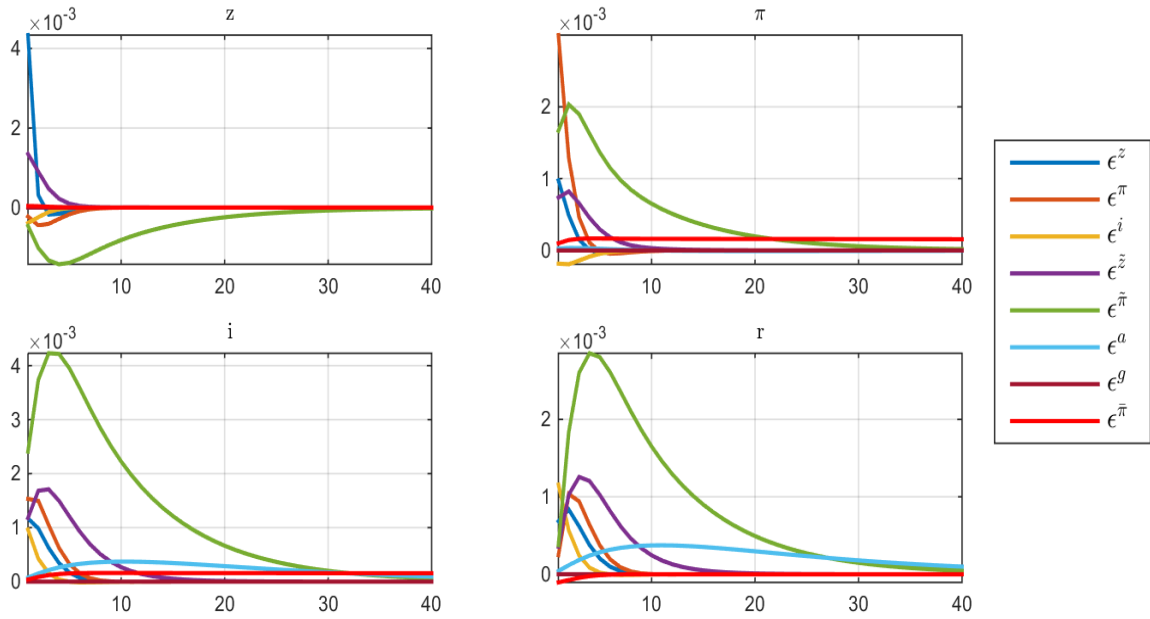
Starting with the responses of the output gap, it seems like the variable is immune to any shocks in the economy. Undoubtedly, the most prominent shocks are transitory and persistent demand shocks that enter directly into the Euler equation and persistent inflation shock. The effect of transitory demand shock (ϵ^z), although its standard deviation is quite high, dies down very quickly while that of the persistent demand shock ($\epsilon_t^{\tilde{z}}$) lasts for about 5 quarters. The persistent inflation (supply) shock depress the output gap up to the medium term horizon. Also note, here that the idiosyncratic shock to potential output growth (ϵ^g), even they lift the level of output, cannot affect the output gap. Nonetheless, we can see a little effect from the shock to common trend growth (ϵ^a); this happens via the real interest rate channel.

Compared to the output gap, the inflation is more sensitive to shocks in the model. The positive demand shocks and inflation shocks elevate the inflation and fade out in different phrase. The shock to the inflation target ($\epsilon^{\bar{\pi}}$) is likely to shift the whole path of the inflation forever. The effect on the monetary policy can be viewed in the term of negative response of the inflation to the shock to policy rate (ϵ^i). Again, this variable is not affected by the idiosyncratic shock to potential growth but response slightly to shock to common trend growth.

Figure 2: Shock loadings
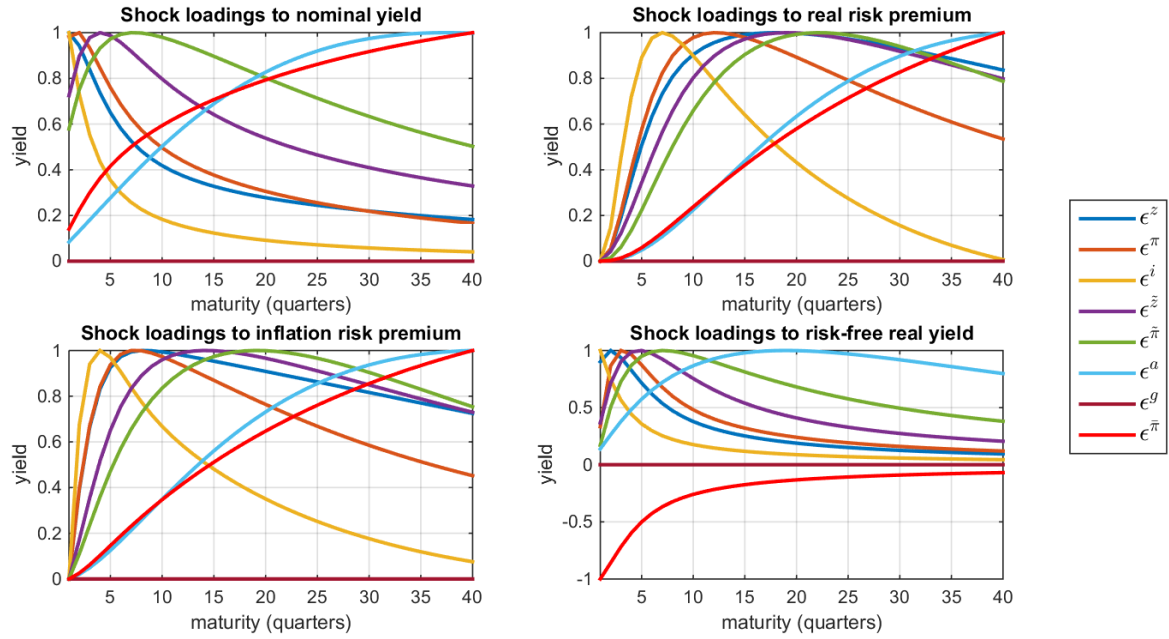
The responses of the short rate, in turn, are the consequences of the output gap and the inflation determined by the policy rule as well as the responses of the short real rate relate to those of interest rate and inflation. A shift in interest rate path due to the inflation target shock happens the same way as inflation. These future paths of nominal and real interest rate are the main forces that shape the nominal yield curves as depicted in equation (15) and (19).

Looking more carefully to the finance counterpart of the model, in Figure 2, one can interpret the responses to the nominal curve at different maturities as loadings of each shock. Since the sizes of the shocks are their standard deviations, some shocks have very little standard deviations and are difficult to compare in the same picture; thus, the figure shows responses that are normalized to be peaked at "1". At the short end, the nominal yields are mostly explained by the shocks to the real economy, i.e. shocks to output gap (both transitory and persistent shocks), shocks to inflations (both transitory and persistent shocks), and monetary policy shock. The response to monetary policy shock illustrates that the central bank can influence the yield curve only at the short end.

For medium-term yields, the contributions to yield variation are the mix between persistent shocks, shock to trend growth, and shock to the inflation target. Moreover, at the long end, the contributions of shock to trend growth and shock to the inflation target explain the most of the yield movement. Besides, the idiosyncratic shock to potential growth has no implication to yield curve in the model.

The shock loadings to real risk premium and inflation risk premium are illustrated in next two plots. The contribution profiles are similar to those of the nominal yield except no risk premia occurs in the first maturity. The study also finds that shock loadings to inflation risk premium are relatively peaked closer to the short end as compared to those of real risk premium.

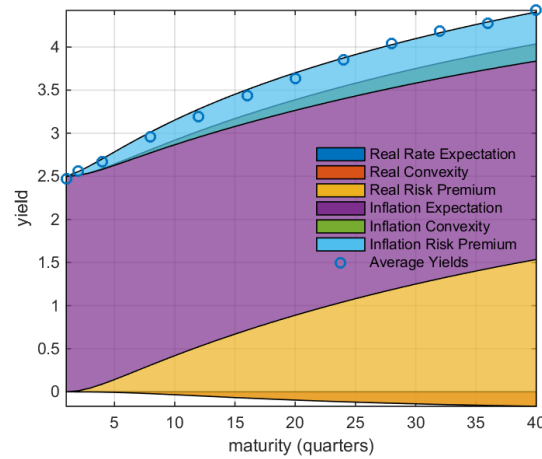Figure 3: Hypothetical yield curve in steady state

Figure 3 shows a hypothetical yield curve in an ideal state of the economy where all variables are in the steady state, and all shocks disappear. As a result, the expectation paths of real interest rate and inflation are kept at their steady-state levels, and this make the term structure of the real rate and inflation expectations constant at all maturities. The first stack which is the real rate expectation is invisible because its steady-state value is zero. However, the nominal curve exhibits upward sloping due to the piling up effects of risk premia where real risk premium is relatively thick comparing to inflation risk premium. Also, note two little downward slope minus areas which are the forces of convexities pressing down the curve. The steady-state curve is well fitted to the properties of nominal yields mean over the sample period. Altogether, at any point in time, we can view a nominal yield curve that deviates from the steady-state one as an outcome of the state and the shocks that taken place in the economy.

4.2 Cross-sectional Analysis

A snapshot of the yield curve as of 2014Q2 is used to illustrate the dynamics of the underlying variables that generate such a yield curve and its components. At first glance, as shown in Figure 4, Thai government bond yield curve is slightly flat at the short end at the end of 2014Q2. We can imply roughly from the curve that the investors may expect the short rate to be maintained for a while or even some rate cuts in the short run. However, by observing that yields from 2-year maturity onward are steeper, one can foresee rate hikes in the near future using the expectation hypothesis.

The above explanation coincides with the economic conditions that challenge Thailand at that time. Figure 5 displays the developments of the underlying variables, both observed and unobserved, and the expectations based on model simulation. Thai economy was facing a sudden drop in output due to its political unrest situation in the first quarter but began to recover in the next quarter when the situation was somewhat resolved, so the impact was expected to fade away in a couple of quarters. This slump is captured by transitory and persistent output gap shocks as we can address negative output shock (ϵ_z) and negative persistent output shock ($\tilde{z}$) in 2014Q2, in which these shocks dragged the output gap and inflation away from their trends. The policy rate (i), in respond to the shocks, is expected to be cut for a bit more and to be normalized within a year. Moreover, some positive inflation shocks are detected causing

inflation slightly overshoots when moving back to the target although the output gap is entirely closed.

Figure 4 Cross-sectional decomposition of the nominal curve
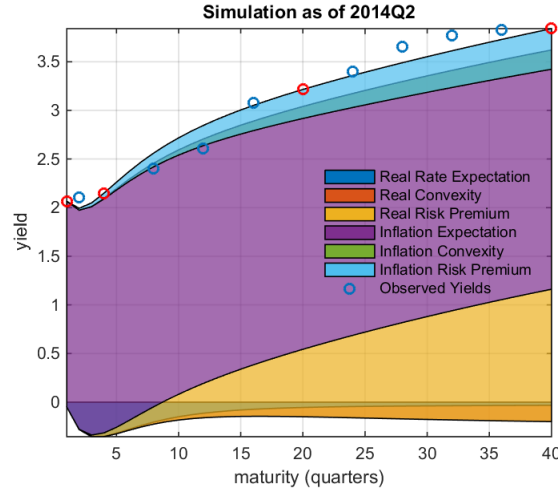
According to the underlying fundamentals described above, we can explain the revealed term structure of components in Figure 4 in a consistent way. The real interest rate, which consists of the risk-free real interest rate and the real risk premium, falls into negative territory at the short end due to a temporary decline in the expected real interest rate. The inflation expectations gradually expand from 2.1% at the short end to 2.5% at 10-year maturity as the inflation is expected to move from around 2% at 2014Q2 back to the target of around 2.5%. The risk premia are relatively small since the investors are in relatively bad state that requires less compensate for uncertainty.

Figure 5 Developments of underlying variables (Actual data in the shaded area)
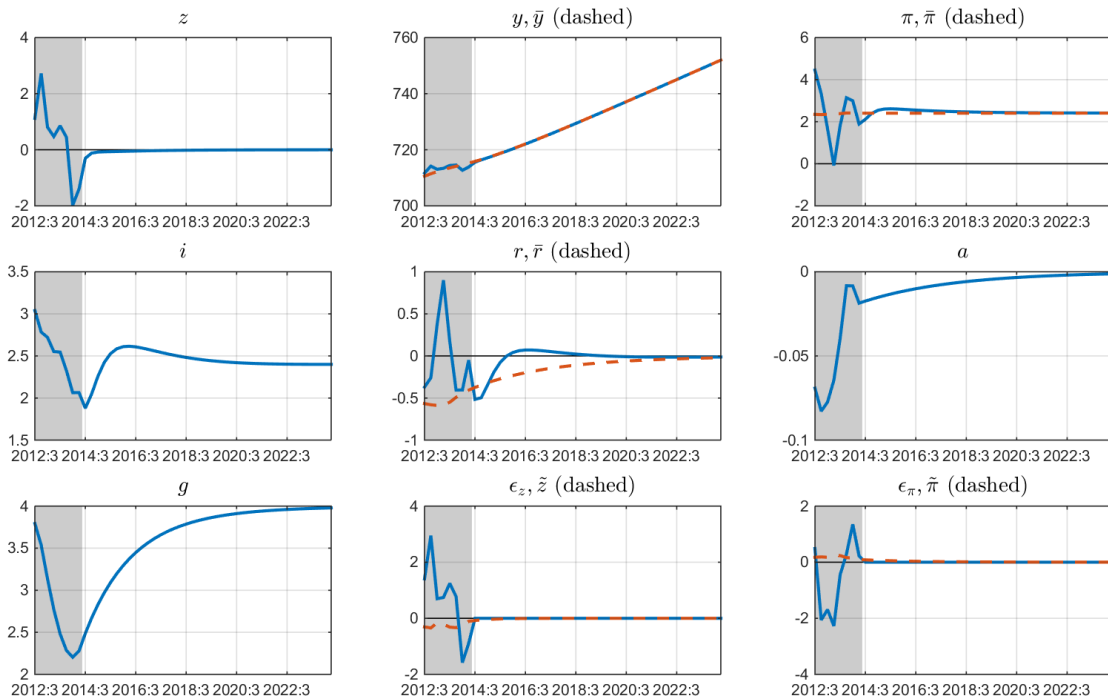
4.3 Extracted Yield Curve Components

The analysis on the extracted time-series will be discussed in this subsection. Figure 6 plots the time-series of the macroeconomic variables and the revealed unobserved components; here we can assess how unobserved trends are tracked with economic background of the past. Starting with the potential output ($\bar{y}$), Thailand seems to have a break in this variable. The potential growth (g) had begun to drop slowly since 2004 and was at its trough during the global financial crisis in 2009. The weak and unstable recovery was reflected in a stagnant potential growth after the crisis; and Thailand unlikely meets her per-crisis level again. The natural rate of interest ($\bar{r}$), which share the same trend as potential output growth slowly declined and fell below zeros since 2006. This in turn partially lower the level of nominal interest rate over time. The inflation and inflation target will be discussed later.

Figure 6 Time-series of underlying variables
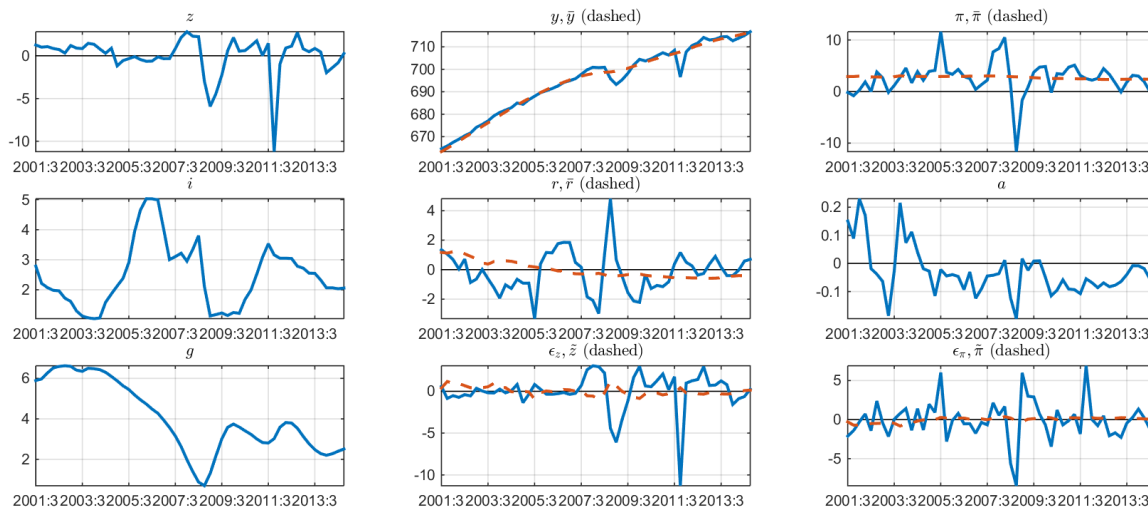
The decomposition of 10-year yield is shown in Figure 7. It can be broadly seen that risk premia movements are the majority of yield's volatility; the real rate expectation contributes the second, and the inflation expectation is relatively stable while convexities are constant. More specifically, real risk premium is slightly more volatile and tends to co-move with the inflation risk premium. The risk premia drop substantially over the studying period which reflects decreasing demand for compensation on the uncertainty. This fall primarily accounts for the downward trend in the long-term yield[10]. What also needed to be remarked was Thailand's monetary policy success in keeping the long-term real interest rate to the very low levels during two 'very easing' cycles in 2004 and 2009.

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[10] The phenomenon of declining risk premium is also exhibited by the U.S.; see (Bernanke, 2013).

Figure 7 Decompositions of 10-year yields
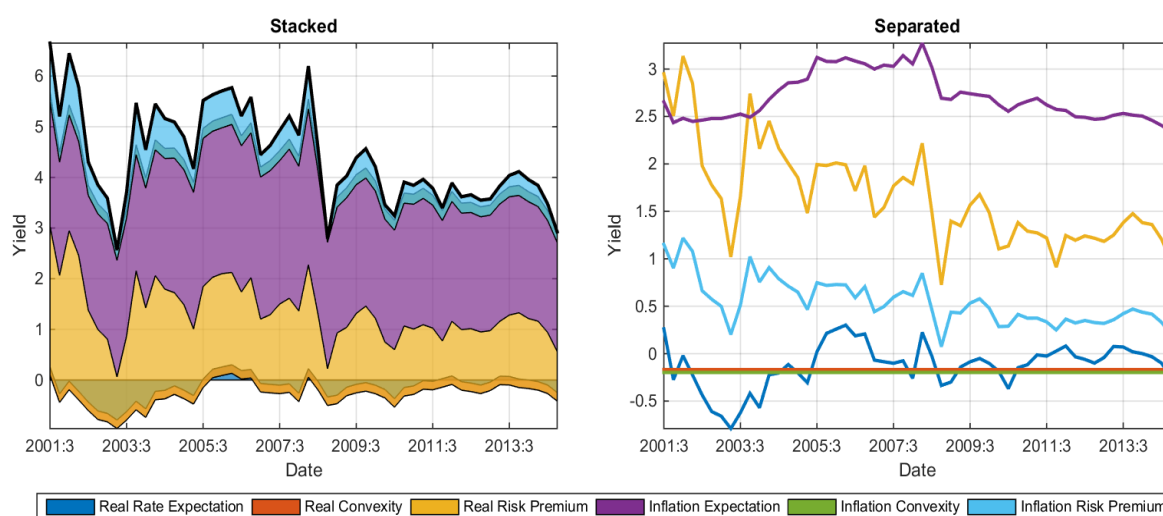
We will finally pay a close attention to two essential long-term inflation measures extracted from the yield curve and compare them with two other measures. Figure 8 plots the family of inflation expectations: model-implied inflation target, model-implied 10-year inflation rate, survey-based inflation forecast, and break-even inflation[11]. Roughly speaking, long-term inflation expectation is close to the target for most of the time. During the early years of Thailand's adoption of inflation targeting in 2000, the target seems to be relatively high and volatile; this can be viewed as a period for the central bank to build up the credibility. Moreover, high and volatile inflation target and long-term inflation could be caused by the wider range of target in the first phase of the inflation targeting regime[12]. However, the level and the volatility of both measures decrease continuously over time, especially for the last five years; thus shows that Thailand's long-term inflations are very much anchored.

Figure 8 also plots two model-independent measures to assure model's accuracy. The survey-based inflation forecast is taken from Asia Pacific Consensus Forecast. The model-implied expected inflation well tracked the survey-based inflation forecast in general, but they started to decouple after 2009. However, the model-implied expected inflation is closely matched to break-even inflation calculated from inflation-linked bond in the early period of its first issuance in 2011Q3. In recent years, unfortunately, this measure is not comparable since its maturity is far less than ten years, and it faced a dramatical drop in 2013 when it was rarely traded in the market.

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[11] Since the number of Inflation-linked bonds (ILB) in Thailand is very limited, so this measure is derived from only 10-year ILB (ILB217A) and note that the tenor of breakeven inflation is shortened in every period since ILB is running down its maturity.

[12] The Bank of Thailand has narrowed the target range from 0.0-3.5 percent per annum to 0.5-3.0 percent per annum since 2010.

Figure 8 Target, 10-year expected, survey-based, and break-even inflations
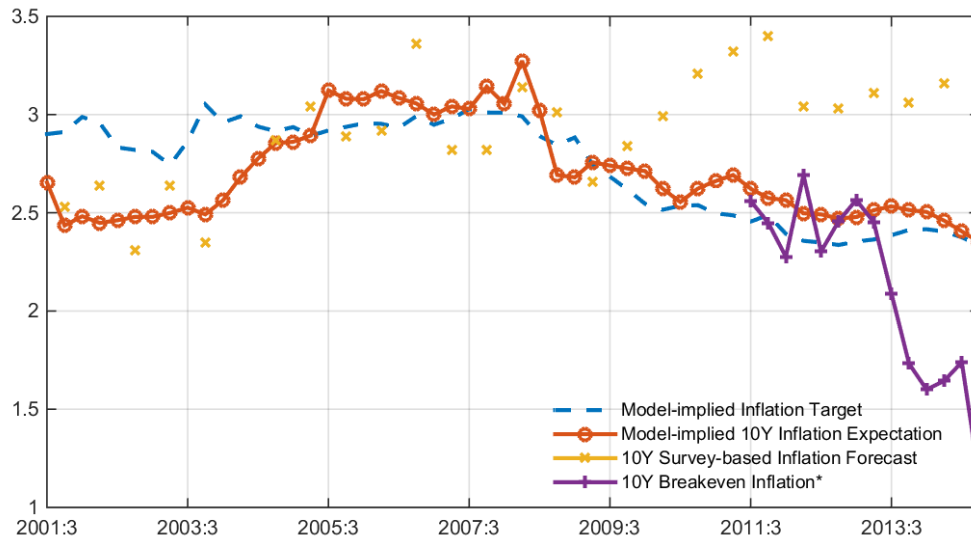
5. Conclusions

The long-term inflation expectation is one of the most important measures in modern central bank's practices owing to its strong influence on current inflation. In this paper, a macro-finance term structure model is used to extract the time-series of inflation expectations and associated components: the real interest rate and risk premia, from the government bond yield curves. Based on New-Keynesian framework, the model is designed to use altogether with macroeconomic variables in order to identify such components. The Kalman filtering and Bayesian inference are employed for the estimation.

All in all, the results, which are in line with those from studies using similar type of model such as Hördahl (2008), Rudebusch (2010), and Rudebusch and Wu (2008), show that Thailand's long-term yields are primarily moved by the real interest rate while the inflation expectations and inflation risk premium are less volatile. This study also finds that the level and volatility of long-term inflation expectations were decreasing since the adoption of inflation targeting in Thailand and are very much anchored over the past five years.

Appendix

A. Pricing Nominal Bonds

The prices of bonds are affine functions of the states (equation (18)) where the coefficients satisfy difference equations in (20). We now start with the price of one-period bond at time t to develop such relations:

$$P_t^1 = E_t(M_{t+1}). \tag{A1}$$

Recall that our nominal pricing kernel (M_{t+1}) is a log normal process (equation (15)), we substitute M_{t+1} with $\exp\left(-i_t - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\epsilon_{t+1}\right)$ and apply the property of the mean of log-normal random variables[13] and the link between short rate and state variables (i.e. $i_t = \delta_i'X_t$):

$$P_t^1 = E_t\left[\exp\left(-i_t - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\epsilon_{t+1}\right)\right]$$

$$= \exp\left(-i_t - \frac{1}{2}\lambda_t'\lambda_t\right)E_t[\exp(-\lambda_t'\epsilon_{t+1})]$$

$$= \exp\left(-i_t - \frac{1}{2}\lambda_t'\lambda_t + \frac{1}{2}\lambda_t'\lambda_t\right)$$

$$= \exp(-i_t)$$

$$= \exp(-\delta_i'X_t). \tag{A2}$$

Considering associated affine pricing equation for one-period bond, $P_t^1 = \exp(A_1 + B_1'X_t)$, one can yield the initial values for the recursions $A_1 = 0$ and $B_1' = -\delta_i$. Then in the next step, we move to pricing bonds in all maturities. This can be computed recursively with pricing $n + 1$ maturity bond using the expected value of the discounted bond price with maturity n at time $t + 1$.

$$P_t^{n+1} = E_t(M_{t+1}P_{t+1}^n) \tag{A3}$$

Plug into the equation the pricing kernel and the exponentially affine pricing function, we can solve for the price of n+1 period bond with the same manner as above:

$$P_t^{n+1} = E_t\left[\exp\left(-i_t - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\epsilon_{t+1} + A_n + B_n'X_{t+1}\right)\right]$$

$$= E_t\left[\exp\left(-i_t - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\epsilon_{t+1} + A_n + B_n'(\mu + \Phi X_t + \Sigma\epsilon_{t+1})\right)\right]$$

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[13] That is $E(e^x) = e^{\mu + \frac{1}{2}\sigma^2}$ if x is normal with mean μ and variance σ^2.

$$= \exp\left(-i_t - \frac{1}{2}\lambda_t'\lambda_t + A_n + B_n'(\mu + \Phi X_t)\right)E_t[exp(B_n'\Sigma - \lambda_t')\epsilon_{t+1}]$$

$$= \exp\left(-i_t - \frac{1}{2}\lambda_t'\lambda_t + A_n + B_n'(\mu + \Phi X_t) + \frac{1}{2}(B_n'\Sigma - \lambda_t')(B_n'\Sigma - \lambda_t')'\right)$$

$$= \exp\left(-i_t + A_n + B_n'(\mu + \Phi X_t) - B_n'\Sigma\lambda_t' + \frac{1}{2}B_n'\Sigma\Sigma'B_n\right).$$

By substituting for the market price of risk and short rate, the price of the bond will be:

$$P_t^{n+1} = \exp\left(A_n + B_n'(\mu - \Sigma\lambda_0) + \frac{1}{2}B_n'\Sigma\Sigma'B_n + \left(-\delta_i + B_n'(\Phi - \Sigma\lambda_1)\right)X_t\right). \tag{A4}$$

Finally, matching the coefficients to exponential affine pricing functions gives:

$$A_{n+1} = A_n + B_n'(\mu - \Sigma\lambda_0) + \frac{1}{2}B_n'\Sigma\Sigma'B_n$$

$$B_{n+1}' = -\delta_i + B_n'(\Phi - \Sigma\lambda_1) \tag{A5}$$

and note that the term $\frac{1}{2}B_n'\Sigma\Sigma'B_n$ is the source of convexity when converting bond prices to yields.

B. Parameter Estimation Details

Table B1: Estimation details of macro module

Parameter	Prior			Posterior
	Distribution	Mean	S.D.	Mode
α_1	normal	0.1	0.02	0.136841
α_2	normal	0.3	0.02	0.354367
α_3	normal	0.35	0.02	0.263358
$\rho_{\tilde{z}}$	normal	0.75	0.02	0.731723
$\rho_{\tilde{\pi}}$	normal	0.63	0.02	0.886753
β_1	normal	0.85	0.02	0.352103
β_2	normal	0.55	0.02	0.152207
Φ_z	normal	0.55	0.02	0.381455
Φ_π	normal	3.5	0.02	3.4726
$\rho_{\tilde{r}}$	normal	0.9	0.02	0.878994
ρ_g	normal	0.8	0.02	0.869464
θ_g	normal	0.9	0.02	0.911387
ρ_a	normal	0.9	0.02	0.933843
σ_z	normal	0.003895	5.00E-05	0.004408
σ_π	normal	0.00175	5.00E-05	0.002201
$\sigma_{\tilde{z}}$	normal	0.0011	2.00E-05	0.001123
$\sigma_{\tilde{\pi}}$	normal	0.000238	2.00E-05	0.000419
σ_a	normal	1.50E-06	2.00E-05	9.10E-05
σ_g	normal	0.000375	2.00E-05	0.000419
$\sigma_{\bar{\pi}}$	normal	0.000158	2.00E-05	0.000163

Table B2: Estimation of price of risk parameters (posterior mode)

λ_0 $\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ λ_1

λ_0	λ_1											
0.00	0.00	0.00	0.00	-1.29	0.00	-0.65	-3.95	-0.22	-2.59	0.00	0.00	0.00
0.00	0.00	0.00	0.00	-3.44	0.00	-0.04	-1.72	-1.36	-3.35	0.00	0.00	0.00
0.00	0.00	0.00	0.00	-0.21	0.00	0.00	-5.27	0.00	-0.44	0.00	0.00	0.00
0.00	0.00	0.00	0.00	-4.90	0.00	-0.41	0.00	-3.55	-6.68	0.00	0.00	0.00
0.00	0.00	0.00	-7.54	-5.08	0.00	-3.19	0.00	-12.02	-28.03	0.00	0.00	0.00
-0.10	0.00	0.00	0.00	0.00	-14.90	-1.63	-3.46	0.00	-10.37	0.00	0.00	-3.58
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	-1.74	-0.48	-1.35	0.00	-3.13	0.00	0.00	-0.54

* All lambdas have the same prior that is a normal distribution with mean = 0 and standard deviation = 2.5.

References

Ang, A., and Piazzesi, M. (2003). A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables. Journal of Monetary Economics *50*, 745–787.

Ang, A., Bekaert, G., and Wei, M. (2008). The Term Structure of Real Rates and Expected Inflation. Journal of Finance *63*, 797–849.

Beechey, M.J. (2008). Lowering the anchor: how the Bank of England's inflation-targeting policies have shaped inflation expectations and perceptions of inflation risk (Board of Governors of the Federal Reserve System (U.S.)).

Bekaert, G., Cho, S., and Moreno, A. (2010). New Keynesian Macroeconomics and the Term Structure. Journal of Money, Credit and Banking *42*, 33–62.

Bernanke, B.S. (2013). Long-Term Interest Rates. Speech Delivered at "Annual Monetary/Macroeconomics Conference: The Past and Future of Monetary Policy," Sponsored by Federal Reserve Bank of San Francisco, San Francisco, California, March 1.

Chernov, M., and Mueller, P. (2012). The term structure of inflation expectations. Journal of Financial Economics *106*, 367–394.

Christensen, J.H.E., Lopez, J.A., and Rudebusch, G.D. (2010). Inflation Expectations and Risk Premiums in an Arbitrage-Free Model of Nominal and Real Bond Yields. Journal of Money, Credit and Banking *42*, 143–178.

D'Amico, S., Kim, D.H., and Wei, M. (2010). Tips from TIPS: the informational content of Treasury Inflation-Protected Security prices (Board of Governors of the Federal Reserve System (U.S.)).

Dewachter, H., Iania, L., and Lyrio, M. (2014). Information in the yield curve: A Macro-Finance approach (National Bank of Belgium).

Diebold, F.X., and Li, C. (2006). Forecasting the term structure of government bond yields. Journal of Econometrics *130*, 337–364.

Finlay, R., and Wende, S. (2012). Estimating Inflation Expectations with a Limited Number of Inflation-Indexed Bonds. International Journal of Central Banking *8*, 111–142.

Gimeno, R., and Marqués, J.M. (2009). Extraction of financial market expectations about inflation and interest rates from a liquid market (Banco de España).

Hördahl, P. (2008). The inflation risk premium in the term structure of interest rates. BIS Quarterly Review.

Hördahl, P., and Tristani, O. (2010). Inflation risk premia in the US and the euro area (European Central Bank).

Hördahl, P., and Tristani, O. (2012). Inflation Risk Premia In The Term Structure Of Interest Rates. Journal of the European Economic Association *10*, 634–657.

Joyce, M.A.S., Lildholdt, P., and Sorensen, S. (2010). Extracting inflation expectations and inflation risk premia from the term structure: A joint model of the UK nominal and real yield curves. Journal of Banking & Finance *34*, 281–294.

Laubach, T., and Williams, J.C. (2003). Measuring the Natural Rate of Interest. The Review of Economics and Statistics *85*, 1063–1070.

Lemke, W. (2008). An affine macro-finance term structure model for the euro area. The North American Journal of Economics and Finance *19*, 41–69.

Nelson, C.R., and Siegel, A.F. (1987). Parsimonious Modeling of Yield Curves. The Journal of Business *60*, 473–489.

Rudebusch, G.D. (2010). Macro-Finance Models Of Interest Rates And The Economy. Manchester School *78*, 25–52.

Rudebusch, G., and Wu, T. (2008). A Macro-Finance Model of the Term Structure, Monetary Policy and the Economy. Economic Journal *118*, 906–926.