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The Output Euler Equation and Real Interest Rate Regimes

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Pym Manopimoke

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Pym Manopimoke*

Abstract

Output Euler equations (OEE) for the US deliver slope estimates that are not significantly different from zero. This finding is counterintuitive as it implies a zero elasticity of intertemporal substitution (EIS) and aggregate demand movements that are nonresponsive to the short-term real interest rate. This paper shows that failure to account for regime changes in the dynamics of the real interest rate is responsible for the zero EIS result. In doing so, an empirical investigation is carried out based on an unobserved components framework with Markov-switching parameters that models the underlying process for the real interest rate jointly with the OEE. According to the estimation results, the ex-post real interest rate is a highly persistent process with means, variances and degrees of persistence that are different for the periods 1966-1980, 1980-1985, and 1985-2015. Once these changes in real interest rate behavior are taken into account, estimates for the EIS are 0.1 and are no longer statistically insignificant. This finding is robust to various measures of the output gap as well as alternative specifications for the time-varying natural real rate.

JEL Classification: E12; E32; E43; E52.

Keywords: Aggregate Demand; Output Euler Equation, Markov-Switching, Monetary Policy, New Keynesian IS Curve, Real Interest Rate; Persistence, Unobserved Components Model; Elasticity of Intertemporal Substitution.

*Senior Economist, Puey Ungphakorn Institute for Economic Research, Bank of Thailand. Address: 273 Samsen Road, Watsamphraya, Phra Nakhon District, Bangkok, Thailand. Tel. +662-356-7874. E-mail: pymm@bot.or.th

1. Introduction

During recent decades, New Keynesian macroeconomic models have become a popular framework for monetary analysis. In its simplest form, the New Keynesian model comprises of the output Euler equation (OEE), which generalizes the consumption Euler equation to output, and an inflation equation, which resembles the expectation-augmented Phillips curve. While both equations are equally important for macroeconomic analyses, the dynamics of aggregate demand as captured by the output Euler equation has been severely understudied, especially when compared to its aggregate supply counterpart.

This paper aims to fill this gap in the literature by focusing on an important empirical puzzle facing the New Keynesian output equation. According to theory, the slope of the OEE relates aggregate demand to the real interest rate, and captures the key channel in which monetary policy is able to affect the real economy through the mechanism of intertemporal substitution in consumption. However, when brought to the data, a robust finding is that the OEE slope is zero, which challenges the standard view on the effectiveness of the short-term interest rate as a monetary policy instrument. Furthermore, with theory implying that the OEE slope is the negative of the elasticity of intertemporal substitution (EIS), the empirical challenge facing the OEE becomes all the more perplexing since it infers that agents do not substitute consumption over time.

While a plethora of studies investigate this so-called EIS puzzle, the majority of investigations thus far have been carried out with consumption Euler equations. For example, Mankiw (1981), Hall (1988) and Campbell and Mankiw (1989) are early studies that examine the EIS puzzle, and report that there is virtually no evidence for intertemporal substitution in US consumption data. Subsequent work by Campbell (2003) and Yogo (2004) estimate the EIS for several countries while carefully accounting for statistical problems stemming from weak instruments, yet they arrive at similar conclusions[1]. However, as pointed out by Bilbiie and Straub (2012), studying the EIS puzzle within the context of the consumption Euler equation can be difficult as it involves issues related to the non-separability of consumption and leisure in the utility function. The shift in focus to the OEE can avoid such issues, and at the same time, also advance our knowledge on how the monetary transmission channel should be properly modeled in the New Keynesian output relation[2].

[1]These findings have been challenged by a few studies, particularly those employing microeconomic data to show that the zero EIS result is due to limited asset market participation in bonds and stock markets by the majority of households in the economy (Attanasio and Weber, 1994; Atkeson and Ogaki, 1996; Vissing-Jørgensen, 2002). Nevertheless, the general consensus in the macroeconomic literature still seems to be that the average EIS is zero for the US economy.

[2]Identification of the EIS parameter is also crucial towards studying various other macroeconomic topics. For example, Summers (1984), King and Rebelo (1990) and Trostel (1993) show that the size of the EIS parameter is important in determining the macroeconomic effects of capital income taxation. Jones et al. (2000) argue that the degree of intertemporal substitution is a key determinant of whether uncertainty boosts or slows down long-run economic growth.

In fact, studies that examine the EIS puzzle through the lens of the OEE, while currently limited, have met with greater success. For example, Goodhart and Hofmann (2003) estimates the New Keynesian output equation for G7 countries and finds a significant link between output and interest rates once the model is augmented to include additional determinants of aggregate demand, such as asset prices and monetary aggregates. More recently, Bilbiie and Straub (2012) puts forth an interesting explanation that the zero OEE slope result stems from an EIS parameter that hides a structural break. More specifically, they show via a split sample analysis that the EIS estimate is a convolution of two non-zero coefficients with opposite signs across the samples 1965-1979 and 1982-2003. Then, they go on to justify the incorrect sign for the EIS parameter in the earlier period with institutional evidence of limited asset market participation in the US.

While the key argument of Bilbiie and Straub (2012) is compelling, the EIS is a deep structural parameter of the economy, and evidence on its time-varying nature remains scarce and is subject to considerable debate (Atkeson and Ogaki, 1996; Attanasio and Browning, 1996; Chiappori and Paiella, 2011). On the other hand, there is more concrete evidence that over the postwar period, the means, variances, and degrees of persistence of the US real interest rate has undergone multiple regime shifts (Garcia and Perron, 1996; Rapach and Wohar, 2005; Manopimoke, 2009). Given that the real interest rate is the main driving variable for output in the aggregate demand relation, changes in its times series behavior could deliver important implications for estimates of the OEE. For example, the closed-form specification for the OEE implies that the output gap is driven by current and future forecasts of the real interest rate gap. Predictions of the real interest rate gap, in turn, depends on the statistical properties of the real interest rate, such as its persistence and its long-run average rate. Therefore, any changes in these statistical properties of the real interest are most likely to affect the output Euler relation, and if not properly accounted for when estimating the OEE, inferences on the EIS parameter could turn out to be highly misleading.

This paper examines precisely the abovementioned hypothesis. More specifically, it investigates whether the EIS puzzle can be explained by the failure of past studies to account for changes in the dynamic properties of the US real interest rate when estimating the OEE. In doing so, this paper develops an unobserved components (UC) model for the closed-form OEE that is modeled jointly with the underlying real interest rate process, which is allowed to undergo regime changes at unknown dates. In the UC framework, the key variables for the OEE, which are the output and real interest rate gaps, are expressed as functions of their time-varying natural rates. Since these variables are unobserved, they are treated as latent state variables in the empirical model[3]. Dealing with gap constructs in this way can be seen as more desirable than having to rely on statistical detrending methods as in past studies. These statistical approaches often impose strong priors on the smoothness of the trend and cycle, delivering estimates that can be subject to considerable measurement error. Estimates of the latent state variables in the UC framework on the other hand, depends on principle economic relationships as specified by the OEE, which may help minimize measurement error and provide gap estimates that contain more economic content.

As a preview of the empirical results, this paper finds that the US real interest rate underwent two regime changes around 1980 and 1985. These structural shifts separate real interest rate behavior into three distinct regimes, with means, variances and degrees of persistence that are different. Based on the empirical results, the long-run mean of the real interest rate was exceptionally high during the early to mid 1980s, while the persistence and variability of shocks to the real interest rate gap underwent a significant decline after the Great Moderation in 1985. An important finding from the joint estimation of the OEE and the real interest process is that once changes in the statistical properties of the real interest rate have been properly accounted for, the EIS parameter becomes statistically significant and is estimated with the correct sign throughout the postwar period. In particular, the estimated magnitude of the EIS parameter is 0.1 across the three regimes, and is surprisingly robust against alternative measures of the output gap.

This paper is structured as follows. Section 2 provides some background on the OEE and outlines the empirical model based on its closed-form specification. Section 3 introduces the UC model that is a joint specification of the OEE and the real interest rate process with Markov-switching parameters. Section 4 presents the estimation results and Section 5 provides some further discussion to gain added insight on the empirical findings. Section 6 concludes.

2. The Output Euler Equation

The simplest form of the intertemporal OEE, also known as the New Keynesian IS curve, is a microfounded, optimization-based relation that takes on the following form:

$$x_t = E_t x_{t+1} - \sigma \tilde{r}_t + \eta_t, \tag{1}$$

[3]In some studies, the real interest rate is not detrended at all before entering the OEE due to difficulties associated with estimating the natural real rate. With the real interest rate being a highly persistent process whereas the output gap a stationary one, the resulting persistence mismatch problem could potentially explain why existing OEEs struggle to establish significant slope estimates. As emphasized in the New Keynesian Phillips curve (NKPC) literature, accounting for variations or shifts in the long-run properties of economic variables have been deemed important when estimating the New Keynesian class of models. As shown in Tinsley and Kozicki (2003), Cogley and Sbordone (2008) and Kim et al. (2014), among others, the empirical implications of the NKPC can change markedly once variations in the long-run properties of the inflation process are properly accounted for.

where x_t is the output gap, $E_t x_{t+1}$ is the expectation formed at time t of the one-period-ahead output gap, $\tilde{r}_t$ is the real interest rate gap and η_t is the aggregate demand shock. The real interest rate gap is defined as the difference between the real interest rate and its natural level, or $\tilde{r}_t = r_t - r_t^*$, where r_t is the real interest rate and r_t^* is the natural real rate of interest. According to the 'Wicksellian' concept for the natural real interest rate, r_t^* is the unobservable real rate that is consistent with output equalling potential and stable inflation (Woodford, 2003; Kiley, 2015). In practice, it may vary over time with structural factors such as the rate of output trend growth rate and time preferences. The real interest rate gap, on the other hand, is related to short-run fluctuations in the business cycle, and reflects the existence of nominal rigidities in the economy.

In a closed economy without capital, investment in durable goods and a government, consumption equals output and the OEE slope is the aggregate elasticity of intertemporal substitution (EIS). Under the assumption of CRRA utility, the EIS parameter σ is the inverse of the coefficient of relative risk aversion. Theory predicts that the slope of the OEE is negative, because following an increase in the short-term interest rate, the representative agent that practices intertemporal substitution will substitute consumption (output) today for consumption (output) tomorrow, ultimately lowering aggregate demand. Note that this mechanism of intertemporal substitution is an important channel in which monetary policy can affect the economy through its influence on short-term interest rates.

While theoretically sound, the OEE in (1) provides a remarkably poor description of the key dynamic features of the aggregate data. This is because the forward-looking element, $E_t x_{t+1}$, fails to completely capture the persistent responses of output to monetary policy shocks. To this end, 'hybrid' specifications that augment the OEE with lags of the output gap have become widespread, as these 'backward-looking' elements help the model match the high degree of inertia in the data (Svensson 1999; Fuhrer, 2000; Rudebusch, 2002). The presence of output lags in the OEE while seemingly ad hoc, are typically justified by theoretical models related to habit formation, in which consumers' utility depends partly on current output relative to past output (Fuhrer, 2000). However, although these backward-looking elements afford the OEE to be better reconciled with the data, many economists are still puzzled as to why future output expectations do not play a more important role in determining current output through the forward-looking element in the OEE. However, as pointed out by Fuhrer and Rudebusch (2004) and Fuhrer and Olivei (2004), obtaining accurate estimates of OEE parameters are generally difficult, as weak identification is often an issue in conventional GMM estimation.

Another empirical challenge that plagues the OEE is that the model typically fails to deliver slope estimates that are significantly different from zero. This finding is counterintuitive because it implies a zero EIS, which suggests that agents do not substitute consumption over time. While a number of papers acknowledge this empirical irregularity in their estimates of the OEE, most studies merely treat it as an anomaly and offer no further explanations for their findings. Such studies include Fuhrer and Rudebusch (2004), whom report zero EIS estimates for the OEE based on a variety of output detrending methods, real interest rate definitions and instrument sets while using generalized method of moments (GMM) and maximum likelihood (ML) methods. Fuhrer and Olivei (2004) reach similar conclusions even when an improved GMM method that better deals with the problem of weak identification is used to estimate the model. In a recent study, Stracca (2010) estimates the OEE and several possible extensions using panel data from 22 OECD countries to increase the statistical power of the empirical exercise, but still fails to establish a significant link between the output gap and the real interest rate.

To investigate why OEEs typically deliver zero EIS estimates, this paper considers an approach that sidesteps the difficulty in dealing with the forward-looking element, $E_t x_{t+1}$. This can be achieved by forward iteration of (1), arriving at the following closed-form specification:

$$x_t = \alpha_1 x_{t-1} + \alpha_2 x_{t-2} - \sigma \sum_{j=0}^{\infty} E_t \tilde{r}_{t+j} + \eta_t. \tag{2}$$

As is standard in the literature, two lags of the output gap have been added to the above specification to help the model capture the high degrees of inertia in the data.

According to (2), the current output gap now depends on the sum of current and future forecasts of the real interest rate gap, which approximates a long-run real interest rate. Thus it is apparent that the central bank's primary impact on the economy comes about not only through the level at which it sets current overnight rates, but also from the way it affects private-sector expectations about the likely future path of short-term overnight rates. Therefore, the future behavior of real rates become an important determining factor of current business cycle conditions, which has led many policymakers to stress the importance of credibility and commitment in their conduct of monetary policy.

3. The Unobserved Components Framework with Markov-switching Parameters

A key challenge for estimating the closed-form OEE in (2) is obtaining an expression for the infinite sum term, which contains multistep forecasts of future real interest rate gaps. This expression can be computed in a number of ways depending upon how the dynamics of the natural real rate and the real interest rate gap are specified. One way to define the dynamics of the real interest rate in conjunction with the OEE in (2) is according to the UC framework as outlined below:

Baseline UC Model

Output Gap Equation:

$$x_t = \alpha_1 x_{t-1} + \alpha_2 x_{t-2} - \sigma \sum_{j=0}^{\infty} E_{t-1}\tilde{r}_{t+j} + v_t, v_t \sim iid.N(0, \sigma^2_{v,D_t}), \tag{3}$$

Real Interest Rate Equations:

$$r_t = r_t^* + \tilde{r}_t, \tag{4}$$

$$r_t^* = (1-\rho_{S_t})r_{S_t}^{LR} + \rho_{S_t} r_{t-1}^* + e_t, e_t \sim iid.N(0, \sigma^2_{e,S_t}), \tag{5}$$

$$\tilde{r}_t = \phi_{S_t}\tilde{r}_{t-1} + \varepsilon_t, \varepsilon_t \sim iid.N(0, \sigma^2_{\varepsilon_{S,t}}). \tag{6}$$

This joint specification for the output gap and real interest rate, which will henceforth be referred to as the baseline UC model, can be described as follows. First, (3) follows the closed-form OEE in (2), but with one slight modification. For feasible estimation of the model, the expectational element $E_t(\cdot)$ in the infinite sum term is replaced with $E_{t-1}(\cdot)$, otherwise current forecasts of the real interest rate gap may be correlated with the error term. Accordingly, the error term in the output gap equation becomes $v_t = \eta_t - \sigma \sum_{j=0}^{\infty}(E_t\tilde{r}_{t+j} - E_{t-1}\tilde{r}_{t+j})$. Since the second element in v_t is related to economic agents' revision on expectations of current and future real interest rate gaps, the correlation between v_t and ε_t, which is the innovation to the real interest rate gap, are allowed to be nonzero during estimation.

To complete the specification for the output equation, the variance of shocks to the output gap is defined as:

$$\sigma^2_{v,D_t} = (1-D_t)\sigma^2_{v,1} + D_t\sigma^2_{v,2}, \tag{7}$$

$$D_t = \begin{cases} 0, & \text{if } t \leq 1984Q3 \\ 1, & \text{otherwise,} \end{cases} \tag{8}$$

where the presence of the dummy variable helps capture any decline in the variance of shocks to real output during the onset of the Great Moderation in 1984Q3 (see Kim and Nelson, 1999; McConnell and Perez-Quiros, 2000).

Next, (4) is decomposes the real interest rate into two components: the natural real interest rate r_t^*, and the real interest rate gap $\tilde{r}_t$. In (5), r_t^* is described as a stationary time-varying process with unconditional long-run mean r^{LR} and persistence parameter ρ, which makes r_t^* relevant in a shorter-term context. One way to interpret r_t^* in this model is based on r_t^* being the neutral monetary policy stance consistent with the short-term goals of the Fed, to be set period by period under the situation where the economy has not necessarily settled at its long-run level. Accordingly, the natural real rate in the baseline UC framework is treated as an exogenous process, thus its innovation e_t is assumed to be uncorrelated with all other shocks in the system.

It is also worth mentioning that the chosen specification for r_t^* is purely statistical. This is similar in spirit to the empirical specifications of Larsen and McKeown (2002), Basdevant et al. (2004), and Cour-Thimann et al. (2004). Another popular approach that is more in line with theory is to let r_t^* vary with real fundamentals, such as the determinants of trend GDP growth (Giammarioli and Valla, 2003; Laubach and Williams, 2003; Neiss and Nelson, 2003). While this definition for r_t^* is explored later for robustness checks, the baseline UC model chooses to remain rather agnostic about how r_t^* is related to other economic variables due to the high degree of specification uncertainty when estimating the natural real interest rate.

Finally, an expression for the real interest rate gap in (6) defines $\tilde{r}_t$ as an autoregressive process (AR) of order 1 with persistence parameter ϕ. Based on this specification for $\tilde{r}_t$, the sum of multi-step forecasts of the real interest gap can now be computed as:

$$\begin{aligned}
\sigma \sum_{j=0}^{\infty} E_{t-1}\tilde{r}_{t+j} &= \sigma E_{t-1}(\tilde{r}_t + \tilde{r}_{t+1} + \tilde{r}_{t+2} + \ldots) \\
&= \sigma(\phi_{S_t} + \phi_{S_t}^2 + \phi_{S_t}^3 + \ldots)\tilde{r}_{t-1} \\
&= \frac{\sigma\phi_{S_t}}{(1-\phi_{S_t})}\tilde{r}_{t-1},
\end{aligned}$$

which when substituted into (3), yields:

$$x_t = \alpha_1 x_{t-1} + \alpha_2 x_{t-2} - \frac{\sigma\phi_{S_t}}{(1-\phi_{S_t})}\tilde{r}_{t-1} + v_t, v_t \sim iid.N(0, \sigma_{v,D_t}^2). \tag{3'}$$

According to the expression above, the link between aggregate demand and the real interest rate gap depends on both the EIS parameter and the persistence parameter of the real interest rate gap. This specification makes it apparent that changes in the times series properties of the real interest rate can have important implications for the link between the output gap and the real interest rate. In fact, based on the literature on real interest rate dynamics, there is widespread evidence of regime-switching behavior in the mean, variance and persistence of the US real interest rate (Huizinga and Mishkin, 1986; Bai and Perron, 2003; Neely and Rapach, 2008; Manopimoke, 2009; Kim and Kim

2013)[4]. To account for the possibility of regime-switching in the real interest rate, as well as to examine how changes in its statistical properties may affect estimates of the OEE, all parameters that govern real interest rate behavior in the baseline UC model: $r^{LR}, \rho, \phi, \sigma_e$ and σ_ε, are allowed to undergo regime changes at unknown dates. In the empirical specification, regime-switching dynamics are governed by a first-order Markov-Switching process S_t, which represents the state in which the economy is in at time t.

When specifying the UC model with regime-switching parameters, the number of regimes in the real interest rate process is assumed to be unknown. It is not until the model is estimated that the number of states in the UC model is determined via a series of diagnostic tests[5]. However, as a simple exposition, should there be three recurring regimes in the real interest rate process, S_t will follow a three-state Markov process with transition probability matrix:

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix}, \tag{9}$$

where $P_{ij} = Pr[S_t = j|S_{t-1} = i]$ and $\sum_{j=1}^{3} P_{ij} = 1$ for $i = 1,2,$ and 3. Note that in the Markov-switching model, changes in the behavior of the real interest rate need not be recurring, as the model can also accommodate for one-time discrete shifts or structural breaks. In such a case, P_{21}, P_{31}, and P_{32} will equal zero and P_{33} will be an absorbing state with probability equal to 1.

3.1 An Alternative Measure for the Output Gap

One shortcoming to the baseline UC model is that it treats the output gap, x_t, as an observable variable. Accordingly, x_t will need to be proxied by estimates of the output gap, which can be obtained by, for example, applying statistical detrending methods to real GDP. Although this approach is standard in the literature, it is well-known that statistical constructs of x_t can be subject to considerable measurement error. Therefore, this paper also considers estimating the output gap as a latent state variable in the UC framework. Note that this approach will produce estimates for

[4]Many studies argue that accounting for regime changes in the long-run mean of the US real interest rate series is critical towards being able to fully capture its high degree of persistence. This is because with the real interest rate being a highly persistent process, many studies fail to reject the null hypothesis of a unit root (Rose, 1988; King et al., 1991; Gali, 1992; Mishkin, 1981; Koustas and Serletis, 1999). However, Perron (1990) argues that one may mistaken the high levels of persistence in a times series process for a unit root if regime-switching in the mean of the process is unaccounted for. Garcia and Perron (1996), among other studies, show that this is the case. In particular, the authors find that once regime-switching in the long-run mean of the ex-post real interest rate have been accounted for, the US real interest rate series between 1961 to 1986 essentially becomes a random white noise process.

[5]Selecting the number of states in this way is arguably more desirable when compared to the approach of Bilbiie and Straub (2012). These authors assume that there is only one structural break in the OEE and rely on the Wald test for GMM estimators to identify the timing of change. This paper makes no such restrictions on the number of states that can occur, and estimates the dates of regime change endogenously with other model parameters.

the output gap that is consistent with the OEE.

To allow the output gap to enter the empirical model as an unobserved variable, the baseline UC specification in (3)-(6) is augmented with the following set of equations, resulting in a trivariate UC model for output, real interest rate, and inflation:

Trivariate UC model:

Output Equations:

$$y_t = \tau_t + x_t, \tag{10}$$

$$\tau_t = \delta_t + \tau_{t-1} + w_t, w_t \sim iid.N(0, \sigma^2_{w,D_t}) \tag{11}$$

$$\delta_t = \delta_{t-1} + \eta_t, \eta_t \sim iid.N(0, \sigma^2_\eta) \tag{12}$$

Inflation Equation:

$$\pi_t = c_1\pi_{t-1} + c_2\pi_{t-2} + c_3\pi_{t-3} + kx_{t-1} + z_t, z_t \sim iid.N(0, \sigma^2_{z,\tilde{D}_t}). \tag{13}$$

Each equation can be interpreted as follows. In (10), the equilibrium real output y_t is decomposed into a stochastic trend component τ_t, and a cyclical component x_t. The dynamics of x_t follows the OEE as outlined in (3). As for stochastic trend output, it follows a random walk with double drift as shown in (11), where the drift term δ_t in (12) corresponds to a time-varying output trend growth rate. Time variation in δ_t is an essential feature to help capture any slowdowns in productivity which may have occurred in 1974, or during the onset of the recent Great Recession. Finally, to ensure that τ_t is estimated as the country's productive capacity that is consistent with stable inflation, a reduced form Phillips curve is added to the UC specification. As shown in (13), inflation depends on its first three lags which acts as proxies for inflation expectations. It is also driven by the output gap, which explains short-run fluctuations in inflation over the business cycle.

As before, shocks to the output equations undergo a one-time change in 1984Q3 to account for the Great Moderation:

$$\sigma^2_{v,D_t} = (1 - D_t)\sigma^2_{v,1} + D_t\sigma^2_{v,2}, \tag{14}$$

$$\sigma^2_{w,D_t} = (1 - D_t)\sigma^2_{w,1} + D_t\sigma^2_{w,2}. \tag{15}$$

where D_t follows (8). Heteroskedasticity in the inflation process can also be captured in a similar fashion. During the Great Inflation of the 1970s, US inflation was significantly more volatile compared to other periods. The variance of shocks to the inflation process thus follows:

$$\sigma^2_{z,\tilde{D}_t} = \tilde{D}_{1t}\sigma^2_{z,2} + \tilde{D}_{2t}\sigma^2_{z,2} + \tilde{D}_{3t}\sigma^2_{z,3}, \tag{16}$$

where

$$\tilde{D}_{1t} = \begin{cases} 1, & \text{if } t_0 \leq t \leq 1971Q1, \\ 0, & \text{otherwise,} \end{cases} \tag{17}$$

$$\tilde{D}_{2t} = \begin{cases} 1, & \text{if } 1971Q2 \leq t \leq 1981Q1 \\ 0, & \text{otherwise} \end{cases} \tag{18}$$

$$\tilde{D}_{3t} = \begin{cases} 1, & \text{if } 1981Q2 \leq t \leq T \\ 0, & \text{otherwise.} \end{cases} \tag{19}$$

Last, to estimate the baseline and trivariate UC models, the models are to be written in state-space form and estimated with maximum-likelihood methods based on Kim's (1994) approximate Kalman filter. Since the trivariate UC model is a generalization of the baseline UC model, only the state-space representation of the former one is outlined in Appendix A.

4. Empirical Results

4.1 Data

For the baseline UC model, four measures of the output gap are used as proxies of x_t, which are based on the following definitions of potential output: (1) the Congressional Budget Office's (CBO) official estimate of potential output; (2) the Hodrick-Prescott (HP) filter of log real GDP; (3) a deterministic quadratic trend for log real GDP; and (4) a segmented deterministic linear trend for log real GDP with a breakpoint in the first quarter of 1974. As for the trivariate UC model, y_t is the log of real GDP. The inflation measure in the Phillips curve equation is calculated as $\pi_t = 400 \times (lnP_t - lnP_{t-1})$, where P_t is the GDP chain-weighted price index.

The relevant real interest rate measure for the analysis of the OEE in both UC models is the ex-ante real interest rate (EARR), defined as the difference between the nominal short-term interest rate and current period inflation expectations. However, due to the unobserved nature of inflation expectations, it is often assumed that agents have rational expectations, and the ex-post real interest rate (EPRR) is employed for the analysis instead. To construct the EPRR, this paper subtracts the current period inflation rate from the nominal interest rate. To be consistent with the Phillips curve equation, the relevant inflation measure for the EPRR is constructed from the GDP chain-weighted price index. The nominal interest rate is the quarterly average of the overnight federal funds rate.

For the empirical analysis, all data is taken from the Federal Reserve Economic Database (FRED). The study is based on quarterly data that spans 1966Q1 to 2015Q3. The start of the sample marks the beginning of the period in which the federal funds rate is recognized as a primary instrument of monetary policy. As mentioned by Fuhrer and Rudebusch (2004), the federal funds rate did not exceed the discount rate in the period prior to 1965, rendering it as an invalid instrument of monetary policy.

4.2 Empirical Results

This section carries out an empirical investigation of the EIS puzzle as follows. First, both the baseline and trivariate UC models are estimated with no-regime switching parameters to obtain benchmark results that further analyses can be compared against, as well as to verify the EIS puzzle. Next, to determine the appropriate number of states, the UC models are estimated again with incremental number of regimes, and their performances are evaluated with residual diagnostic tests. In carrying out the residual diagnostic tests, it is kept in mind that remaining serial correlation in the residuals signal model misspecification, whereas the presence of serial correlation in their squares suggest remaining ARCH effects. Model misspecification tests for Markov-switching models are typically carried out in this way, since formal hypothesis tests such as the likelihood ratio, Wald and Lagrange multiplier are invalid in the presence of regime changes. In particular, these tests do not have standard asymptotic distributions due to the problem of identically zero scores under the null and the presence of nuisance parameters under the alternative (see Hansen, 1992). Finally, note that unless specified, diagnostic test results and any plots for the empirical findings are based on the trivariate UC model, as findings from the baseline UC models are quantitatively similar.

Table 1 reports the estimation results for the UC models with no regime-switching dynamics. For the output equation, the estimated sum $\alpha_1 + \alpha_2$ is close to one for all output gap measures, suggesting that the transitory component of output is highly persistent. There is also strong evidence of the Great Moderation, as observed by the significant decline in the variance of shocks to US output after 1984Q3. As for the real interest rate equation, both the natural real interest rate and the real interest rate gap are highly persistent. The estimate of the long-run steady-state level r_{LR} is associated with a large standard error, but this is not surprising given that estimation of the natural real rate typically faces considerable challenges. Turning to examine the estimated variance of shocks, both components of the real interest rate are, in general, equally volatile. This finding contrasts with Neiss and Nelson (2003), whom report a more smooth natural real rate. Last, the parameter estimates for the inflation equation in the trivariate UC model are reported separately in

Table 2 due to space considerations. As shown, US inflation is a highly persistent process as suggested by the sum $c_1 + c_2 + c_3$, and as expected, the variability of shocks to inflation were higher during the Great Inflation once compared to other periods[6].

While most parameters in Table 1 are of the correct sign and display reasonable magnitudes, the key parameter of interest, σ, is not statistically significant, making a strong case for the EIS puzzle. However, further scrutiny of the results suggest that the UC models with no regime-switching may be misspecified, because parameter estimates describing the EPRR process differ to a considerable degree across the various output gap specifications. Furthermore, based on the residual diagnostic tests on the real interest rate series' residuals and their squares, the test results in the first block of Table 3 provides strong evidence that the UC model with no changes in regimes is not an appropriate fit to the data. In particular, based on the reported p-values associated with the Ljung-box test statistics, the null of no serial correlation in the model's squared residuals can be firmly rejected at the one percent level.

With misspecification in the no regime-switching case, the UC models are estimated again, but this time the parameters in the real interest rate equations follow a two-state Markov-switching process. The filtered probabilities associated with the latent state variable S_t are plotted in Figure 1. As shown, there is convincing evidence that the dynamics of the US real interest rate underwent a significant change during the Great Moderation period in the mid 1980s. Examining the parameter estimates for the two-state model in Table 4, it can then be seen that once this regime shift is taken into account when estimating the OEE, the EIS parameter σ now becomes statistically significant with the correct sign. In addition, the magnitude of σ is equal to or close to 0.1 across all output gaps measures, which is an interesting finding given that measures of the various output gaps differ to some considerable degree. Compared to existing studies, $\sigma = 0.1$ is a reasonable estimate. Based on aggregate consumption data, Hall (1988), Mankiw (1981) and Yogo (2004) estimate the EIS to be around 0.2. By extending the New Keynesian output equation to include asset prices and monetary aggregates, Goodhart and Hofmann (2003) report a statistically significant EIS estimate of 0.04[7].

Next, the remaining parameter estimates in Table 4 are analyzed. Estimates for the output equation are similar to the no regime-switching case, except that now there is more evidence of time-variation in the output trend growth rate as signified by the estimate of σ_η. The long-run

[6]The estimation results for the inflation equation in subsequent analyses are also reported in Table 2. However, since they are similar to the no regime-switching case, these results are not discussed further in the remainder of this section.

[7]Admittedly, an EIS estimate of 0.1 is much smaller than what is required by theory. Calibrated macroeconomic models designed to match growth and business cycle facts typically require that the EIS be close to one (Kydland and Prescott, 1982; Weil, 1989). While small empirical estimates for the EIS still remains a puzzle, many studies explain that in aggregate, the measures of the EIS are low because of limited participation in the stock and bond markets by the majority of households in the economy (Vissing-Jørgensen, 2002; Guvenen, 2006).

means of the EPRR process are still estimated with considerable uncertainty over both regimes. However, compared to the no regime-switching case, estimates of the persistence parameters as well as the variability of real interest rate shocks are now less disparate across the various output gap measures. Finally, when examining the difference in parameter estimates across the two regimes, it can be observed that the structural change in the mid 1980s has been in large part driven by the significant decline in the level of the long-run mean, as well as the persistence and the variability of shocks to the real interest rate gap[8].

Although the two-state UC models appear to describe the joint dynamics of output and the real interest rate well, it is still necessary to evaluate the empirical fit of the UC models. According to the residual diagnostic test results in the second block of Table 3, the two-state model appears well-specified. However, judging from the filtered probabilities in Figure 1, allowing for a third state in the UC model may be necessary. This is because while the filtered probabilities show that a regime change occurred around the mid 1980s, prior to this period the distinction between states 1 and 2 are not necessarily clear cut. During this time, although there is a higher probability that the model is in state 1, the probability that the model is in state 2 is also nontrivial, being as high as 0.7 in the early 1980s, which is period well-known for macroeconomic change.

The filtered probabilities associated with the three-state UC model is plotted in Figure 2. Compared to the two-state case, the transition between the three regimes are now sharper. As expected, the model identifies an additional regime change in the early 1980s, separating the dynamics of the EPRR into three distinct regimes: 1966-1980, 1980-1985 and 1985-2015[9]. Huizinga and Mishkin (1986), Garcia and Perron (1996), Rapach and Wohar (2005), and Manopimoke (2009) also identify similar regimes for the EPRR process, and interprets them as a monetary phenomenon[10]. This interpretation seems appropriate given that the first shift in the early 1980s closely coincided with the beginning of Paul Volcker's chairmanship at the Federal Reserve Bank, which is known to have brought about a monetary policy stance that is more focused on achieving stability in inflation and economic activity. The timing of the second regime shift in 1985 occurred during the onset of the Great Moderation. While the driving force behind the Great Moderation is still a subject of debate, a number of influential studies argue that improved monetary policy was an important contributing factor (Clarida et al., 2000; Lubik and Schorfheide, 2004; Boivin and Giannoni, 2006; Boivin et

[8]On the other hand, the persistence of the natural real rate ρ, and the variability of its shocks σ_e, remain relatively constant over the two regimes. In the three-state model that will be discussed next, σ_e also does not vary significantly across regimes, and therefore it is treated as a constant in all UC models. However, ρ varies somewhat in the three-regime model, thus the dynamics of ρ are left unconstrained.

[9]Note that there is a brief period during the global financial crisis in which the model switches back to regime 1, but this regime change is largely ignored because it disappears in the model's estimates of smoothed probabilities.

[10]However, these authors also report an additional regime shift in 1973. This is not surprising since their measure of the EPRR is based on CPI inflation which is influenced more heavily by the sharp rises in the price of oil that occurred during the Great Inflation period.

al., 2010)[11].

Table 5 contains the remaining estimation results that are associated with the three-state model. Similar to the two-state case, the key parameter of interest, σ, is statistically significant. It is also estimated with a magnitude that is about 0.1 across all the output gap measures. For the estimated output dynamics, they are similar to the two-state case. On the other hand, there are noticeable differences in the implied behavior of the real interest rate series. Compared to the two-state case, the amount of uncertainty attached to the estimates of the long-run real interest rate means are smaller, especially during the first two regimes that lead up to the Great Moderation. As for the levels of the long-run means, they vary between medium, high and low levels respectively. Since the high mean state corresponds to a high variance state for the real interest rate gap and vice versa, the findings in this paper in large part echoes the results of Garcia and Perron (1996), whom also find a strong positive relationship between the long-run mean and the variability of shocks to the EPRR process.

Turning to analyze the degree of persistence in the natural real interest rate, deviations of r_t^* from its long-run mean has become larger since the Great Moderation, as ρ increases in magnitude from approximately 0.7 to 0.9. On the other hand, the persistence of the real interest rate gap while pronounced in the pre 1985 period, fell dramatically to zero after the Great Moderation. This finding stands in contrast with Garcia and Perron (1996), whom argue that deviations of the EPRR from its regime-switching long-run mean is essentially a random white noise process throughout the entire postwar period. The main difference between the model proposed in this paper and that of Garcia and Perron is that this paper allows for time variation in the natural real rate of interest, whereas the model of Garcia and Perron (1996) assumes a constant long-run steady-state level that undergoes infrequent but distinct regime shifts.

Changes in real interest rate behavior can be best observed from Figure 3, which contains a plot of the estimated natural real interest rate and the actual EPRR. As shown, while the two series loosely comoved in the same general direction prior to the mid 1980s, the actual real rate series began to track the natural real interest rate closely after the Great Moderation. From the plot, the implied behavior of the natural real interest rate in the post 1985 period is surprisingly similar to those of Barksy et al. (2014). Based on an extension of the well-known Smets and Wouters (2003) DSGE model with price and wage rigidities, these authors along with Justiniano et al. (2013) explain that a policy path in which the actual EPRR tracks the natural real rate can significantly stabilize the output gap and decrease the variability of wage and price inflation. Based on this line of reasoning, the behavior of the actual and natural real rate processes as implied by the UC models

[11] The other prominent view is the "good-luck" hypothesis, which suggests that the substantial decline in macroeconomic volatility during the Great Moderation was largely the result of smaller shocks affecting the economy (Canova and Gambetti, 2005; Primiceri and Justiniano, 2006; Sims and Zha, 2006).

in the post Great Moderation period may be a signal of improved monetary policy.

Last, the empirical fit of the three-state UC models are evaluated with residual diagnostic tests. In the final block of Table 3, one can verify that similar to the two-state case, the three-state model is also well-specified. While earlier interpretations of the filtered transition probabilities seem to favor the three-state model, two information criterions are employed to formally select between the two models. The first test is based on the Akaike information criterion (AIC), which is calculated as $lnL - k$, where lnL denotes the log-likelihood and k corresponds to the number of parameters in the model. For the two and three-state models, the AIC is computed as -417.59 and -423.45 respectively. The second test is based on the Bayesian information criterion (BIC), which is defined as $-2lnL + k \cdot ln(n)$ where n denotes the sample size. It is calculated as -591.63 and -638.26 for the two and three-state models respectively. Since the model with the lowest AIC and BIC is preferred, the best-fitting specification for the joint dynamics of output and the real interest rate appears to be the three-state Markov-switching UC model.

The bottom line result from the empirical analysis in this section is that estimations of the OEE in previous studies could be misleading. The findings from the Markov-switching UC models suggest that the EIS puzzle stems from failure to account for regime-switching in the dynamics of the real interest rate. To ensure that this result is not tied to a certain specification of the real interest rate, alternative specifications for the natural real interest rate have been considered as robustness checks. One specification ties the dynamics of the natural real interest rate to the time-varying growth rate of output as outlined in Appendix B. In large part, this model is similar to the UC model of Laubach and Williams (2003), but is different in the sense that it allows for regime-switching in the parameters of the real interest rate. Similar to the main empirical findings in this section, the Laubach and Williams variant with no regime-switching replicates the EIS puzzle, while once allowing three states in the model yields estimates of σ that are statistically significant and are estimated at around 0.08 for all output gap measures.

5. Further Discussion of Results

Notice that the closed-form OEE in (3') is statistically indistinguishable from the following reduced form IS relation:

$$x_t = c + \alpha_1 x_{t-1} + \alpha_2 x_{t-2} - \beta r_{t-1} + w_t. \tag{20}$$

when $\beta = \frac{\sigma\phi}{1-\phi}$. Therefore, to check whether the estimated magnitudes of σ from the UC models are reasonable, implied estimates of β from the three-state Markov-switching UC model can be compared against actual estimates of β from the reduced form IS relation above. When estimating the reduced form model, to allow for the possibility that β may vary gradually over time rather than undergo discrete regime shifts, a rolling estimation over the sample 1966Q1-2015Q3 is carried out. The rolling estimation is based on a 15-year fixed window and the data used for x_t is the CBO output gap. As before, r_t is the EPRR and is calculated as the difference between the federal funds rate and GDP chain-weighted inflation.

Figure 4 contains the rolling estimates of β. Overall, the magnitude of β appears reasonable, given that reduced form IS models typically yield estimates of the slope that are between 0.05 and 0.10 (Estrella and Fuhrer, 1999; Rudebusch and Svensson, 1999; Rudebusch, 2002). The rolling estimation results suggest that β underwent a distinct decline during the early 1980s. This finding implies that since then, the output gap has become less sensitive to movements in the real interest rate, which is a finding that is consistent with the implications drawn from many VAR analyses. For example, Boivin and Giannoni (2002, 2006) employ a split sample analysis and find that exogenous monetary policy shocks as obtained by a recursive identification scheme had a smaller effect on aggregate demand in the post Volcker period. Based on factor augmented VARs and a DSGE model, Boivin et al. (2010) show that the effect of monetary policy shocks on real GDP has become more muted since the early 1980s.

Implied values of β that are calculated from the estimated parameters of the trivariate UC model in Table 5 are 0.083, 0.108 and 0.005 in the three regimes respectively. These values are roughly in line with the reduced form estimates of β that are shown in Figure 4, providing more support for the empirical relevance of the three-state Markov-switching UC model. Furthermore, the implications that can be drawn from the results in this section is that a zero β estimate may not necessarily stem from the lack of evidence for intertemporal substitution as is typically understood. With β being a convolution of both the EIS parameter and persistence parameter of the real interest rate gap, the slope of the IS relation may appear small simply because of exceptionally low degrees of persistence in the real interest rate gap, which appeared to be the case for most of the post mid 1980s period.

6. Conclusion

While the mainstay of much macroeconomic research relies on the New Keynesian output equation, a disconcerting result is that the model fails to capture many key dynamic properties of the data. One important problem with empirical estimates of the OEE is that it fails to establish a significant link between output and the real interest rate, which also implies the counterintuitive result of a zero elasticity of intertemporal substitution. Also known as the EIS puzzle, this finding is a key challenge to longstanding economic theory and any standard macroeconomic model that relies on the mechanism of intertemporal substitution for monetary analysis.

In the New Keynesian literature, comprehensive assessments of estimation issues facing the

OEE have been surprisingly rare. This paper aims to fill this gap by showing that the EIS puzzle is due to the failure of past studies to take into account the changing times-series properties of the real interest rate when estimating the OEE. In doing so, this paper develops a joint specification for output and the real interest rate, where the long-run mean, variance, and persistence of the real interest rate series are allowed to undergo regime changes at unknown dates. Time variation in potential output and the natural real rate of interest are features that are also carefully accounted for when estimating the empirical model based on an unobserved components framework.

The best-fitting model for the joint behavior of output and real interest rates characterizes real interest rate dynamics into three distinct regimes: 1966-1980, 1980-1985, and 1985-2015. While the results from the UC model with no regime-switching fails to establish any link between output and the real interest rate, allowing the real interest rate to undergo three recurring regimes deliver estimates of the EIS that are statistically significant. Furthermore, estimates of the EIS parameter are consistently 0.1 regardless of the output gap measure used. These findings thus cautions future work against assuming constant dynamics for macroeconomic variables that are subject to regime change, otherwise predictions from the relevant macroeconomic models may end up highly misleading.

Apart from providing an explanation for the EIS puzzle, this paper also contributes to the longstanding literature that deals with measurement issues of the real interest rate. Of particular interest are the findings that since the Great Moderation in 1985, the variance and persistence of the real interest rate gap declined significantly, and the natural real rate of interest started to track the actual real interest rate closely. With the real interest rate being a key measure of the monetary stance, these results undoubtedly contribute to our understanding on historical monetary policy conduct in the US. However, to gain added intuition on the driving variables behind real interest rate behavior, future research along the lines of Neely and Rapach (2008), that examines how real interest rate persistence may be tied to a particular monetary policy regime in place is highly encouraged.

References

Attanasio, O., Browning, M., 1995. Consumption over the life cycle and over the business cycle. American Economic Review 5, 1118-1137.

Atkeson, A., Ogaki, M., 1996. Wealth varying intertemporal elasticities of substitution: evidence from panel and aggregate data. Journal of Monetary Economics 38, 507-534.

Attanasio, O.P. Weber, G., 1993. Consumption growth, the interest rate and aggregation. Review of Economic Studies 60, 631-649.

Bai, J., Perron, P., 2003. Computation and analysis of multiple structural change models. Journal of Applied Econometrics 18(1), 1-22.

Barksy, R., Justiniano, A., and Melosi, L., 2014. The natural rate and its usefulness for monetary policy making. American Economic Association 104 (5), 37-43.

Basdevant, O.N., Bjorksten and Karagedikli, O. 2004. Estimating a time varying neutral real interest rate for New Zealand. Reserve Bank of New Zealand Discussion Paper 2004/01.

Bilbiie, F.O., Straub, R., 2012. Changes in the output Euler equation and asset markets participation. Journal of Economic Dynamics and Control 36(11), 1659-1672.

Boivin, J., Giannoni, M., 2002. Assessing changes in the monetary transmission mechanism: A VAR approach. Federal Reserve Bank of New York Economic Policy Review 8(1), 97-111.

Boivin, J., Giannoni, M., 2006. Has monetary policy become more effective? Review of Economics and Statistics 88(3), 445-462.

Boivin, J., Kiley, M.T., Mishkin, F.S., 2010. How has the monetary transmission mechanism evolved over time? NBER Working Paper No. 15879.

Campbell, J.Y., 2003. Consumption-based asset pricing. In G. M. Constantinides, M. Harris, and R. Stulz (eds), Handbook of the Economics of Finance, Vol. 1B, Amsterdam: Elsevier, Chapter 13, 801-885.

Campbell, J.Y., Mankiw, 1989. Consumption, income and interest rates: Reinterpreting the time series evidence. NBER Macroeconomic Annual 1989 edited by O. Blanchard and S. Fischer, 185-216.

Canova, F., Gambetti, L., 2005. Structural changes in the US economy: Bad luck or bad policy? CEPR Discussion Paper No. 5457.

Chiappori, P-A., Paiella, M. 2011. Relative risk aversion is constant: evidence from panel data.

Journal of the European Economic Association 9(6), 1021-1052.

Clarida, R., Gali, J., Gertler, M., 2000. Monetary policy rules and macroeconomic stability: Evidence and some theory. The Quarterly Journal of Economics 115(1), 147-180.

Cogley, T., Sbordone, A.M., 2008. Trend inflation, indexation, and inflation persistence in the New Keynesian Phillips curve, American Economic Review 98(5), 2101-2126.

Cour-Thimann, P., Pilgaard, R., Stracca, L. 2004. The output gap and the real interest rate gap in the euro area, 1960-2003. paper presented at the Bank of Canada workshop on neutral interest rates, 9-10 September, 2004.

Estrella, A., Fuhrer, J.C., 1999. Are deep parameters stable? The Lucas Critique as an empirical hypothesis. Computing in Economics and Finance 621, Society for Computational Economics.

Fuhrer, J.C., 2000. Habit formation in consumption and its implications for monetary-policy models. American Economic Review 90, 367-90.

Fuhrer, J.C., Olivei, G.P., 2004. Estimating forward-looking Euler equations with GMM estimators: An optimal instruments approach. Federal Reserve Bank of Boston No. 04-2 Working Paper.

Fuhrer, J.C., Rudebusch, G.D., 2004. Estimating the Euler equation for output. Journal of Monetary Economics 51(6), 1133-1153.

Gali, J., 1992. How well does the IS-LM model fit postwar U.S. data? Quarterly Journal of Economics 107(2), 709-738.

Garcia, R., Perron, P., 1996. An analysis of the real interest rate under regime shifts. The Review of Economics and Statistics 78(1), 111-125.

Giammarioli, N., Valla, N., 2003. The natural rate of interest in the Euro area. ECB Working Paper No. 233.

Goodhart, C., and Hofmann, B. 2003. The IS curve and the transmission of monetary policy: Is there a puzzle? Bonn ZEI Working Paper No. B 13-2003.

Guvenen, F., 2006. Reconciling conflicting evidence on the elasticity of intertemporal substitution: A macroeconomic perspective. Journal of Monetary Economics 53(7), 1451-1472.

Hall, R.E., 1988. Intertemporal Substitution in Consumption. Journal of Political Economy 96, 339-357.

Hansen, B.E., 1992. The likelihood ratio test under nonstandard conditions: Testing the

Markov trend model of GNP. Journal of Applied Econometrics 7, S61-S82.

Huizinga, J., Mishkin, F.S., 1986. Monetary policy regime shifts and the unusual behavior of real interest rates. Carnegie-Rochester Conference Series on Public Policy 24(1), 231-274.

Jones, L.E., Manuelli, R.E., Stacchetti, E., 2000. Technology (and policy) shocks in models of endogenous growth. Staff Report 281, Federal Reserve Bank of Minneapolis.

Justiniano, A., Primiceri, G.E., Tambalotti, A., 2013. Is there a tradeoff between inflation and output stabilization? American Economic Journal: Macroeconomics, 1-31.

Kiley, M.T., 2015. What can the data tell us about the equilibrium real interest rate? Finance and Economics Discussion Series 2015-077.

Kim, C-J., 1994. Dynamic linear models with Markov-switching. Journal of Econometrics 60(1-2), 1-22.

Kim, C.-J., Kim J., 2013. Bayesian inference in regime-switching ARMA models with absorbing states: The dynamics of the ex-ante real interest rate under structural breaks. Discussion paper series No. 1306.

Kim, C.-J., Manopimoke, P., Nelson, C.R., 2014. Trend inflation and the nature of structural breaks in the New Keynesian Phillips curve. Journal of Money, Credit and Banking 46(2-3), 253-266.

Kim, C.-J., Nelson, C.R., 1999. Has the US economy become more stable? A Bayesian approach based on a Markov-switching model of the business cycle. Review of Economics and Statistics, 81(4), 608-616.

King, R.G., Rebelo, S.T., 1990. Public policy and economic growth: Developing neoclassical implications. The Journal of Political Economy 98(5), S127-S150.

King, R.G., Plosser, C.I., Stock, J.H., Watson, M.W., 1991. Stochastic trends and economic fluctuations. American Economic Review 81(4), 819-840.

Koustas, Z.,Serletis, A., 1999. On the Fisher effect. Journal of Monetary Economics 44(1), 105-130.

Kydland, F.E., Prescott, E.C., 1982. Time to build and aggregate fluctuations. Econometrica 40, 1345-1370.

Laubach, T., Williams, J., 2003. Measuring the natural rate of interest. Review of Economics and Statistics, 85 (4), 1063-1070.

Larsen, J.D.J., McKeown, J. 2002. The information content of empirical measures of real interest rate and output gaps for the United Kingdom. BIS papers No 19.

Lubik, T. A., Schorfheide, F., 2004. Testing for indeterminacy: An application to U.S. monetary policy. American Economic Review 94(1), 190-219.

Mankiw, N.G. 1981. The permanent income hypothesis and the real interest rate. Economics Letters 7, 307-311.

Manopimoke, P., 2009. An analysis of the real interest rate and monetary policy under regime shifts. Thammasat Economic Journal 27(2).

McConnell, M.M., Perez-Quiros, G.P., 2000. Output fluctuations in the United States: What has changed since the early 1980s? American Economic Review, 90, 1464-1476.

Mishkin, F.S., 1981. The real rate of interest: An empirical investigation. Carnegie Rochester Conference Series on Public Policy 15, 151-200.

Neely, C.J., Rapach, D.E., 2008. Real interest rate persistence: Evidence and implications. Federal Reserve Bank of St. Louis Review 90(6), 609-641.

Neiss, K., Nelson, E. 2003. The real rate gap as an inflation indicator. Macroeconomic Dynamics 7, 239-262.

Perron, P., 1990. Testing for a unit root in a time series with a changing mean. Journal of Business and Economic Statistics 8(2), 153-162.

Primiceri, G., Justiniano, A., 2006. The time varying volatility of macroeconomic fluctuations. The American Economic Review 98(3), 604-641.

Rapach, D.E., Wohar, M.E., 2005. Regime changes in international real interest rates: Are they a monetary phenomenon? Journal of Money, Credit and Banking 37(5), 887-906.

Rose, A.K., 1988. Is the real interest rate stable? Journal of Finance 43(5). 1095-1112.

Rudebusch, G.D., Svensson, L.E.O.,1999. Policy rules for inflation targeting. Chapter in NBER book Monetary Policy Rules, Edited by John B. Taylor, 203-262.

Rudebusch, G.D., 2002. Assessing nominal income rules for monetary policy with model and data uncertainty. Economic Journal 112, 1-31.

Sims, C.A., Zha, T., 2006. Were there regime switches in US monetary policy? American Economic Review 96(1), 54-81.

Smets, F., Wouters, R., 2003. An estimated dynamic stochastic general equilibrium model of the euro area. Journal of the European Economic Association 1(5), 1123-1175.

Stracca, L., 2010. Is the New Keynesian IS curve structural? European Central Bank Working Paper Series No. 1236.

Stock, J. H., Watson, M., 1998. Median unbiased estimation of coefficient variance in a time-varying parameter model. Journal of the American Statistical Association 93(441), 349-358.

Summers, L.H., 1984. The after-tax rate of return affects private savings. American Economic Review 74(2), 249-253.

Svensson, L. 1999. Inflation targeting: some extensions. Scandinavian Journal of Economics 101, 337-361.

Tinsley, P.A., Kozicki, S., 2003. Alternative sources of lag dynamics of inflation. Computing in Economics and Finance 92, Society for Computational Economics.

Trostel, P.A., 1993. The effect of taxation on human capital. The Journal of Political Economy 101(2) 327-350.

Vissing-Jørgensen, A., 2002. Limited asset market participation and the elasticity of intertemporal substitution. Journal of Political Economy 110(4), 825-853.

Weil, P., 1989. The equity premium puzzle and the risk-free rate puzzle. Journal of Monetary Economics 24, 401-421.

Woodford, M. 2003. Interest and prices: foundations of a theory of monetary policy. Princeton University Press.

Yogo Y., 2004. Estimating the elasticity of intertemporal substitution when instruments are weak. The Review of Economic and Statistics 86(3), 797-810.

Appendix A

The corresponding state-space representation of the UC model with unobserved output gap can be written as:

Measurement equation

$$
\begin{bmatrix} y_t \\ r_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & k & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tau_t \\ x_t \\ x_{t-1} \\ \delta_t \\ r_t^* \\ \tilde{r}_t \\ z_t \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ c_1\pi_{t-1} + c_2\pi_{t-2} + c_3\pi_{t-3} \end{bmatrix}
$$

Transition equation

$$
\begin{bmatrix} \tau_t \\ x_t \\ x_{t-1} \\ \delta_t \\ r_t^* \\ \tilde{r}_t \\ z_t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ (1-\rho_{S_t})r_{S_t}^{LR} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \alpha_1 & \alpha_2 & 0 & 0 & \frac{-\sigma\phi_{S_t}}{1-\phi_{S_t}} & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho_{S_t} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \phi_{S_t} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \tau_{t-1} \\ x_{t-1} \\ x_{t-2} \\ \delta_{t-1} \\ r_{t-1}^* \\ \tilde{r}_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} w_t \\ v_t \\ 0 \\ \eta_t \\ e_t \\ \varepsilon_t \\ z_t \end{bmatrix}
$$

$$
\begin{bmatrix} w_t \\ v_t \\ \eta_t \\ e_t \\ \varepsilon_t \\ z_t \end{bmatrix} \sim i.i.d.N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{w,D_t}^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{v,D_t}^2 & 0 & 0 & \rho\sigma_{v,D_t}\sigma_\varepsilon & 0 \\ 0 & 0 & \sigma_\eta^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_e^2 & 0 & 0 \\ 0 & \rho\sigma_{v,D_t}\sigma_\varepsilon & 0 & 0 & \sigma_\varepsilon^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{z,\tilde{D}_t}^2 \end{bmatrix} \right),
$$

where the dynamics of D_t and $\tilde{D}_t$ are as defined in (8) and (17)-(19) respectively.

Appendix B

Building upon the approach of Laubach and Williams (2003), an alternative specification for the UC model with the natural real interest rate expressed as a function of the time-varying trend output growth rate can be written as follows:

Output Equations:

$$y_t = \tau_t + x_t$$

$$\tau_t = \delta_t + \tau_{t-1} + w_t, w_t \sim iid.N(0, \sigma^2_{w,D_t})$$

$$\delta_t = \delta_{t-1} + \eta_t, \eta_t \sim iid.N(0, \sigma^2_\eta)$$

Real Interest Rate Equations:

$$r_t = r^*_t + \tilde{r}_t,$$

$$r^*_t = c_0\delta_t + d_t$$

$$d_t = \psi_{S_t} d_{t-1} + e_t, e_t \sim iid.N(0, \sigma^2_e)$$

$$\tilde{r}_t = \phi_{S_t}\tilde{r}_{t-1} + \varepsilon_t, \varepsilon_t \sim iid.N(0, \sigma^2_{\varepsilon_{S,t}}),$$

Inflation Equation:

$$\pi_t = c_1\pi_{t-1} + c_2\pi_{t-2} + c_3\pi_{t-3} + kx_{t-1} + z_t, z_t \sim iid.N(0, \sigma^2_{z,\tilde{D}_t}).$$

The above specification is similar to the trivariate UC model as described by (3), (10)-(13), with the main difference being that the natural real interest rate is now a function of the trend output growth rate and d_t. The dynamics of d_t is assumed to follow an AR(1) process, and it captures other determinants of the natural real interest rate such as households' rate of time preference. Note that alternative specifications for d_t such as a random walk process is also considered, but they yield results that are qualitatively similar.

The shocks in the above specification are defined in the same way as the trivariate UC model. As before, S_t follows a first-order Markov-Switching process, and the number of regimes in the UC model are determined via a series of residual diagnostic tests. As described in Laubach and Williams (2003), a pile-up problem can occur during estimation causing the innovations to the trend output growth rate, σ_η to be biased towards zero. The median unbiased estimator of Stock and Watson (1998) is thus used to obtain estimates of the ratio $\lambda_g = \sigma_\eta/\sigma_w$, which is then imposed during estimation. In general, the estimation method follows the sequential steps as described by Laubach and Williams (2003). The estimation results are available from the author upon request.

Table 1: **Estimation Results for the UC Models with No Regime Change**

Parameters	Output Gap Measures CBO	HP-Filter	Quadratic	Segmented	UC
	Output Equation				
σ	0.077	0.000	0.126	0.139	0.067
	(0.047)	(0.000)	(0.129)	(0.171)	(0.047)
α_1	1.233***	1.091***	0.994***	1.016***	1.759***
	(0.083)	(0.067)	(0.186)	(0.213)	(0.067)
α_2	-0.267***	-0.253***	-0.075	-0.054	-0.811
	(0.081)	(0.067)	(0.148)	(0.173)	(0.069)
$\sigma_{v,1}$	0.990***	0.966***	0.900***	0.908***	0.180**
	(0.091)	(0.083)	(0.142)	(0.162)	(0.089)
$\sigma_{v,2}$	0.508***	0.487***	0.372***	0.384**	0.000
	(0.037)	(0.031)	(0.113)	(0.185)	(0.037)
$\rho_{v,\varepsilon}$	-0.007	0.418***	0.216	0.178	-0.999***
	(0.093)	(0.128)	(0.513)	(0.688)	(0.000)
$\sigma_{w,1}$	-	-	-	-	0.885***
	-	-	-	-	(0.087)
$\sigma_{w,2}$	-	-	-	-	0.490***
	-	-	-	-	(0.038)
σ_η	-	-	-	-	0.052
	-	-	-	-	(0.040)
	Real Interest Rate Equation				
r^{LR}	1.559	1.742	1.824**	1.777*	2.076
	(1.492)	(1.720)	(0.895)	(1.012)	(1.511)
ϕ	0.562***	0.968***	0.814***	0.798***	0.397***
	(0.140)	(0.023)	(0.151)	(0.179)	(0.124)
ρ	0.963***	0.364***	0.898***	0.909***	0.964***
	(0.025)	(0.137)	(0.041)	(0.051)	(0.023)
σ_e	0.726***	0.955***	1.240***	1.235***	0.696***
	(0.167)	(0.106)	(0.139)	(0.226)	(0.119)
σ_ε	1.000***	0.723***	0.419	0.436	0.971***
	(0.132)	(0.116)	(0.366)	(0.600)	(0.104)
Likelihood	-332.546	-313.826	-334.628	-337.516	-411.807

Note: Standard errors are in parentheses. *,**,*** denotes statistical significance at the 10, 5 and 1 percent levels respectively.

Table 2: **Estimation Results for the Inflation Equation in the Trivariate UC Model**

	No-Change	Two-State	Three-State
k	0.099	0.187**	0.139*
	(0.065)	(0.085)	(0.081)
c_1	0.556***	0.544***	0.538***
	(0.074)	(0.073)	(0.078)
c_2	0.166**	0.157*	0.164*
	(0.079)	(0.080)	(0.093)
c_3	0.233***	0.256***	0.258***
	(0.072)	(0.073)	(0.075)
$\sigma_{z,1}$	1.074***	1.076***	1.100***
	(0.233)	(0.237)	(0.245)
$\sigma_{z,2}$	1.539***	1.522***	1.561***
	(0.182)	(0.047)	(0.183)
$\sigma_{z,3}$	0.773***	0.766***	0.769***
	(0.048)	(0.047)	(0.048)

Note: Standard errors are in parentheses. *,**,*** denotes statistical significance at the 10, 5 and 1 percent levels respectively.

Table 3: **Residual Diagnostic Tests for the Trivariate UC Model**

Lag	Standardized Residuals	Squared Standardized Residuals
	No Regime Changes	
1	0.308	0.000
4	0.162	0.000
8	0.374	0.000
12	0.379	0.000
16	0.589	0.000
20	0.538	0.000
	Two Markov-switching Regimes	
1	0.670	0.238
4	0.329	0.716
8	0.614	0.378
12	0.847	0.628
16	0.924	0.822
20	0.958	0.853
	Three Markov-switching Regimes	
1	0.567	0.100
4	0.283	0.437
8	0.654	0.258
12	0.870	0.545
16	0.844	0.590
20	0.880	0.690

Note: Reported are the p-values associated with the modified Ljung-Box Portmanteau test statistic under the null of no serial correlation.

Table 4: **Estimation Results for the Two-State Markov-Switching UC Models**

Parameters	CBO	HP-filter	Quadratic	Segmented	UC
	Output Gaps				
	Output Equation				
σ	0.100**	0.082**	0.099*	0.096**	0.105***
	(0.046)	(0.040)	(0.054)	(0.047)	(0.033)
α_1	1.270***	1.119***	1.293***	1.173***	1.135***
	(0.072)	(0.069)	(0.070)	(0.101)	(0.086)
α_2	-0.302***	-0.261***	-0.316***	-0.187**	-0.217***
	(0.071)	(0.069)	(0.070)	(0.092)	(0.081)
$\sigma_{v,1}$	0.975***	0.934***	0.980***	0.967***	0.946***
	(0.089)	(0.085)	(0.090)	(0.092)	(0.089)
$\sigma_{v,2}$	0.523***	0.478***	0.530***	0.481***	0.481***
	(0.034)	(0.030)	(0.034)	(0.042)	(0.035)
$\rho_{v,\varepsilon}$	-0.029	-0.028	-0.029	0.026	-0.005
	(0.113)	(0.109)	(0.105)	(0.125)	(0.002)
$\sigma_{w,1}$	-	-	-	-	0.000
					(0.023)
$\sigma_{w,2}$	-	-	-	-	0.000
					(0.026)
σ_η	-	-	-	-	0.094**
					(0.035)
	Real Interest Rate Equation				
r_1^{LR}	4.292	4.603	4.392	3.819	6.278
	(5.690)	(4.089)	(5.096)	(4.030)	(6.400)
r_2^{LR}	0.636	0.491	0.547	0.746	0.468
	(1.867)	(1.650)	(1.854)	(1.719)	(0.368)
ϕ_1	0.521***	0.509***	0.517***	0.533***	0.518***
	(0.134)	(0.080)	(0.176)	(0.132)	(0.029)
ϕ_2	0.120	0.080	0.090	0.850***	0.143
	(0.461)	(0.341)	(0.405)	(0.057)	(0.266)
ρ_1	0.981***	0.981***	0.978***	0.974***	0.975***
	(0.034)	(0.033)	(0.035)	(0.036)	(0.033)
ρ_2	0.956***	0.956***	0.956***	0.951***	0.950***
	(0.028)	(0.028)	(0.028)	(0.028)	(0.028)
σ_e	0.661***	0.703***	0.665***	0.748***	0.703***
	(0.092)	(0.096)	(0.091)	(0.056)	(0.093)
$\sigma_{\varepsilon,1}$	1.552***	1.553***	1.556***	1.535***	1.566***
	(0.183)	(0.182)	(0.187)	(0.181)	(0.180)
$\sigma_{\varepsilon,2}$	0.295**	0.254*	0.290**	0.144	0.270*
	(0.137)	(0.144)	(0.132)	(0.095)	(0.151)
Likelihood	-307.200	-292.589	-308.773	-306.337	-390.592

Note: Standard errors are in parentheses. *,**,*** denotes statistical significance at the 10, 5 and 1 percent levels respectively.

Table 5: **Estimation Results for the Three-State Markov-Switching UC Models**

	Output Gaps				
Parameters	CBO	HP-filter	Quadratic	Segmented	UC
	Output Equation				
σ	0.111**	0.083**	0.110***	0.103**	0.100**
	(0.047)	(0.040)	(0.038)	(0.041)	(0.044)
α_1	1.277***	1.119***	1.295***	1.328****	1.156***
	(0.068)	(0.068)	(0.068)	(0.068)	(0.089)
α_2	-0.306***	-0.259***	-0.319***	-0.331***	-0.242***
	(0.069)	(0.069)	(0.068)	(0.069)	(0.086)
$\sigma_{v,1}$	0.973***	0.936***	0.976***	0.978***	0.955***
	(0.089)	(0.084)	(0.090)	(0.090)	(0.093)
$\sigma_{v,2}$	0.529***	0.480***	0.533***	0.546***	0.488***
	(0.033)	(0.030)	(0.034)	(0.035)	(0.037)
$\rho_{v,\varepsilon}$	0.000	0.071	0.000	0.010	0.039
	(0.080)	(0.116)	(0.085)	(0.014)	(0.073)
$\sigma_{w,1}$	-	-			0.000
					(0.006)
$\sigma_{w,2}$	-	-	-	-	0.000
					(0.036)
σ_η	-	-	-	-	0.084**
					(0.003)
	Real Interest Rate Equation				
r_1^{LR}	0.747*	0.835*	0.781*	0.772**	1.152
	(0.442)	(0.481)	(0.452)	(0.381)	(1.391)
r_2^{LR}	6.370***	6.182***	6.304***	6.527***	6.525***
	(0.810)	(0.775)	(0.845)	(0.827)	(1.407)
r_3^{LR}	0.289	0.338	0.305	0.290	0.300
	(2.033)	(2.783)	(1.990)	(1.925)	(3.633)
ϕ_1	0.513***	0.509***	0.505***	0.586***	0.455**
	(0.114)	(0.156)	(0.054)	(0.170)	(0.229)
ϕ_2	0.483**	0.500***	0.503***	0.502***	0.520***
	(0.238)	(0.039)	(0.028)	(0.079)	(0.088)
ϕ_3	0.033	0.000	0.000	0.000	0.048
	(0.300)	(0.004)	(0.000)	(0.000)	(0.274)
ρ_1	0.661**	0.679**	0.675**	0.338	0.847***
	(0.316)	(0.345)	(0.281)	(0.892)	(0.206)
ρ_2	0.696***	0.656**	0.725***	0.716***	0.855***
	(0.236)	(0.306)	(0.210)	(0.209)	(0.124)
ρ_3	0.960***	0.959***	0.960***	0.959***	0.960***
	(0.027)	(0.032)	(0.027)	(0.027)	(0.037)
σ_e	0.664***	0.677***	0.667***	0.668***	0.664***
	(0.088)	(0.088)	(0.084)	(0.084)	(0.096)
$\sigma_{\varepsilon,1}$	1.274***	1.264***	1.273***	1.269***	1.255***
	(0.169)	(0.172)	(0.170)	(0.185)	(0.210)
$\sigma_{\varepsilon,2}$	1.855***	1.849***	1.870***	1.850***	1.909***
	(0.330)	(0.327)	(0.337)	(0.321)	(0.344)
$\sigma_{\varepsilon,3}$	0.386***	0.370***	0.379***	0.376***	0.390***
	(0.115)	(0.095)	(0.090)	(0.090)	(0.110)
Likelihood	-303.085	-287.766	-304.420	-307.334	-388.450

Note: Standard errors are in parentheses. *,**,*** denotes statistical significance at the 10, 5 and 1 percent levels respectively.

Figure 1: Filtered Probabilities for the Two-State Markov-Switching Trivariate UC Model

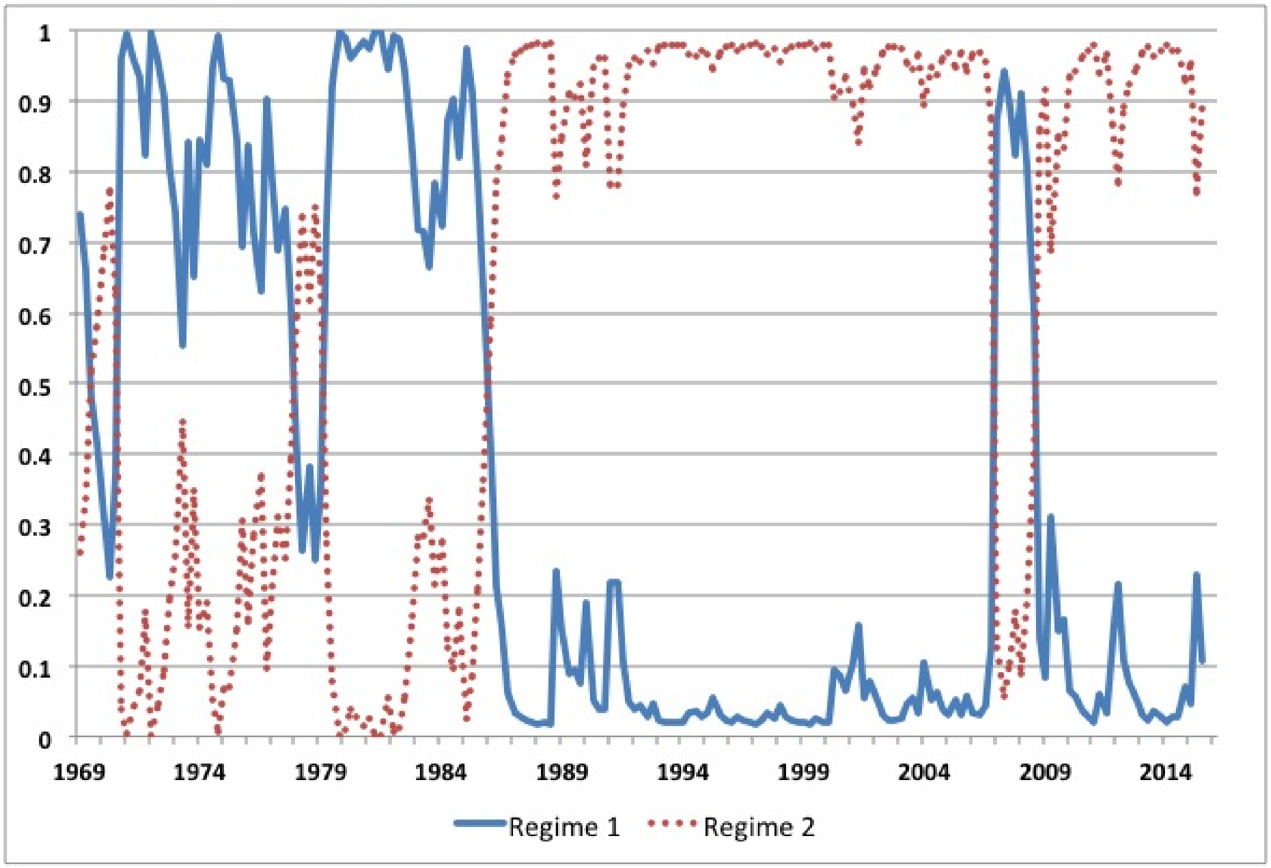


Figure 2: Filtered Probabilities for the Three-State Markov-Switching Trivariate UC Model

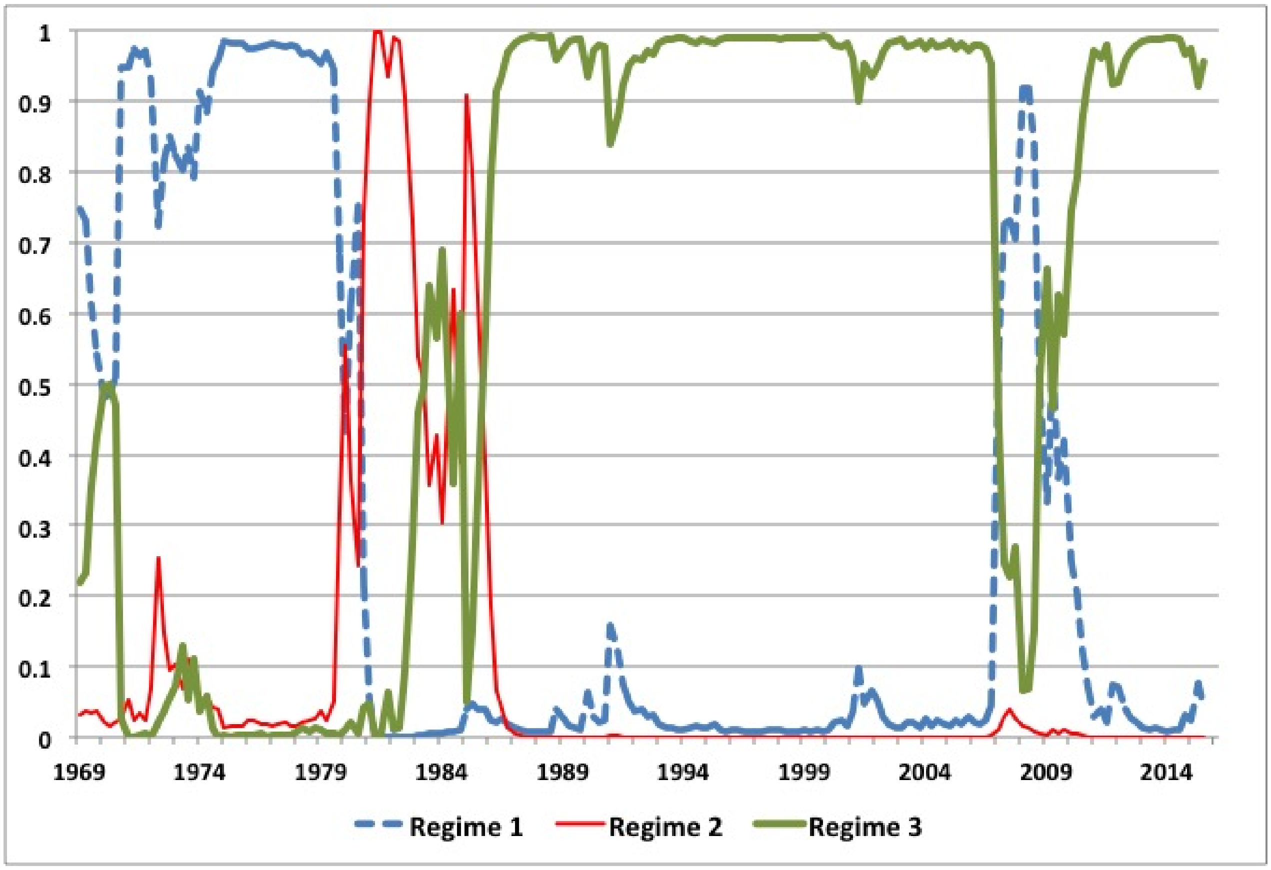


Figure 3: The Ex-Post Real Interest Rate and the Estimated Natural Real Rate from the Three-State Markov-Switching Trivariate UC Model

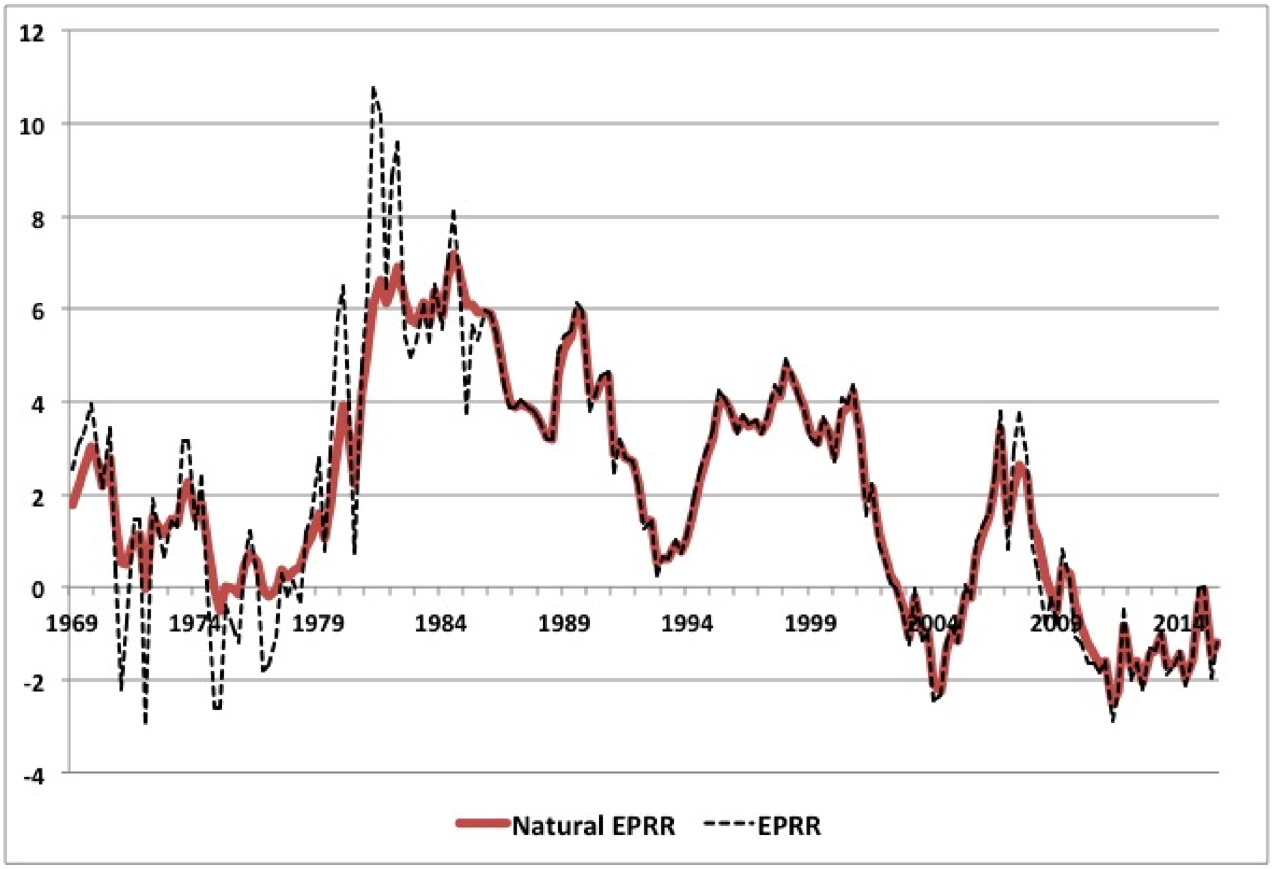
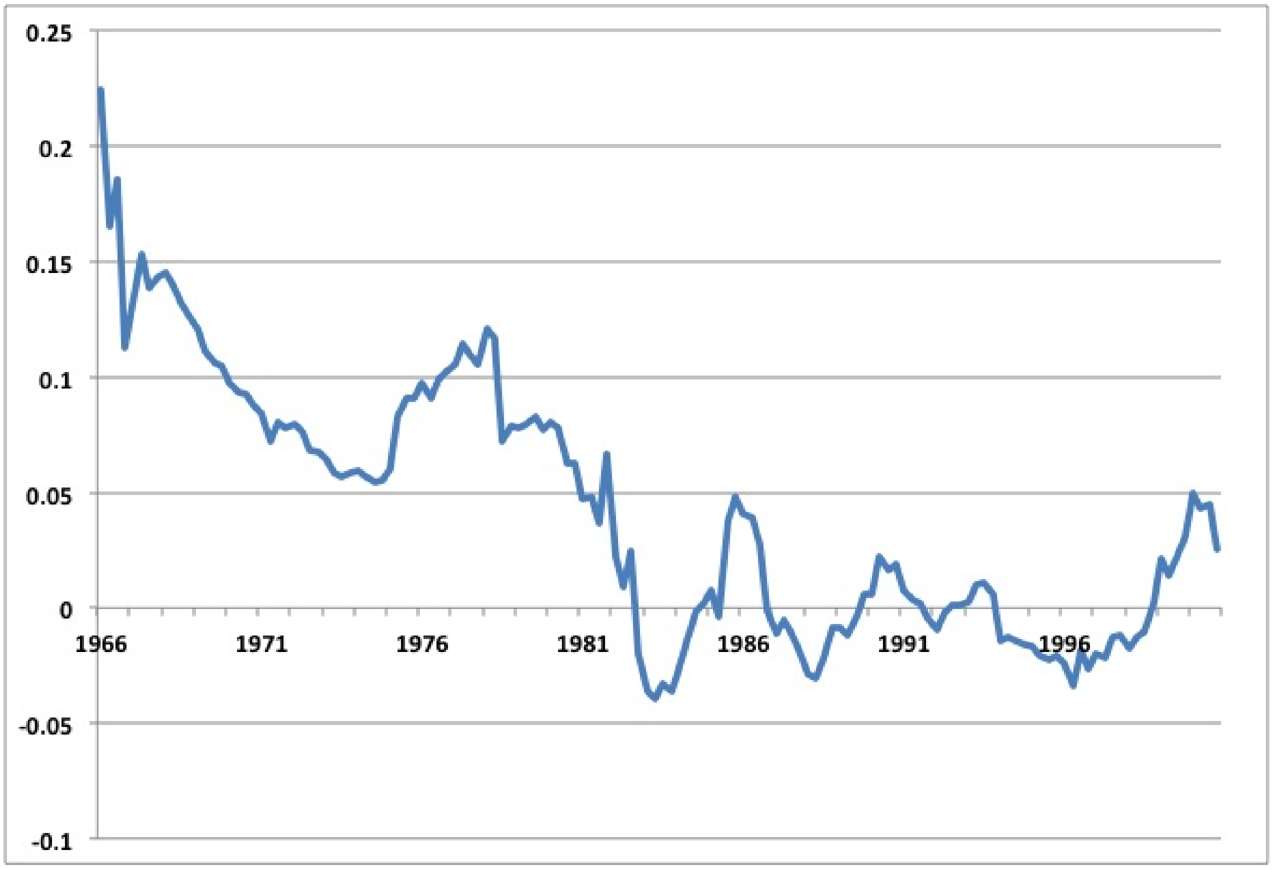


Figure 4: Rolling Estimates of β from the Reduced Form IS Curve



Note: Plotted are 15-year rolling estimates of β from $x_t = c + \alpha_1 x_{t-1} + \alpha_2 x_{t-2} + \beta r_{t-1} + e_t$, where the year marks the beginning of the estimation window.