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by

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Age-Dependent Risk Aversion: Re-evaluating Fiscal Policy Impacts of Population Aging*

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Abstract

The integration of age-dependent increasing risk aversion (IRA) into an overlapping generations model (OLG) with risk-sensitive preferences provides a more comprehensive understanding of risk aversion, life-cycle behavior, and welfare under uncertainties. A quantitative analysis shows that IRA individuals accumulate more precautionary savings and adjust working hours to mitigate income shocks. However, this mitigation of uncertainty entails a cost of reduced resources, which could have otherwise been used to increase overall consumption of goods and leisures. Three alternative policies to address the challenges posed by aging are evaluated: increasing a payroll tax rate, reducing pension benefits, and extending the retirement age. The results show that individuals who expect to become more risk averse in old age may prefer the payroll tax rate increase, as the other two options results in relatively higher income uncertainty, which contradicts the results of previous studies that assumed constant risk aversion (CRA).

Keywords: Overlapping generations model, Fiscal sustainability, Demographic changes, Increasing risk aversion, Non-expected utility

JEL Codes: D15, D81, E62, J11

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Empirical evidence shows that individuals tend to become more risk averse as they age[1]. Furthermore, it is well-documented that decision making is influenced by an individual's consideration of future uncertainties. This study incorporates these two aspects into an overlapping generations (OLG) model with risk-sensitive preferences. The model provides important insights into the channels through which increasing risk aversion affects behavior and welfare in the face of uncertainty.

Regarding the first aspect, this paper challenges the conventional assumption of constant and age-independent risk aversion, which is unrealistic in an OLG model that aims to account for age-specific behaviors. The willingness to take risk is a fundamental aspect of economic model as it affects key mechanisms such as an individual's consumption, saving, and welfare under uncertainties. These mechanisms in turn influence prices and macroeconomic variables. Multiple studies (Donkers et al. (2001), Barsky et al. (1997), Dohmen et al. (2017), Roalf et al. (2011), Bakshi and Chen (1994) and Pålsson (1996)) have found a clear pattern that risk aversion tends to increase with age . This may be due to the fact that older individuals are less capable of earning extra income, making them less resilient to income shocks compared to their younger counterparts. Although few studies have explored the change in risk aversion and its macroeconomic impact, this assumption is rarely considered in the OLG literature[2].

Regarding the second aspect, this paper advocates for the inclusion of future uncertainty in an OLG model. In fact, economics has long recognized the importance of the theory of choice under uncertainty. When individuals are risk-averse, their precautionary savings increase as independent income shocks or background risks increase (Kimball, 1989). In the study by Tallarini (2000), he finds that risk aversion increases the welfare cost of uncertainties in a business cycle model. These findings highlight that individuals value not only the levels of consumption and leisure but also the certainty of achieving their plans. Risk-averse individuals prefer a sure outcome over a gamble with equal expected value, and higher level of uncertainty will lower welfare to varying degree depending on their level of risk aversion at any given time.

This paper integrates the two often overlooked aspects into a heterogeneous-agent OLG model with idiosyncratic wage and mortality shocks to assess the behavioral and welfare impacts of three alternative policies: increasing the payroll tax rate, reducing pension benefits, and extending the retirement age. This study considers two risk

[1] It is worth noting that factors such as education, employment status, immigration status, income and wealth can also impact an individual's risk aversion. This paper, however, focuses solely on the aspect of age.

[2] The study by DaSilva et al. (2019) incorporates preferences with increasing risk aversion in a partial equilibrium three-period OLG model and demonstrates that it produces results for equity premium, savings and portfolio shares that are more consistent with U.S. data. However, such study does not emphasize behavioral responses and welfare under uncertainty.

aversion assumptions: constant risk aversion (CRA) and age-dependent increasing risk aversion (IRA). Instead of the standard time-separable utility function[3], this paper employs risk-sensitive preferences to disentangle the degree of risk aversion from the elasticity of intertemporal substitution (EIS), enabling variations in risk aversion without making assumptions on EIS. Furthermore, this form of preferences exhibits the translation-invariant property and is monotonic (Bommier et al., 2017), which aids in providing consistent comparative statics regarding the impacts of varying risk aversion on precautionary savings and welfare[4]. The utility function used here enables an intuitively understanding of the trade-off between future uncertainty and instantaneous utility, given different levels of risk aversion.

The results of this study suggest that IRA individuals tend to mitigate old age uncertainties by increasing their precautionary savings and adjusting their hours worked in response to income shocks. This increased risk aversion influences the optimal level of resources that individuals are willing to use to protect themselves against lifetime volatility, resulting in lower lifetime consumption and leisure. This study shows that IRA and CRA groups may experience similar levels of consumption over a lifetime. However, the former group will work more hours and have more savings to counteract volatility. In addition, a stronger focus on controlling uncertainty may result in higher welfare costs when expected uncertainty is high.

This paper contributes to the existing literature on structural OLG models, building upon the foundational work of Auerbach and Kotlikoff (1987). The studies in this field encompass a wide array of topics[5], but two of the most relevant to this paper are Nishiyama (2015) and Kitao (2014), which examine best policy response to maintain fiscal balance in an aging population. These studies conclude that reducing the replacement rate and increasing the normal retirement age result in higher welfare for future generations compared to increasing the payroll taxes. However, this paper argues that welfare ranking of policy reforms may change when aspects of age-dependent risk aversion and future uncertainties are incorporated. The results show that IRA

[3]The time-separable utility framework has several limitations that render it inappropriate for studying age-dependent risk aversion. One of its main drawback is the inability to specifically examine risk aversion, as there is an inverse relationship between the elasticity of intertemporal substitution (EIS) and relative risk aversion (Miao (2014) and Karantounias (2018)). Assuming age-varying risk aversion therefore imposes an unrealistic behavior on the EIS, which is hardly justified. Additionally, time-separable specification assumes that individuals are risk-neutral in future utility, rendering them indifferent between gambles with the same expected payoffs.

[4]See detailed discussion in Bommier et al. (2017) and Bommier et al. (2020)

[5]Studies in this field have explored a variety of topics, including social security privatization (e.g., Kotlikoff et al., 1999; Nishiyama and Smetters, 2007), the welfare and macroeconomic effects of various tax reforms (e.g., De Nardi et al., 1999; Huggett and Ventura, 1999; Altig et al., 2001; Vogel et al., 2017), and the design of optimal fiscal policies (e.g., İmrohoroglu et al., 1995; Gottardi et al., 2015), for instance. Each of these studies built on the original Auerbach and Kotlikoff (1987)'s model, adding more advanced model specifications and assumptions, such as idiosyncratic wage shocks, endogenous human capital accumulation, and more realistic government and population structures.

is linked to a decline in welfare under a reduced replacement rate and extended retirement age due to their comparatively higher old-age uncertainty compared to the payroll tax increase.

A limitation of this study is the difficulty in estimating the age-dependent risk aversion. This study assumes a linear increase in risk aversion with age, serving as a framework for considering the relationship between risk aversion, life-cycle behaviors and welfare. However, explicit quantitative estimation requires determining the risk aversion of individuals at different ages, which is beyond the scope of this paper. Previous literature has used various methods to estimate risk aversion[6], but eliciting risk aversion values for use in a non-expected utility model remains challenging because it is a function of discounted future consumption and leisure plans (Chen et al., 2013) and an individual's ability to adjust labor supply in response to shocks (Swanson, 2018).

The paper by L. P. Hansen et al. (2008) presents an analytical solution to calculate the relative risk aversion value where the intertemporal substitution is unit elastic. However, the elicitation process still faces challenges, such as the lack of availability of age-dependent consumption time series, partly due to the difficulty in distinguishing between the effects of age, time, and cohort. There is also the issue of endogenous labor supply that needs to be considered. Incentivized experiment face similar problems as it is challenging to find economic experiments that control for the traits of different ages and cohorts. Appendix A provides an example of estimating risk aversion values from findings of Albert and Duffy (2012)), but their applicability in a quantitative framework remains a topic of discussion.

The rest of the paper is organized as follows. Section 1 describes the model structure and provides a brief explanation for the use of risk-sensitive preferences. Section 2 explains the methods used to calibrate the model parameters. In section 3, numerical results are compared based on the two risk aversion assumptions and three

[6] The estimation of risk aversion has been approached quantitatively in studies on the concept of risk aversion in Arrow (1965) and Pratt (1964). This approach evaluates risk aversion as the elasticity of marginal utility with respect to wealth and can be derived from the portfolio choice data. Most studies have found risk aversion to increase with age (Morin and Suarez (1983),Pålsson (1996), Bellante and Green (2004)), with the exception of those that do not categorize housing as a risky asset, which find risk aversion decreases until age 65 and then increases (Riley Jr and Chow (1992), Halek and Eisenhauer (2001)). Another method to estimate risk aversion is through an incentivized experiment that analyze an individual's decision-making towards different pay-off structures. One popular approach is the lottery choice menu proposed by Holt and Laury (2002). In their study, Albert and Duffy (2012) apply the lottery choice approach to two different age groups and show that older adults are more risk-averse than younger adults, consistent with another experiment conducted by Dohmen et al. (2011) based on a 10-point scale, as well as the study by Roalf et al. (2011). Self-reported questionnaires, such as the German Socio-Economic Panel, also have been used to evaluate an individual's willingness to take risk. For further references, see Schildberg-Hörisch (2018), which reviews several studies that document systemic changes in risk aversion as individuals progress from children into adulthood and old age.

policy options. The model examines redistribution of life-cycle variables and welfare impacts. Section 4 concludes.

1 The model

The model economy consists of overlapping-generation individuals whose utility is characterized by risk-sensitive preferences. Time is discrete and the price of the consumption good is the numeraire and is normalized to 1. The government is assumed to be able to credibly commit to its policies and operate a pay-as-you-go social security system.

1.1 Households

In a one-year period, denoted by t, the economy is made up of individuals who enter the labor market at age $j = 1$ (corresponding to an actual age of 20 years old). These individuals work until they reach the retirement age of $J_R = 47$ (corresponding to an actual age of 67 years old) and live until the age of J = 81 (corresponding to an actual age of 100 years old). Living individuals face a probability of surviving from age $j-1$ to age j at time t, denoted by ξ_t^j. Prior to entering the labor market, these individuals complete their education, which can be at one of the three levels: high school graduate or less, some college or associate degree, or bachelor's degree and higher (denoted by $s \in \{L, M, H\}$).

Upon entering the labor market, individuals are endowed with one disposable unit of time to be allocated to leisure $(1 - l_t^j)$ or supplying labor l_t^j. They also have no initial wealth $(a = 0)$ and earn a wage rate w_t per each efficiency unit when supplying labor. Workers pay progressive income tax on their earnings up to the maximum taxable amount, y_t^s, and ,once retired, receive social security benefits based on Average Indexed Monthly Earnings (AIME) that take into consideration the 35 years of individual's top earnings (see details in Section 1.1.2).

Demographics are assumed to be in a steady state, and the population of age j at time t, denoted by N_t^j, can be written as $N_t^{j+1} = (1+n)\xi_t^{j+1}N_t^j$, where n is the population growth rate. The total population at time t, denoted by N_t, is the sum of all age cohorts alive at that time. The share of the age-j cohort in the total population at time t is represented by m_t^j, with the condition that $\sum_{j=1}^{J} m_t^j = 1$.

1.1.1 Labor productivity

Labor earnings are determined by $w_t h^j l_t^j$, where the productive efficiency unit h^j is assumed to be a function of deterministic age-earning profile o_j, an inherent productivity effect θ_s that depends on education levels, and an idiosyncratic productivity component $\eta^j \in H$ that follows a first-order Markov process, represented by:

$$h^j = \begin{cases} o^j \cdot \exp(\theta_s + \eta^j) & \text{for } j < J_R \\ 0 & \text{for } j \geq J_R \end{cases}$$
$$\eta^{j+1} = \rho\eta^j + \epsilon^{j+1} \quad \text{with} \quad \epsilon^{j+1} \sim N(0, \sigma_\epsilon^2), \tag{1}$$

where ρ represents the auto-correlation from the stochastic productivity of age j to age $j+1$. Individuals also receive saving income determined by an asset holding at the beginning of the period a_t^j and an interest rate r_t.

1.1.2 Retirement benefits

The government operates a pay-as-you-go pension system where pension benefits (denoted by pen) are a function of AIME which takes into account up to 35 years of a worker's maximum indexed earnings. According to Kitao (2014) and French (2005), AIME (or earning points, e_t, in this paper) can be approximated as

$$e_{t+1} = \begin{cases} e_t + \frac{y_{L,t}}{35} & \text{for } 20 \leq j \leq 55 \\ e_t + \max\left\{0, \frac{y_{L,t} - e_t}{35}\right\} & \text{for } 55 < j < J_R, \end{cases} \tag{2}$$

$$\tilde{y}_t = \min\{w_t h^j l_t^j, y_t^s\}, \tag{3}$$

where $\tilde{y}_t$ is the covered earnings up to maximum taxable amount, y_t^s.

Pension benefits can be approximated using a concave piece-wise linear function of earning points. Given the actual values of turning points of \$132,900 and maximum taxable earnings from the US Social Security Administration (SSA) in 2019, the pension benefits can be written as

$$\text{pen}(e_t) = \begin{cases} 0.9 \times e_t & \text{if } e_t \leq \$11,112 \\ \$10,001 + 0.32 \times (e_t - \$11,112) & \text{if } \$11,112 < e_t \leq \$66,996 \\ \$27,884 + 0.15 \times (e_t - \$66,996) & \text{if } \$66,996 < e_t. \end{cases} \tag{4}$$

Both turning points and maximum taxable earnings are assumed to be indexed to the average income growth.

1.1.3 Progressive tax

The progressive tax structure is derived from the paper by Keane and Wasi (2016) with certain modifications. Taxable income, represented by TI, is calculated from

$$\text{TI} = \max\{w_t h^j l_t^j - \tau^s \cdot \tilde{y}_t + r_t a_t^j - SD, 0\} \tag{5}$$

where SD is the standard deduction, which was \$12,200 in 2019, and tau_s is the payroll tax rate. The progressive tax structure is calculated from

$$\ln(\text{Tax}) = -3.9543 + 1.22563 \cdot \ln(\text{TI}) \tag{6}$$

with coefficients that were estimated from the above study.

1.1.4 Preferences

This study employs a special case of a recursive non-expected utility by Epstein and Zin (1989) and Weil (1990)(EZW), in which the intertemporal elasticity of substitution is equal to one. According to Tallarini (2000), this form of utility function can be transformed into a risk-sensitive preference similar to the specification used by L. P. Hansen and Sargent (1995). This approach has useful properties for the current analysis, as its monotonic property provides a clear and consistent interpretation of comparative statics on precautionary savings under varying levels of risk aversion. Additionally, it enables an evaluation of the impact that future uncertainties have on individual decision-making.

Assume that individuals derive utility from consuming a composite good consisting of consumption c_t^j and leisure $(1-l_t^j)$, which is expressed in the form of Cobb-Douglas utility function

$$U(c_t^j, 1-l_t^j) = (c_t^j)^{\nu}(1-l_t^j)^{1-\nu}, \tag{7}$$

where ν is the taste parameter of consumption. Let V_t^j be a value function of an individual aged j at time t. According to the approach described in Tallarini (2000)[7] , when assume unit of elasticity of substitution, the recursive risk-sensitive preferences

[7]We can transform EZW preferences into risk-sensitive preferences by making the assumption of unit elasticity of substitution. Given a Cobb-Douglas utility function and income shock process, the recursive utility with unit elasticity of substitution is written as

$$V_t^j = \left[(c_t^j)^{\nu}(1-l_t^j)^{1-\nu}\right]^{1-\beta}\left[\mathbb{E}_t({V_{t+1}^{j+1}}^{1-\gamma^j} \mid \eta_t^j)^{\frac{1}{1-\gamma^j}}\right]^{\beta}, \tag{8}$$

where γ is a coefficient of relative risk aversion.

Take logs and rearrange gives

$$\frac{\ln V_t^j}{1-\beta} = \left(\nu \ln c_t^j + (1-\nu)\ln(1-l_t^j)\right) + \frac{\beta}{(1-\gamma^j)(1-\beta)} \ln \mathbb{E}_t({V_{t+1}^{j+1}}^{1-\gamma^j} \mid \eta_t^j). \tag{9}$$

By transforming $\tilde{V} = \frac{\ln V_t}{(1-\beta)}$ and $\psi^j = -(1-\beta)(1-\gamma^j)$, the above equation can be re-written as

$$\tilde{V}_t^j = \left(\nu \ln c_t^j + (1-\nu)\ln(1-l_t^j)\right) - \frac{\beta}{\psi^j} \ln \mathbb{E}_t(\exp(-\psi^j \tilde{V}_{t+1}^{j+1}) \mid \eta_t^j) \tag{10}$$

can be written as

$$V_t^j = \big(\nu \ln c_t^j + (1-\nu)\ln(1-l_t^j)\big) - \frac{\beta}{\psi^j}\ln \mathbb{E}_t(\exp(-\psi^j V_{t+1}^{j+1})\big|\, \eta_t^j) \tag{11}$$

which resembles risk-sensitive preferences of L. P. Hansen and Sargent (1995) and Weil (1993). The risk-sensitivity parameter ψ^j measures degrees of risk aversion towards future utility of age j agents and β is a discount factor.

To understand how the risk aversion parameter influences the value of the certainty equivalent (CE) of the risk-sensitive preferences in (11), we can use Taylor expansions to draw implications:

$$\frac{1}{\psi^j}\ln(\mathbb{E}(\exp(-\psi^j V_{t+1}^{j+1}))) \xrightarrow{\text{Taylor expansion}} \mathbb{E}(V_{t+1}^{j+1}) - \frac{\psi^j}{2}Var(V_{t+1}^{j+1}). \tag{12}$$

The equation suggests that individuals consider both the expected value and the variance of the next period's value function. This means that they are concerned with the dispersion of their lifetime utility. The second term in the equation can be interpreted as a penalty for the randomness of the future value function, with a degree of penalty depending on the value of parameter ψ^j. When $\psi^j > 0$, individuals exhibit risk-averse behavior towards future utilities. When $\psi \to 0$, their risk-sensitive preferences in (11) convert to von Neumann-Morgenstern expected utility, where the coefficient of relative risk aversion has a direct relationship with the intertemporal elasticity of substitution.

1.1.5 Heterogeneity and state variables

At time t, individuals are characterized by the states $x_t = \{j, a, s, \eta, e\}$ and the aggregate state of the economy is characterized by $X_t = \{\phi_t(x_t)\}$, where $\phi_t(x_t)$ represents distributions of individuals with states x_t and satisfies $\int d\phi_t(x_t) = N_t$. The government policy schedule is given by $\Psi_t = \{G_t, D_t, \text{Tax}, \text{pen}, \tau^s, \tau^c, y^s\}$ where G_t and D_t are government spending and government debt. The population at time t is characterized by $\Phi_t = \{(N_t^j)_{j=1}^J, (\xi_t^j)_{j=1}^J\}$ which is assumed to be deterministic and known to all individuals.

1.1.6 Household's optimization problem

To simplify the notation, I will omit the time subscript t and states x_t unless it is necessary for clarity. Each individual chooses a stream of consumption (c), labor supplies (l), and next-period savings (a') to solve the following dynamic programming problem:

$$V = \max_{\{c,l,a'\}} \Big\{ (\nu \ln c + (1-\nu)\ln(1-l)) - \frac{\beta \xi'}{\psi^j}\ln \mathbb{E}_t(\exp(-\psi^j V')\big|\, \eta)\Big\}, \tag{13}$$

subject to the following constraints

$$a' = \frac{1}{1+\mu}\left[(1+r)a + whl + q + \text{pen}(e) - T - c\right], \tag{14}$$

$$T = \tau^c c + \tau^s \tilde{y} + \text{Tax(TI)}, \tag{15}$$

$$c > 0, \quad 0 \leq l \leq 1, \quad a > 0, \tag{16}$$

where the labor-augmenting productivity growth is represented by μ.

From the budget constraint, individuals pay consumption tax ($\tau^c c$), payroll tax rate (τ_s) on the covered earnings $\tilde{y} = \min\{whl, y^s\}$, and progressive tax (Tax(TI)) on working and capital income calculated from eq (5)-(6). Without annuity markets, the assets of the deceased are equally distributed among living individuals as accidental bequests q.

Each individual of every age j at every time period t choose policy functions $a'(x_t)$, $c(x_t)$ and $l(x_t)$ to solve equations (13)-(16).

1.2 The distribution of the households

Let $\phi(x_t)$ represent the distribution of individuals under states x_t. Since individuals are assumed to enter the economy at the age of 20 with no assets, they are distributed solely according to their intrinsic productivity which can be written as

$$\int_{A\times S\times H\times E} d\phi(x_t = \{j = 20, a, s, \eta, e\}) = \int_{A\times S\times H\times E} d\phi(x_t = \{j = 20, 0, s, 0, 0\}) = N_t^{20}. \tag{17}$$

where $a \in A, s \in S, \eta \in H, e \in E$

The distributions of heterogeneous individuals above the age of 20 are determined by the policy function $a^{j'}$, the accumulation of earning points to the next period $e'(x_t)$, and the transition probability of earning shocks $\pi(\eta'|\eta)$ from the current state η to the next-period state η':

$$\phi(x' = \{j', a', s, \eta', e'\}) = \int_{A\times S\times H\times E} a^{j'}(x) \cdot e'(x) d\phi(x_t = \{j, a, s, \eta, e\}) \cdot \pi(\eta'|\eta). \tag{18}$$

1.3 Firms

The model assumes the representative firm that hires capital K_t and labor L_t to produce a single type of output with a constant returns to scale Cobb-Douglas production function $F(K_t, L_t) = \Omega_t K_t^{\alpha} L_t^{1-\alpha}$ where Ω_t is a total factor productivity and α and $1-\alpha$ are the output elasticity of capital and labor respectively. The law of motion for capital is characterized by

$$(1+n_t)(1+\mu)K_{t+1} = (1-\delta)K_t + I_t, \tag{19}$$

where capital depreciates at a fixed rate δ and I_t is the amount of investment in the period t.

The firm maximizes its profit by choosing K_t and L_t, while taking an interest rate r_t and a wage rate w_t as given. The firms' profit maximization problem is

$$\max_{\{K_t, L_t\}} \Omega_t K_t^{\alpha} L_t^{1-\alpha} - (r_t + \delta)K_t - w_t L_t, \tag{20}$$

and the profit maximising conditions are

$$(1-\alpha)\Omega_t \left(\frac{K_t}{L_t}\right)^{\alpha} = w_t, \tag{21}$$

$$\alpha\Omega_t \left(\frac{L_t}{K_t}\right)^{1-\alpha} = r_t + \delta. \tag{22}$$

The factor market clearing condition requires $K_t = \tilde{K}_t$ and $L_t = \tilde{L}_t$, where $\tilde{K}_t$ and $\tilde{L}_t$ are supplies of capital and labor.

1.4 Government

The government maintains two separate balanced budgets: one for the general government spending and another for the pension system.

For the general government spending, the model assumes that the government spends an exogenous amount as a constant share (g) of GDP and repays debt with interest rate $(1 + r_t)D_t$. On the revenue side, the government raises tax from labor income, capital income, and consumption. Also, it issues new debt D_{t+1} which grows at the population growth rate n_t and the labor-augmenting productivity growth rate μ. The progressive income tax function is endogenously scaled so that the general government budget is balanced. The general government budget is balanced when

$$G_t + (1 + r_t)D_t = \sum_{j=20}^{J} \int_{A\times S\times H\times E} \Big[\text{Tax}(x) + \tau^c \cdot c(x)\Big]\phi(x) \cdot m_t^j + (1 + n_t)(1 + \mu)D_{t+1}. \tag{23}$$

On the pension system, the government sets the payroll tax rate so that the total payroll tax revenue collected matches the total social security benefits spending in in the benchmark year

$$\sum_{j=1}^{J} \int_{A\times S\times H\times E} \text{pen}(e_t) \cdot \phi(x) \cdot m_t^j = \sum_{j=1}^{J} \int_{A\times S\times H\times E} \tau^s \cdot \tilde{y} \cdot \phi(x) \cdot m_t^j. \tag{24}$$

The government is also assumed to uniformly redistribute accidental bequests which equal the end-period wealth of deceased individuals to all living individuals.

1.5 Equilibrium conditions

With an assumption of perfect foresight in the intertemporal dynamic OLG model, for the set of aggregate state of the economy X_t, government policy schedule Ψ_t, and population projection Φ_t, the recursive competitive equilibrium consists of individuals' policy functions $\{a'(x_t), c(x_t), l(x_t)\}_{t=0}^T$ for each individual state $x_t = \{j, a, s, \eta, e\}$ and factor prices $\{w_t, r_t\}_{t=0}^T$ that satisfy the following conditions:

1. Aggregate and individual variables are consistent with the distributions $\phi_t(x_t)$ that evolves according to (17) - (18).

2. Individuals' policy functions solve equation (13) subject to constraints (14) - (16).

3. Firm's factor input choices K_t, L_t solve firm's profit optimization (20).

4. The labor income tax and social security tax satisfies the government's budget balances (23) - (24).

5. Accidental bequests are allocated according to

$$q_t \sum_{j=1}^{J} \int_{A\times S\times H\times E} d\phi_t(x)\cdot m_t^j = \sum_{j=1}^{J} \int_{A\times S\times H\times E} (1-\xi')(1+\mu)a'(x_t)\cdot d\phi_t(x)\cdot m_t^j. \quad (25)$$

6. All markets clear

   - Factor market for labor:

$$L_t = \tilde{L}_t \quad (26)$$

   - Factor market for capital:

$$K_t = \tilde{K}_t, \tilde{K}_t + D_t = A_t \quad (27)$$

   - Goods market:

$$Y_t = C_t + I_t + G_t \quad (28)$$

7. The economy is in the stationary state where $\{X_{s+1} = X_s, \Psi_{s+1} = \Psi_s, \Phi_{s+1} = \Phi_s\}_{s=t}^{\infty}$

2 Calibration

The model is calibrated to the United States' economy and one model period corresponds to one year. Model's parameters are normalized in a way that one model unit equals to \$ 54,100 (national average wage in 2019[8]). Table 1 summarizes the values of parameters used in the model.

Table 1: Parameter summary

Parameter		Value	Source/comment
Demographics			
Survival probabilities	ξ_t^j	see 2.1	Social Security Administration
Maximum age	J	100	
Retirement age	J_R	67	Eligible age for medicare
Labor-augmenting prod. growth	μ	1.5%	Average growth rate of per-capita real GDP
Preference			
Discount factor	β	0.995	Target: capital-output ratio of 3.0
Taste parameter of consumption	ν	0.322	Target: actual working time (OECD)
Age dependent risk aversion			
Risk parameter in future utility	ψ	see 2.2	
Labor productivity			
Age earning profile	o^j	see 2.3	G. D. Hansen (1993)
Intrinsic productivity (education)	θ_s	see 2.3	U.S. Bureau of labor Statistics
Stochastic productivity	η^j		
- autocorrelation	ρ	0.96	Target: the variance of log labor earnings
- variance	σ_ϵ^2	0.04	Target: the variance of log labor earnings
Production and technology			
Income share	α	0.41	U.S. Bureau of labor Statistics
Total factor productivity	Ω	0.89 (CRA) 0.94 (IRA)	Target: wage = 1.0 in 2019
Depreciation rate	δ	8%	Target: interest rate = 4.0% in 2019
Government			
Maximum taxable income	y^s	\$132,000	Social Security Administration (2019)
Social security benefit	pen	see 1.1.2	Social Security Administration
Government spending	G	20% of GDP	Average 1975-2018
Government debt	D	60% of GDP	
Consumption tax rate	τ^c	5.54%	Nation average of retail sales taxes
Progressive income tax rate	Tax	see 1.1.3	Keane and Wasi (2016)

2.1 Demographics

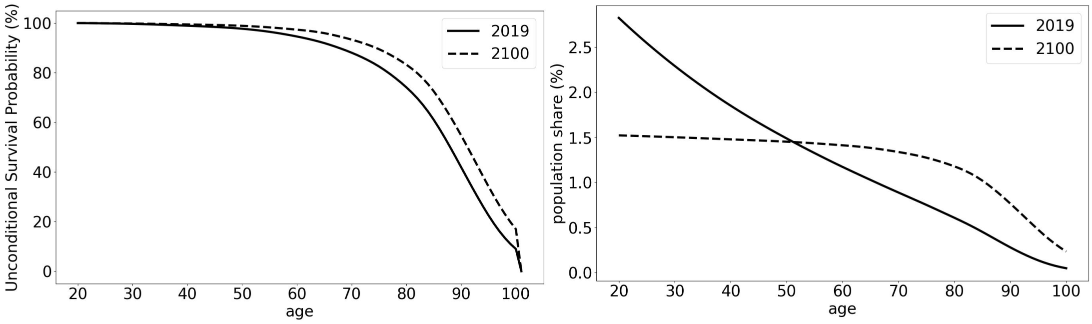
This paper examines two time frames: the benchmark year of 2019[9] and the long run aging scenario of 2100. The assumption is made that demographics will be in a steady state in both periods. Survival probabilities are calculated from the intermediate projection of SSA life table[10]. The population growth rates have been calibrated to match the old-age dependency ratios from the United Nations' medium variant projection of 24.4% in 2019 and 56.5% in 2100. The resulting population growth rates are 2.1% and 0.1% respectively. Figure 1a and 1b show the unconditional survival

[8]Source: the Social Security Administration (SSA)

[9]2019 was chosen to avoid economic irregularities caused by the COVID-19 outbreak that began in 2020.

[10]Life table is from the 2022 annual report of the Board of Trustees of the Federal Old-Age and Survivors Insurance and Federal Disability Insurance Trust Funds. The report contains separate life tables for male and female. The average value is calculated with weights obtained from UN population data.

probabilities and population structures of the two steady states.



(a) Unconditional survival rates

(b) Population share

Figure 1: Population structure

2.2 Preferences

In the CRA scenario, the discount factor β is calibrated to 0.995 in order to achieve a capital-output ratio is 3.0. The consumption preference parameter ν is calibrated to 0.322 to match the average fraction of time spent on working[11]. It is assumed that the IRA scenario uses the same set of preference parameters as the CRA sceario. This allows for a comparison of welfare and behavioral responses between the two cases, with any differences being solely a result of variations in risk aversion. Intuitively, this assumption can be interpreted as individuals being short-sighted towards changes in their own risk aversion in the benchmark year or, alternatively, IRA individuals being a minority in the economy and thus not affecting aggregate variables in the benchmark year.

The primary distinction between the CRA and IRA scenario lies in thier assumption of risk aversion. This paper initially posits assumptions on the values of risk aversion, which are subsequently converted into risk aversion in future utility. In the CRA scenario, the relative risk aversion parameter $\bar{\gamma}$ is set to a commonly used value of 3 in literature. In contrast, the IRA scenario assumes that individuals become increasingly risk averse as they age, in alignment with patterns previously observed in literature. In order to ensure comparability between the two scenarios, relative risk aversion in the IRA scenario is constructed such that its averge values, weighted by age shares, also equals 3. Specifically, the values of age-dependent relative risk aversion are chosen to be linearly increasing with age and satisfy

$$\sum(m^j \cdot \gamma^j) = \bar{\gamma} = 3 \quad \text{and} \quad \tilde{\gamma} = \gamma^{JJ} - \gamma^1 \tag{29}$$

[11] According to the Organization for Economic Co-operation and Development (OECD), US workers spend 32.5% of available time, or 1,777 hours, working ouf of a total of 5,475 hours (15 hours per day, 365 days per year).

where $\tilde{\gamma}$ represents the range of relative risk aversion values of the oldest and the youngest individuals in the economy, with $\tilde{\gamma} = 0$ indicating that individuals of all ages possess the same risk aversion (i.e. it corresponds to the scenario assumed to the CRA case). In order to gain understanding into behavioral differences between the two scenarios, this study arbitrarily assumes $\tilde{\gamma} = 3$ but the conclusion still holds with other positive values of $\tilde{\gamma}$. A range of $\tilde{\gamma} \in [0, 5]$ is then considered when illustrating welfare differences. The values of relative risk aversion are then converted to risk aversion in future utility according to the relationship $\psi^j = -(1-\beta)(1-\gamma^j)$, resulting in a value of $\psi = 0.004$ in the CRA scenario. When assuming $\tilde{\gamma} = 3$, the youngest individuals in the IRA scenario have a value of $\psi = 0.002$ and the oldest individuals have the value of $\psi = 0.008$.

Estimating the exact degree to which older individuals exhibit a higher level of risk aversion compared to the young is beyond the scope of this study. The appendix (see Appendix A) presents one example of how risk aversion values may be estimated based on the experimental findings reported in Albert and Duffy (2012). However, it must be noted that the values obtained are notably high and their applicability in a quantitative framework remains a topic of discussion.

2.3 Productivity

The income share of capital in the Cobb-Douglas production function is 0.41, calculated as 1 minus the 2000-2016 average value of labor income share from the Bureau of Labor Statistics (BLS) (Giandrea and Sprague, 2017). The depreciation rate is assumed to be 8% so that the interest rate in the benchmark year is around 4% in the CRA scenario[12]. The total factor productivity is calibrated to be 0.89 and 0.94 for CRA and IRA scenarios respectively, to normalize the wage rate in each case to 1.0 in the benchmark year.

Table 2: Labor share and relative productivity by education

Education	Share	Relative productivity (High school or less = 1)
High school graduate or less	38%	1.0
Some college or associate degree	26%	1.2
Bachelor's degree and higher	36%	1.9

Labor productivity is determined by 3 components: the age-earning profile, the level of education, and idiosyncratic productivity. The time-invariant age-earning

[12]It is important to note that in the IRA scenario, the interest rate is around 3% in the benchmark year. However, this does not affect the the long run behavioral implications of this study, as the results still hold when applying interest rate of the CRA scenario to the IRA scenario.

profile o^j of working cohorts is based on the estimates of G. D. Hansen (1993) and has been scaled such that average working income of workers in the benchmark year aligns with the actual data. Individuals aged j = 1 are probabilistically assigned to one of three education groups: high school graduate or less, some college or associate degree, and bachelor's degree and higher. The relative education productivity is obtained from BLS, while the share of people in each education group is from the Census Bureau (see Table 2). Lastly, income shock is modeled as a the first-ordered Markov process given by

$$Var[\ln(w_t h^j l^j)] = \sigma_\theta^2 + \sigma_\epsilon^2 \sum_{i=1}^{j} \rho^{2(i-1)}. \tag{30}$$

where the variance of fixed effect σ_θ^2 comes from relative differences in education-specific productivity and the variance σ_ϵ^2 is calibrated to target the variance of log labor earning of 0.3 at the age of 25 and 0.9 at the age of 60, in accordance with the empirical study conducted by Storesletten et al. (2004). Specifically, this requires σ_ϵ^2 of 0.04 and autocorrelation ρ of 0.96 for both risk aversion scenarios. The Rouwenhorst method is used to discretize the Markov process. Given that the shock is idiosyncratic and in accordance with the law of large number, the model is absent of aggregate shock.

2.4 Government

The government spending is assumed to be constant at 20% of the output which is an average rate between the years 2000 to 2019. Additionally, it is assumed that the debt to GDP ratio is maintained at 60% of GDP. The model also assumes a fixed consumption tax rate of 5.54%, which corresponds to the simple average of retail sales taxes across different states in the United States during 2018.

3 Numerical results

This study examines the long run equilibrium implications of population aging. The analysis is organized into four sections. The first section identifies the sources of uncertainty in the model and the ways individuals adjust their behavior in response to these uncertainties. The second and third sections analyze the optimal behavior of CRA and IRA individuals across the life cycle under the baseline scenario and under the three different reform scenarios, respectively. The fourth section evaluates the welfare implications of policy alternatives considered. It is assumed throughout the analysis that all individuals possess perfect foresight and that individuals in the IRA group are cognizant of their increasing risk-averse behavior as they age in the long run.

3.1 Mechanism of hedging against uncertainties across a life cycle

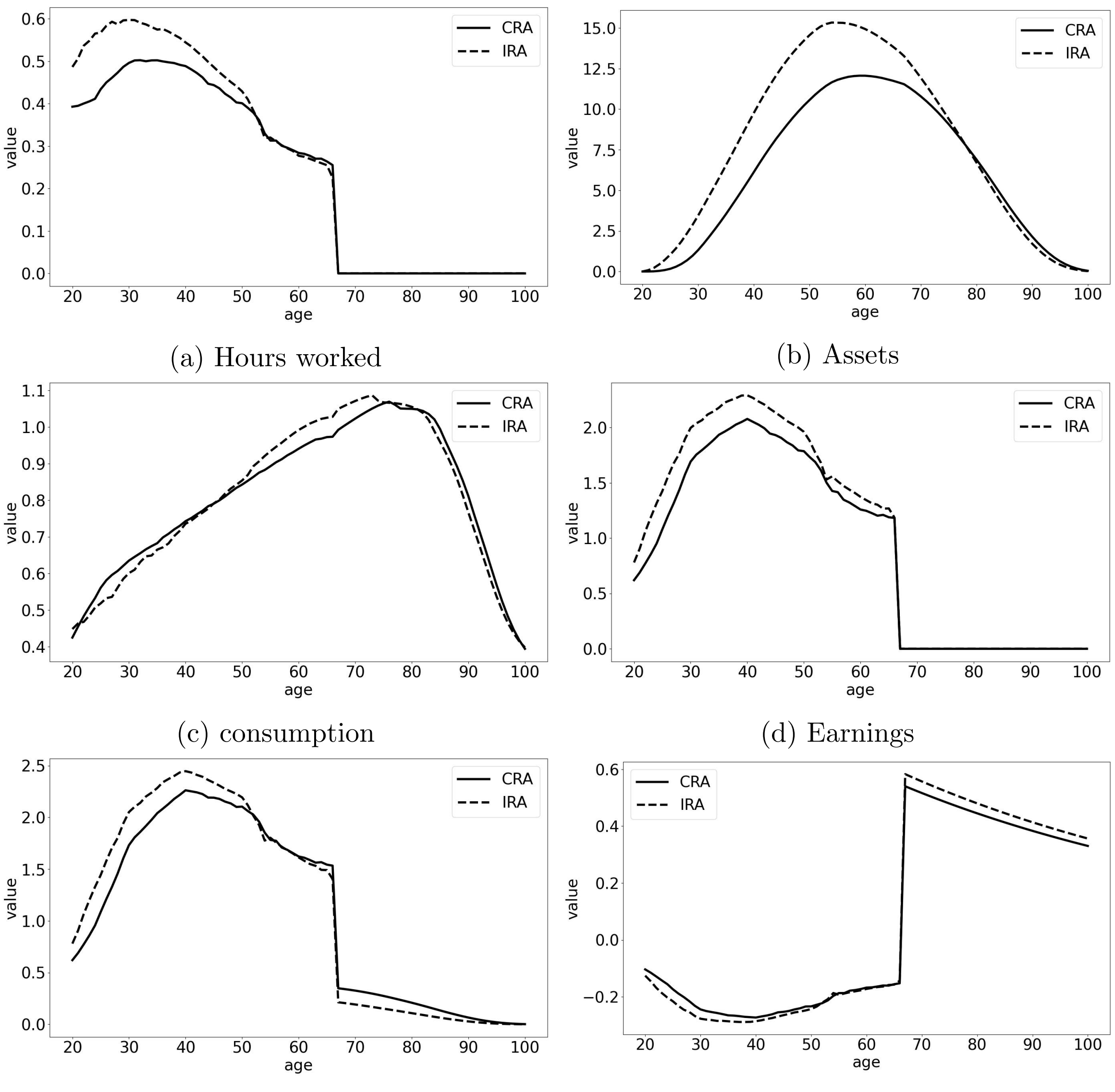
From equation (12), we saw that risk sensitive preferences allow individuals to distinguish and put relative weights between expected value and the variance of next-period value function. Individuals with greater risk aversion, as in the case of the elderly of IRA group in this paper, dislike uncertainties and are willing to leverage available means to keep uncertainties in check.

In the current model, volatility arises from three primary sources. The first source, income shock, is characterized by the income shock process, which capture various event such as unemployment, changes in labor market conditions, and changes in health that can result in unexpectedly low or high income for individuals. The magnitude of these shocks also depends on age preductivity and hours worked. The second source of volatility is pension income, which is calculated based on lifetime earnings history (as described by Equations (2)-(4)). As such, uncertainty surrounding working income gradually translates into uncertainty in pension income, with younger individual experiencing greater uncertainty. Lastly, income volatility also contributes to the uncertainty in level of future savings that individuals can draw upon, especially after retirement. Longer life expectancy leads to higher future uncertainties.

In light of the various sources of uncertainty discussed above, individuals may employ two key strategies to optimally manage the level of expected future uncertainty. The first strategy is precautionary saving, which mitigate the impact of unexpected future events through the accumulation of resources. The second strategy pertains to the adjustment of the number of hours worked in response to fluctuations in earnings, with the goal of maintaining a more consistent level of working income and, through its relationship with earning history, pension income. The degree of risk aversion exhibited by an individual plays a crucial role in determining the extent to which these strategies are employed in response to a certain level of expected uncertainty.

3.2 Long run baseline equilibrium

Absence of fiscal reform, the government budget is not sustainable under projected population aging. A shift in a share of workers towards retirees means that revenues collected from taxes and income of working age population decrease and spending on social security, healthcare and other services increase. Three commonly employed policy alternatives are considered as means to address the fiscal gap. In each scenario, only one policy variable is adjusted to balance the government budget, holding all othe variables constant at their values in the benchmark year. The government spending is assumed to remain at 20% of GDP and debt level at 60% of GDP across all scenarios. The first policy option, baseline case, posits that the government proportionally increases the payroll tax rate to sufficiently cover the increase in pension spending. The second option proposes to inearly scale down the social security benefits while maintaining the payroll tax rate at the current level. The third option extends the retirement age to increase payroll tax revenue and curb pension expenses. The aim of these policy adjustments is to achieve a balanced government budget across all three scenarios. This section will focus on the baseline scenario and alternatives scenarios will be analyzed in the following section.



(a) Hours worked

(b) Assets

(c) consumption

(d) Earnings

(e) Income

(f) Pension

Figure 2: Life cycle behaviors of individuals

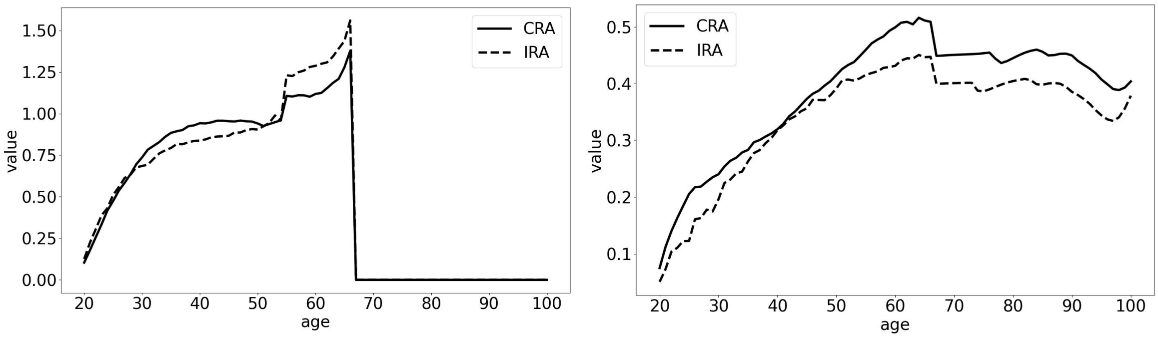
Under the baseline scenario, the payroll tax has to increase for the CRA (IRA) scenario by around 9% from 7.6% (7.2%) in 2019 to 16.7% (16.1%) in 2100 to support the increase in social security benefit spending. Figure 2 compares life cycle behaviors between CRA (solid lines) and IRA (dashed lines) individuals in the long run.

It can been seen from Fig 2 that IRA individuals tend to work longer than the CRA counterpart. This is partly to accumulate more precautionary savings (Fig 2b) and to ascertain the level of pension income early on. In addition, we can see that IRA individuals has lower income volatility (3a) despite more hours worked. Since both groups are subject to the same income shock process and quite similar wage rates, the result suggests that the IRA group adjusts the number of hours worked to smooth out earning shocks – working more under bad shocks and vice versa. Another observed pattern is that high labor supply and low income volatility are concentrated during the first 35 years of work, coincides with the maximum period during which pension benefit is determined. IRA individuals have tendency to hedge against pension uncertainty early on.

The analysis of consumption patterns among CRA and IRA individuals yield interesting finding. Despite IRA group exhibited higher level of lifetime income, consumption level were found to be similar between the two groups. Figure 3 indicates that model-simulated variances of log consumption were lower for both CRA and IRA groups in comparison to the variance of log labor earnings. This suggests that inidividuals in both groups utilize precautionary savings to smooth out consumption over varying states of productivity. However, consumption variances of the IRA group were found to be throughout the life cycle. This can be attributed to the fact that IRA individuals, who have higher concern about uncertainties during old age, tend to allocate a greater proportion of earnings towards savings for future consumption and rely less on uncertain working or pension income. However, it is important to recognize that the act of hedging against future uncertainty is not without cost. Additional resources used for hedging purposes results in a reduction of resources available for optimizing the lifetime consumption and leisure, hence a similar observed consumption between CRA and IRA individuals despite higher lifetime income of the IRA. In other terms, there is a trade-off between the level and volatility of lifetime consumption and leisure.

3.3 Alternative reform scenarios

This section compares the results in the baseline scenario to alternative reforms: benefit reductions and retirement extension.



(a) Volatility of income

(b) Volatility of consumption

Figure 3: Volatility over life cycle

Benefit reduction

% change from baseline	Workers		Retirees	
	CRA	IRA	CRA	IRA
Hours worked	7.2	7.8	N/A	N/A
Assets	6.4	6.9	21.1	27.1
Consumption	13.0	10.4	-14.8	-17.5
Earnings	11.4	11.9	N/A	N/A
Volatility: Income	-2.0	2.5	N/A	N/A
Volatility: Consumption	0.3	-1.1	6.9	6.5

Retirement extension

% change from baseline	Workers		Retirees	
	CRA	IRA	CRA	IRA
Hours worked	-4.4	-4.6	N/A	N/A
Assets	21.0	20.0	2.1	6.6
Consumption	10.3	7.8	15.3	16.6
Earnings	-2.1	-2.3	N/A	N/A
Volatility: Income	4.2	3.8	N/A	N/A
Volatility: Consumption	-2.4	-2.7	7.8	13.1

Table 3: Percentage deviations of life-cycle behavior from the case of contribution rate increase

3.3.1 Benefits reduction

The table 3 presents the deviations of life-cycle variables from the baseline scenario of a payroll tax increase. The individuals are seperated into two groups, workers and retirees, and the deviations are shown for both constant and increasing risk averion assumptions. The top part shows the deviation under the benefit reduction scenario, while the bottom part presents the deviations under the retirement extension scenario. Figures 5 and 6 in the appendix illustrates life-cycle behaviors of the three scenarios under constant and risk aversion assumption.

In the context of a reduction in social security benefits, the government needs to proportionately scale down the benefits calculated using AIME formula by approximately 50% in an aging economy. Compared to the baseline, the result shows higher hours worked which can be explain by three factors. First, as individuals anticipate lower benefits, they will need to become more self-reliant for their retirement spending, which nessitates working more hours to accumulate savings. Secondly, as pension benefits are calculated based on the average income, working more hours not only result in immediate increase in wages, but also in higher retirement benefits. Lowering benefits, therefore, may discourage work to some extent. Thirdly, the payroll tax rate is not as high as it is in the baseline scenario, resulting in higher net wage. This creates a positive income effect that requires working fewer hours to achieve the same working income. The first effect predominates, resulting in a significant increase in labor supply. Disposable income also increases concurrently with the increase in working hours.

In comparison to the baseline scenario, the consumption of retirees is lower as a result of reduced benefits, while the consumption of workers is higher due to the decreased payroll tax and an increase in the number of hours worked. In contrast to the change in its level, the volatility of consumption increases for retirees. Pension serves as a means to mitigate future uncertainty, and its reduction leaves retirees more exposed to potential risks. Furthermore, an increase in the number of working hours and greater income volatility during one's working years also lead to greater savings volatility, which in turn exacerbates the volatility of consumption during retirement.

When comparing the two groups of individuals, it is evident that the IRA group experiences a slightly greater decline in the level of consumption despite working more hours than the CRA group. A partial absence of pension channel to manage earning risk during retirement causes IRA individuals, who seek greater certainty as they age, to keep the consumption volatility relatively close to prior low levels under the baseline scenario. However, this comes at a cost, as reflected by the lower amount of leisure and consumption. It is important to note that although the consumption volality under the IRA scenario deviates more from the baseline than in the CRA scenario, the volatility is still relative lower in absolute terms.

3.3.2 Retirement extension

The extension of the retirement age alleviates the social security deficit associated with an aging population through two mechanisms: by extending the working periods during which the payroll tax revenue is collected and by shortening the periods during which social security benefits are paid. The proposed model calls for individuals to delay their retirement and postpone receipt of pension benefits untill the age of 81. An increase in period of time spent in the labor force leads to a greater accumulation of assets and, subsequently, higher level of consumptions due to the availability of more lifetime resources.

Compared to the baseline, the volatility of consumption is found to increase but for reasons distinct from those associated with a reduction in benefits. Specifically, the postponement of retirement exposes individuals to the stochastic income shock for a longer duration, thereby increasing lifetime uncertainty. The extension of period spent working results in a corresponding extension of period of volatile income and consumption. Additionally, this also amplifies the uncertainty regarding potential future savings. As previously observed, individuals with increasing risk aversion place a significant emphasis on managing uncertainty and are willing to incur higher cost in order to mitigate it, as evidenced by the lower levels consumption in comparison to those with constant risk aversion.

3.4 Welfare analysis

This section evaluates individual's welfare when the government adopts different fiscal reforms. The objective is to understand the differences in welfare of individuals with constant and with age-increasing risk aversion across the three scenarios. Instead of one IRA specification, I show welfare across different ranges of IRA (from 1 to 5). This study employs the method of Hicksian Equivalent Variation (HEV) which asks by what percentage the levels of consumption and leisure in the baseline scenario of the new entrants have to change so that they are equally well-off as those born under alternative reforms[13].

[13]Specifically, HEV is applied to the risk sensitive preference specification. Let Δ be a percentage by which consumption and leisure of a new entrant change that satisfies $V_{RS} = V(\Delta)$, where V_{RS} is the value function under the reform scenario and $V(\Delta)$ is the value function under the baseline scenario of which consumption and leisure across all states are scaled by $(1 + \Delta)$. Note that there is no analytical solution to finding the value of Δ as the preference specification is non-additive. Instead, this study employs the Newton method to solve for the value of Δ, given the preference specification that takes into account lifetime uncertainties.

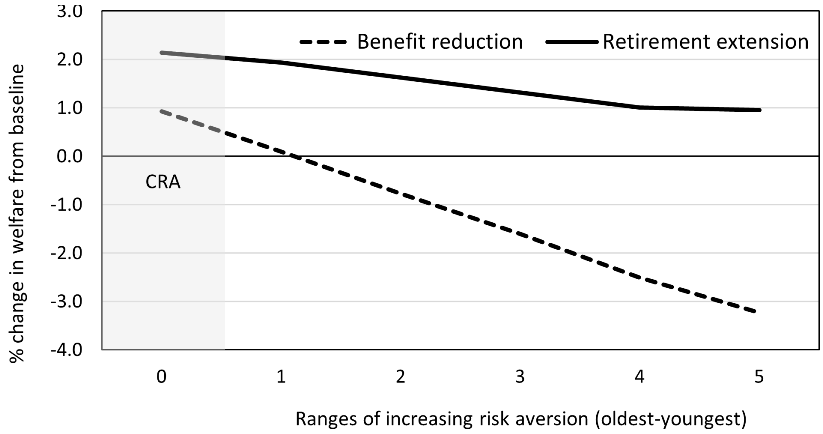


Figure 4: Welfare change from the case of contribution rate increase

Specifically, increases in levels of consumption and leisure caused by the interactions between population and policy feedback help improve welfare and vice versa. However, with the risk-sensitive preferences, the magnitude of changes in future uncertainties also plays an important role, with higher expected uncertainties lowering the overall welfare to the extent depending on how highly risk averse an individual is at the time[14]. Individuals make optimal decisions fully aware of their relative preference between the level and volatility of utility which results in levels of consumption, leisure, and remaining uncertainty.

In comparison to the case of contribution rate increase, CRA individuals experience welfare improvement by 0.9% and 2.1% when the government cuts benefits or extends the retirement age respectively. Improvement in consumption under both scenarios increases welfare although it is somewhat reduced by a lower level of leisure. The findings under the assumption of CRA here are consistent with the studies done by Kitao (2014) and Nishiyama (2015). For the case of IRA, however, we can see the increasing cost of induced uncertainties from a reduction in pension benefits and an extension of retirement age.

Figure 4 plots welfare of these two reforms. Values on the x-axis shows ranges of increasing risk aversion – how much more risk averse the oldest persons are compared to the youngest[15] and values of the y-axis shows how welfare differ from the case of contribution rate increase. When compared reforms to the baseline scenario, it can be observed that welfare deteriorates as individuals become more risk averse as they age. Individuals who expect themselves to be risk averse in the future will be willing to put more resources for the purpose of controlling the level of exposed risk, in turn

[14]Note that both CRA and IRA scenarios use the same risk sensitive preferences and therefore incorporate the effects of changes in uncertainties. The welfare differences in this section are therefore due to the degrees of risk aversion across ages.

[15]For all cases, values of risk aversion are constructed in such a way that the average risk aversion are the same as under the CRA case (see (29)).

sacrificing consumption and leisure. Cutting social security benefits and extending retirement age – both increases level of uncertainty – may not be as strongly preferred to increasing payroll tax as prior studies have suggested. In fact, some people who grow highly risk aversed when aged may be even worse off.

4 Conclusion

The level of risk aversion plays an important role in shaping individuals' behaviors and welfare, and literature indicates that individuals tend to be increasingly risk averse as they age. Additionally, individuals are known to consider future uncertainties when making decision. This study addresses these two aspects by incorporating two elements into an overlapping generations model. Firstly, it allows for age-dependent risk aversion in future utilities, such that individuals become more risk averse as they age. Secondly, it introduces a risk-sensitive preference specification in which certainty equivalence takes into account both the level and volatility of future utilities. The level of risk aversion at different ages can be thought of as the extent to which volatility is penalized.

The proposed model compares two types of individuals: one with a constant level of risk aversion and another with gradually increasingly risk aversion as they age. The model evaluates the long-term effects of an aging baseline scenario, where the social security contribution rate is increased, and two alternative reforms: reducing benefits and extending the retirement age. When compared to individuals with constant risk aversion, it can be observed that those with increasingly risk aversion tend to accumulate more precautionary savings and adjust working hours to absorb instantaneous income shocks, reducing uncertainty in their lifetime consumption. However, mitigating risk is costly as it diverts resources that could have been used to increase overall consumption of goods and leisure.

We can observe differences in behavioral responses of two types of individuals under the impact of two reforms. As both reforms increase the income uncertainties during old age, they are more costly for individuals with increasing risk aversion as they allocate more resources towards heding against risk. The welfare outcomes for individuals with constant risk aversion are consistent with prior literature, which suggests that reducing social security benefits and extending the retirement age result in better social welfare compared to increasing payroll tax rate. However, as individuals become more risk averse with age, the superior welfare results under reform scenarios become less apparent. In fact, for individuals who expect themselves to be highly risk averse when old, they may prefer the increase in payroll tax rate over the two reform scenarios.

The present study serves as a preliminary step to improve our understanding of policy reform, particularly with regards to the impact of varying levels of risk aversion and how uncertainties affect individuals' behavior and welfare. An interesting extension to this model would be to incorporate a risky rate of return on capital. Further analysis could also benefit from an empirical estimation of age-dependent risk aversion in future utility. However, this will be left for future research.

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Appendix

A An example of estimating values of age-dependent of risk aversion

One possible way to estimate values of risk aversion is to utilize the findings from experimental economics. As an example, this section refers to the work by Albert and Duffy (2012) who apply widely-used lottery choice menu proposed by Holt and Laury (2002) to two age groups[16]. In such experiment, participants make 10 decisions, each consists of choosing between paired lottery choices: one with low variance (option A) and another with high variance (option B). Lottery A gives payoffs of either \$16.00 or \$20.00 and lottery B gives either \$1.00 or \$38.50. The probability of earning high payoff in each lottery choice increases from 0.1 to 1.0 in 0.10 increments in each decision. The higher tendency of choosing a safe lottery choice when the chance of getting high payoff is low indicates increasing risk aversion.

Given the experiment results, we can make two assumptions in order to calculate risk aversion values. First, individual's preferences towards the next period value function are assumed to behave in the same manner as towards monetary pay-offs from the paired lottery experiment. Second, certainty equivalence of experiment participants is assumed to follow

$$\frac{1}{\psi} \ln \mathbb{E}_t(e^{-\psi V^+}). \tag{31}$$

The cut-off values of risk aversion can then be calculated to characterise participants who switch from option A to B under certain decisions (Table 4).

The values of risk aversion parameter of young and old adults can then be calculated by using the proportion of safe choices in each of the 10 decisions together with the value of corresponding cut-off points[17]. With this method, the values of risk aversion in future utility are 0.116 for adults aged 32 and 0.231 for adults aged 71.

However, with this approach, the estimated values of risk aversion are quite high compared to the value used in the standard macroeconomic literature. When converting back into relative risk aversion according to the relationship $\psi^j = -(1-\beta)(1-\gamma^j)$ and given he calibrated values of $\beta = 0.995$, relative risk aversion takes values of 24 and 47 for individuals aged 32 and 71 respectively.

[16] In their study, Holt and Laury (2002) estimate risk aversion with CRRA utility function and a hybrid 'power-expo' utility function.

[17] The median values between cut-off points are used in the calculation. For example, $(-0.063 - 0.0158)/2 = -0.039$ is used for individual who choose safe lottery in the first decision but choose a risky lottery in the second decision.

Table 4: Paired lottery experiment

Decision	Prob. of high payoff	Choose option B if	Proportion	
			Young	Old
1	1/10	$\psi < -0.063$	0.000	0.000
2	2/10	$-0.063 < \psi < -0.0158$	0.110	0.030
3	3/10	$-0.0158 < \psi < 0.0174$	0.000	0.050
4	4/10	$0.0174 < \psi < 0.0460$	0.050	0.010
5	5/10	$0.0460 < \psi < 0.0735$	0.120	0.070
6	6/10	$0.0735 < \psi < 0.1022$	0.220	0.070
7	7/10	$0.1022 < \psi < 0.1350$	0.200	0.090
8	8/10	$0.1350 < \psi < 0.1772$	0.110	0.040
9	9/10	$0.1772 < \psi < 0.2448$	0.110	0.160
10	10/10	$\psi > 0.2448$	0.080	0.480

In decision group 10, I assume the probability of high payoff to be 0.99 instead of 1.0 to be able to calculate corresponding ψ value. This results in corresponding range $0.2448 < \psi < 0.4658$.

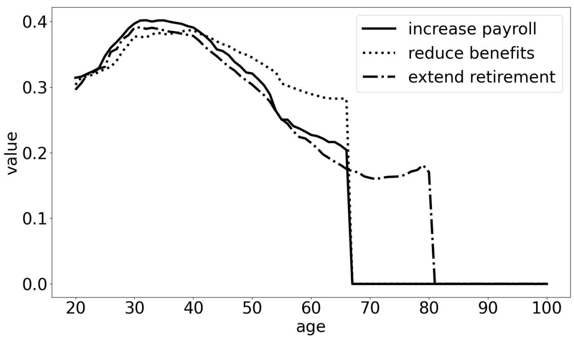
B Computation

The solution method discretises individual states $\{a, s, \eta, e\}$ to simplify the nonlinear dynamic programming problem. Macroeconomic solutions are solved with a Gauss-Seidel procedure by Auerbach and Kotlikoff (1987) and individual policy functions are solved backwards with a value function iteration approach using a minimization routine and interpolation algorithms. The computation steps for the stationary equilibrium can be summarized below

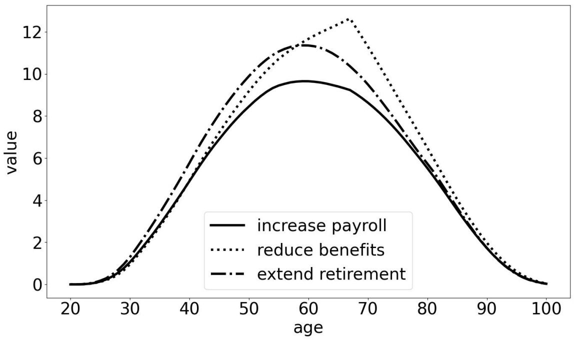
1. Initialize parameters and discretize state space.
2. Calculate price variables according to (21) and (22).
3. Use a value function iteration approach and interpolation algorithm to solve for policy functions $a'(x_t)$, $c(x_t)$ and $l(x_t)$ that solve the individual's dynamic programming problem represented in (13)-(16).
4. Use policy function $a'(x_t)$ together with the accumulation of earning points and the transition probability of earning shocks to solve for individual distributions over state space (Equation 18).
5. Calculate age-specific variables and aggregate variables consistent with policy functions and distribution.
6. Update the government policy schedules.
7. Calculate social security benefits according to Equation (4).

8. Update price variables and iterate from step 2 until all markets clear.

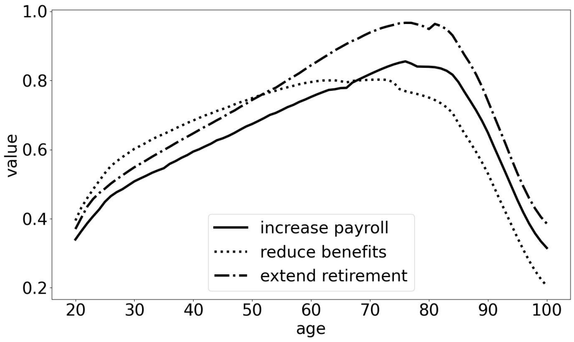
C Other figures



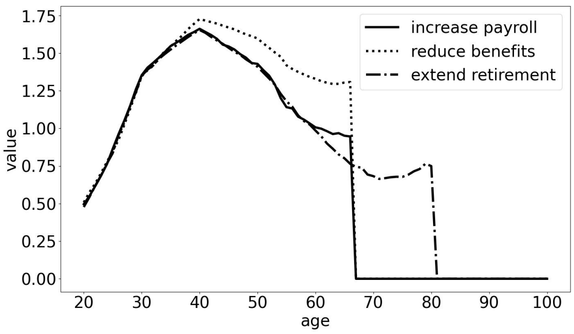
(a) Hours worked



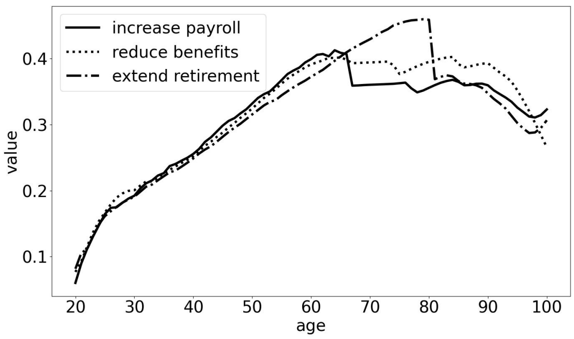
(b) Assets



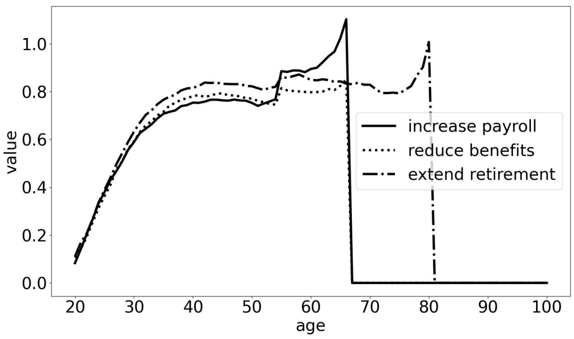
(c) consumption



(d) Earnings

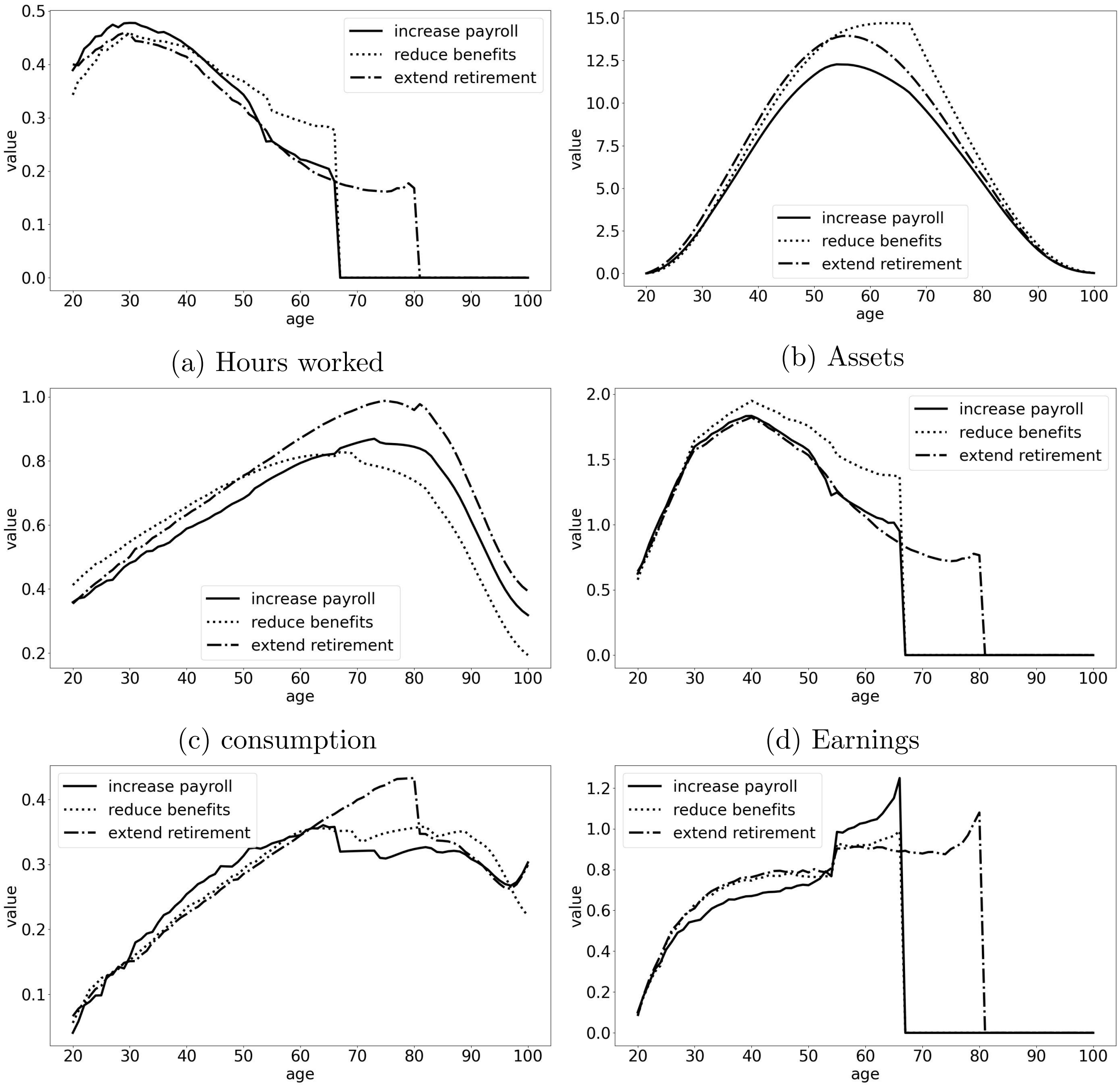


(e) consumption



(f) Earnings

Figure 5: Life cycle behaviors of individuals with constant risk aversion across long run scenarios



(a) Hours worked (b) Assets

(c) consumption (d) Earnings

(e) consumption (f) Earnings

Figure 6: Life cycle behaviors of individuals with increasing risk aversion across long run scenarios