Risk Attitudes in Rent-seeking Contests and Fundraising Lotteries

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I. Introduction

What is a rent-seeking contest?

- A game in which players compete to win a prize (or rent).
- Player i's strategy is to invest x_i to increase the probability of winning the prize.
- Lottery contests VS all-pay auctions

Applications: R&D race, military conflict, litigation, sports

I. Introduction

Gordon Tullock's lottery contest (1980)

- n risk-neutral players with initial income I_i for player i.
- R is the Prize (or rent).
- x_i is player i's rent-seeking investment.
- Player i's objective:

$$
\begin{aligned}
EU_i &= p_i(I_i - x_i + R) + (1 - p_i)(I_i - x_i) \\
&= I_i - x_i + p_i R \\
&= I_i - x_i + \left(\frac{x_i}{\sum_{j=1}^{n} x_j} \right) R
\end{aligned}
$$

I. Introduction

Well-known theoretical predictions:

- Symmetric Nash equilibrium yields optimal investment:

$$x_i^e = \frac{(n-1)R}{n^2}.$$

- Aggregate investment: $X^e = nx^e = \frac{(n-1)R}{n} < R$.
- Rent over-dissipation ($\sum_{i=1}^n x_i > R$) never occurs.
- The contest organizer's profit must be negative.

I. Introduction

Well-known experimental results:

- Recent survey on the experimental evidence of contests
- Most subjects demonstrate **over-investment** ($x_i > x_i^e$).
- **Over-dissipation** is also observed in simultaneous contests with large n.
- Lack of theoretical support for rent over-investment or over-dissipation

I. Introduction

Assume that the rent R is small. Using Taylor's approximation, we can rewrite the equilibrium level of x as

$$x^e \approx \frac{(n-1)R}{n^2}\left[\frac{1+d(\bar{z})\frac{R^2}{24}}{1+r(\bar{z})\frac{(n-2)R}{2n}+d(\bar{z})\frac{R^2}{8}}\right],$$

where

- r is the Arrow-Pratt absolute measure of risk aversion
$$r(\bar{z}) = -u''(\bar{z})/u'(\bar{z})$$
- d is the local measure of downside-risk aversion
$$d(\bar{z}) = u'''(\bar{z})/u'(\bar{z})$$
- $\bar{z}$ is the mid point of the interval $[I-x, I-x+R]$

I. Introduction

Prudence vs downside-risk-aversion measures

- Kimball (1990): Absolute measure of prudence

$$-u'''(x)/u''(x)$$

- Modica and Scarsini (2005): Local downside-risk aversion

$$u'''(x)/u'(x)$$

I. Introduction

Can we find a class of utility functions that support the findings in the lab?

We need utility functions that...

- guarantee that an equilibrium exists
- allow for risk neutrality, risk aversion, and risk lovingness
- are simple enough to derive analytical solutions

I. Introduction

What we do...

- Allow for heterogeneity in risk attitudes
- Derive an equilibrium and prove that it is unique
- Allow for sequential moves (not inlcuded in this presentation)
- Derive optimal prize for the lottery contest organizer
- Find optimal prize when the lottery organizer is a public good provider

II. Rent-Seeking Contests

Assumption 1

$$u_i(w) = \begin{cases} \theta_i e^{\theta_i \alpha_i w} & \text{if } \theta_i \neq 0 \\ w & \text{if } \theta_i = 0 \end{cases}$$

where $\theta_i \in \{-1, 0, 1\}$ *and* $\alpha_i \in (0, \infty)$.

- Generalized CARA utility function

θ_i	$\theta_i\alpha_i$	u_i'	u_i''	u_i'''	$-\frac{u''}{u'}$	$\frac{u'''}{u'}$
-1	$-\alpha_i$	$+$	$-$	$+$	α_i	α_i^2
0	0	$+$	0	0	0	0
1	α_i	$+$	$+$	$+$	$-\alpha_i$	α_i^2

II. Rent-Seeking Contests

Player i's maximization problem:

$$\max_{x_i} EU_i = p_i u_i(I_i - x_i + R) + (1 - p_i) u_i(I_i - x_i)$$
$$= \left(\frac{x_i}{\sum_{j=1}^{n} x_j} \right) u_i(I_i - x_i + R) + \left(1 - \frac{x_i}{\sum_{j=1}^{n} x_j} \right) u_i(I_i - x_i)$$

II. Rent-Seeking Contests

Proposition 1

If Assumption 1 holds, there exists a unique Nash equilibrium in the contest.

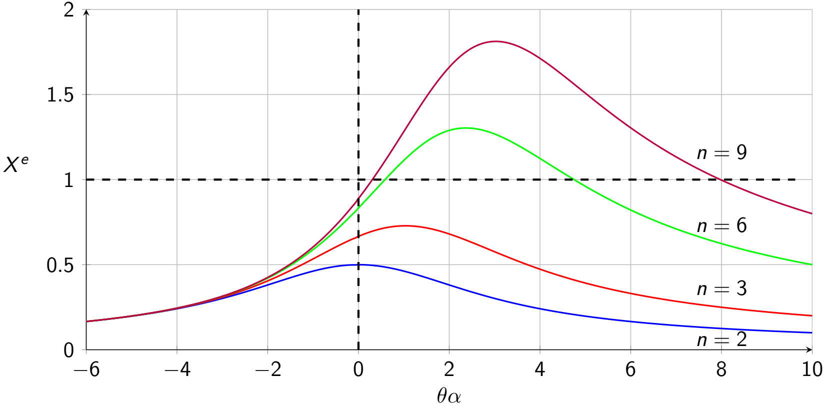
Equilibrium uniquely exists even when each player's contest success function has the form

$$p_i = \frac{f_i(x_i)}{\sum_{j=1}^{n} f_j(x_j)}$$

where $f_i(0) = 0$, $f_i'(x) > 0$, and $f_i''(x) \leq 0$ for all $i = 1, \dots, n$.

II. Rent-Seeking Contests

- Suppose that $R = 1$, $\theta_i = \theta$, and $\alpha_i = \alpha$ for $i = 1, ..., n$.
- Plot of aggregate investment in symmetric equilibrium:



II. Rent-Seeking Contests

Proposition 2

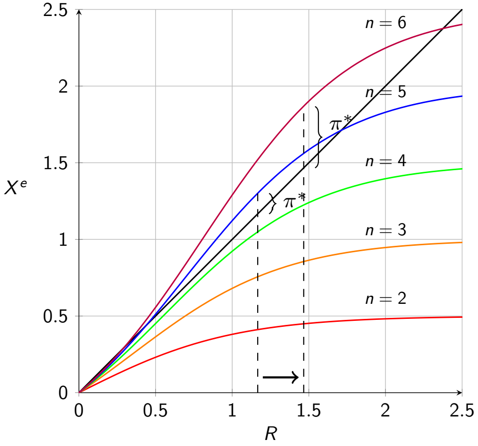
Consider a simultaneous contest with homogeneous players, i.e., $\theta_i = \theta$ and $\alpha_i = \alpha$ for $i = 1, ..., n$

1. *If $\theta \neq 1$ or $n \leq 4$, then $X^e < R$.*
2. *If $\theta = 1$ and n is large enough, then $X^e > R$.*
3. *If $\theta = 1$ and $n \geq 5$, there exists R such that $X^e > R$.*

- $n \leq 4$ is sufficient but not necessary for rent under-dissipation.
- $n \geq 5$ is necessary but not sufficient for rent over-dissipation.

II. Rent-Seeking Contests

- Suppose that $\theta_i = 1$ and $\alpha_i = 2$ for $i = 1, ..., n$.
- Plot of aggregate investment in symmetric equilibrium:



II. Rent-Seeking Contests

Proposition 3

Consider a contest with $n \geq 5$ homogeneous players with $\theta = 1$. We find that

$$R^* = \frac{1}{\alpha}\left[\ln\left(\frac{n-1}{2}\right) + \ln\left(n - 2 + \sqrt{n^2 - 4n}\right)\right]$$

maximizes the contest organizer's profit.

III. Fundraising Lottery Design

Morgan (2000)

- Financing public goods with lotteries
- Quasi-linear utility with risk neutrality in wealth: $u(w, G) = w + h(G)$
- Public good provision: $G = \max\{X^e - R, 0\}$
- Dominates the voluntary contribution mechanism (VCM)

III. Fundraising lottery design

Our model:

Table: Four Types of Players

	$\theta = 1$	$\theta = -1$
$\beta = 0$	risk-loving gambler	risk-averse gambler
$\beta > 0$	risk-loving beneficiary	risk-averse beneficiary

III. Fundraising Lottery Design

Assumption 2

Player i's utility function is given by

$$u_i(w, G) = \theta_i e^{\theta_i(\alpha_i w + \beta_i G)}$$

where w is his final wealth, G is the provision of the public good, $\theta_i \in \{-1, 1\}$, *and* $\alpha_i > \beta_i \geq 0$.

- Lange, List, and Price's (2007) utility:

$$u_i(w, G) = -e^{-(\alpha w + \beta G)}$$

III. Fundraising Lottery Design

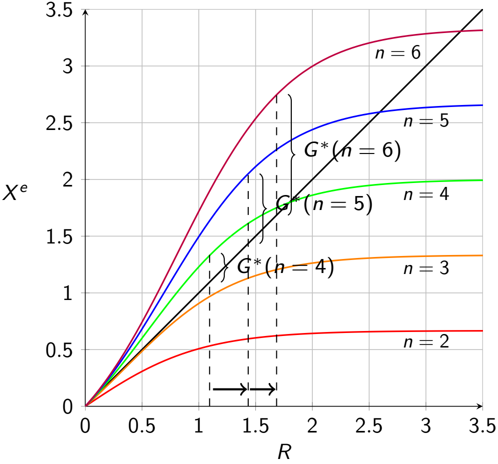
Proposition 4

Suppose that Assumption 2 holds. Let n^+ denote the number of risk-loving players. If n^+ is large enough or β_i is large enough for some i, there exists a unique equilibrium such that $G = X^e - R > 0$.

- It is possible that some risk-averse beneficiaries are free riders in equilibrium.

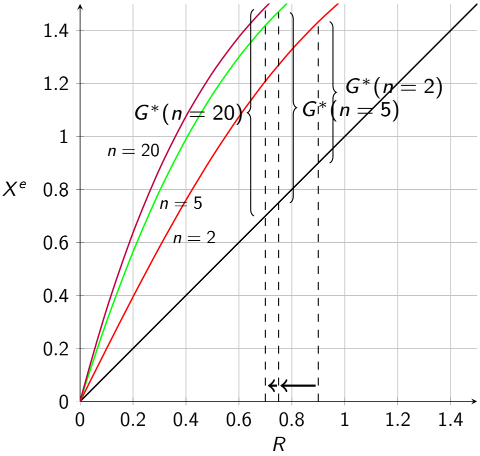
III. Fundraising Lottery Design

- Homogeneous risk-loving players ($\alpha = 2$, and $\beta = \frac{1}{2}$).



III. Fundraising Lottery Design

- Homogeneous risk-averse players ($\alpha = 2$, and $\beta = \frac{3}{2}$).



III. Fundraising Lottery Design

Proposition 5

Suppose that Assumption 2 holds with $\theta_i = \theta$, $\alpha_i = \alpha$, and $\beta_i = \beta$ for all i. If that the lottery organizer chooses R^* so the corresponding public good provided, G^*, is maximized, then

$$\lim_{n\to\infty} R^* = \begin{cases} \infty & \text{if } \theta = 1 \\ \frac{1}{\alpha}\ln\left(\frac{\alpha}{\alpha-\beta}\right) & \text{if } \theta = -1 \end{cases}$$

$$\lim_{n\to\infty} G^* = \begin{cases} \infty & \text{if } \theta = 1 \\ \frac{1}{\alpha}\left[\frac{\beta}{\alpha-\beta} - \ln\left(\frac{\alpha}{\alpha-\beta}\right)\right] & \text{if } \theta = -1 \end{cases}$$

III. Fundraising Lottery Design

- So R^* is our optimal prize.
- Do we really want to maximize G?
- What if we try to maximize EU?

III. Fundraising Lottery Design

Proposition 6

Suppose that Assumption 1 holds with $\theta_i = \theta$, $\alpha_i = \alpha$, and $\beta_i = \beta$ for all i. Suppose that there exist R^* that maximizes G and $\hat{R}$ that maximizes EU.

1. *If $\theta = 1$, then $\hat{R} = \infty$ for all n.*
2. *If $\theta = -1$, then $R^* - \hat{R} > 0$ for all n.*
3. *If $\theta = -1$, then $\lim_{n\to\infty} \left(R^* - \hat{R}\right) = 0$.*

IV. Concluding Remarks

Summary
-Private Lottery

- There is a unique Nash equilibrium in a contest given heterogeneous players (risk-averse/neutral/loving).
- The contest organizer can make a positive profit if the number of risk-loving players is large enough.
- We derive the profit-maximizing prize given a set of homogeneous risk-loving participants.

IV. Concluding Remarks

Summary
-Government Lottery

- There is a unique Nash equilibrium in a contest given heterogeneous risk averters and risk lovers who may or may not benefit from the provided public goods.
- If there are two types: risk-averse beneficiaries and risk-loving gamblers, the benefaciaries may free ride in equilibrium.
- We derive the optimal prize (maximizing public good provision) given a set of homogeneous beneficiaries.

IV. Concluding Remarks

Summary
-Government Lottery given a set of homogeneous beneficiaries

- The optimal prize depends on n, θ, and α.
- If $\theta = 1$, the optimal prize increases with n.
- If $\theta = -1$, the optimal prize may or may not increase with n.
- If $\theta = -1$, the optimal prize is larger than the prize that maximizes the beneficiaries' welfare.

IV. Concluding Remarks

Future research directions

- Unknown types of other players
- Multiple prizes
- Laboratory experiments
- Case Studies:
  -U.S. Lottery: single system for (almost) all states
  -Thailand: Thai Government Lottery, GSB, BAAC

IV. Concluding Remarks

References

- Kimball, M.S., 1990, Precautionary Saving in the Small and in the Large, *Econometrica*, **58**: 53-73.
- Lange, A., J. A. List, and M. K. Price, 2007, Using Lotteries to Finance Public Goods: Theory and Experimental Evidence, *International Economic Review*, **48**: 901-927.
- Modica, S. and M. Scarsini, 2005, A Note on Comparative Downside Risk Aversion, *Journal of Economic Theory*, **122**: 267-271.
- Morgan, J., 2000, Financing Public Goods by Means of Lotteries, *Review of Economic Studies*, **67**: 761-784.
- Tullock, G., 1980, Efficient Rent-Seeking, In Buchanan, J. M., Tollison, R. D., and Tullock, G. (Eds.), *Toward a Theory of the Rent-Seeking Society*, 97-112, College Station: Texas A & M University Press.