Skewness in Expected Macro Fundamentals and the Predictability of Equity Returns: Evidence and Theory

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Introduction

- Long-Run Risks Model: time-varying expected growth rate
  $\rightarrow$ This paper: we look at the cross-section of analysts' forecasts
- At each point in time we look at:
  - Mean of all forecasts
  - Volatility of all forecasts
  - Skewness of all forecasts
- We find:
  1. Evidence of persistence for all the moments
  2. Skewness predicts future Mean
- Questions
  1. How much larger is the premium to compensate for the risk of time-varying moments of the distribution of expected GDP forecasts?
     $\rightarrow$ Bansal and Yaron (2004), Bansal, Kiku, Shaliastovich, and Yaron (2012) look at time varying means and variances
  2. What is the use of this information for forecasting stock market returns?
     $\rightarrow$ Campbell and Diebold (2009) look at first two moments

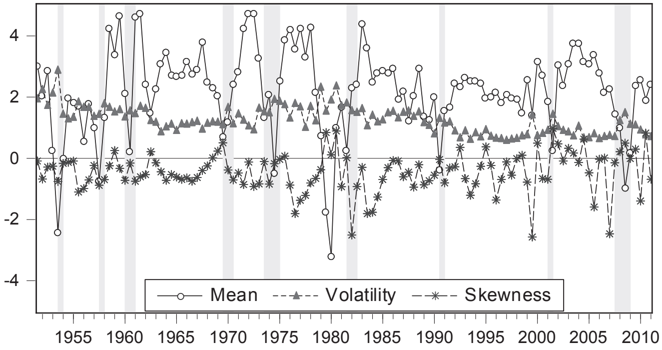
Battle plan

1. The Empirical Evidence
   - A look at the data
   - Time series properties of the cross-section of expected GDP growth
   - Predictive Regression
2. Models
   1. A model with time-varying mean and skewness
   2. A model with time-varying volatility and skewness
3. Comparison with other models .
   - How large is the skewness premium?
   - How different of the dynamic of the conditional skewness of expectant consumption growth?

Data on Expected Real GDP/GNP growth rates

1. Livingston Survey:
   - Time series size: forecasts from 06/1946 to 06/2011, twice per year;
   - Forecast horizon: 6 months and 12 months from now;
   - Cross-sectional size: 19-50+ economists in each period, from 11 sectors (e.g., industry, government, banking, academia, etc).
2. Blue Chip Economic Indicators:
   - Time series size: forecasts from 09/1984 to 06/2011, every month;
   - Forecast horizon: 1, 2, up to 6 quarters ahead;
   - Cross-sectional size: 40-50 economists in each period.

Moments of Expected GDP Forecasts



Transition dynamics of conditional moments

	Mean	Volatility	Third Moment$^{1/3}$
Lagged Mean	0.496 [0.070]	—	—
Lagged Volatility	—	0.886 [0.058]	—
Lagged Third Moment$^{1/3}$	—	—	0.329 [0.077]

Transition dynamics of conditional moments

	Mean	Volatility	Third Moment$^{1/3}$
Lagged Mean	0.480 [0.056]	-0.038 [0.019]	-0.094 [0.055]
Lagged Volatility	0.183 [0.785]	0.818 [0.052]	-0.258 [-0.164]
Lagged Third Moment$^{1/3}$	0.302 [0.093]	-0.085 [0.026]	0.275 [0.068]

Correlation between Predictors

	$V[\cdot]$	$S[\cdot]$	cay	default	term.	DP	VIX^2	VRP	RV	fear
E[·]	0.352	-0.148	-0.044	-0.254	0.247	-0.263	-0.649	-0.206	-0.583	-0.237
V[·]		-0.076	-0.122	0.109	-0.278	0.616	0.622	0.159	0.576	0.231
S[·]			0.044	0.087	-0.013	-0.105	0.346	0.204	0.265	0.168
cay				-0.100	0.274	0.006	-0.133	0.228	-0.248	0.353
default					0.149	0.301	0.814	-0.046	0.875	0.418
term pr.						-0.290	0.134	0.044	0.120	0.434
DP							0.006	-0.155	0.080	0.241
VIX^2								0.325	0.893	0.510
VRP									-0.134	0.427
RV										0.309

Note: This table reports the correlation between equity return predictors. $E[growth]$, $V[growth]$, and $S[growth]$ refer to the median, volatility, and skewness of the cross-sectional distribution of expected GDP growth rate at the beginning of each six months interval from 1952 to 2010. The predictors *cay*, the term premium, the dividend yield, and the default spread are from Lettau and Ludvigson (2005), from 1952 to 2010. VIX^2, VRP and RV are from Bollerslev, Tauchen and Zhou (2010), from1990 to 2010. Fear is from Bollerslev and Todorov (2011) starting in 1996.

Predicting returns

Panel A: Livingston (Up to 98) + Blue Chip, 1951-2010

	[1]	[2]	[3]	[4]	[5]	[6]	[7]
E[growth]	-0.020 (0.006)	-	-	-	-0.023 (0.005)	-	-
V[growth]		0.009 (0.008)	-	-	-	0.005 (0.005)	
S[growth]	-	-	-0.019 (0.003)	-	-	-	0.012 (0.021)
$S[\cdot]^{1/3} \cdot V[\cdot]^{1/2}$	-	-	-	-0.021 (0.004)	-0.026 (0.003)	-0.023 (0.003)	-0.035 (0.023)
Adj. R^2	0.034	-0.002	0.028	0.045	0.093	0.039	0.038

Predicting returns

Panel A: Livingston (Upto 98) + Blue Chip, 1951-2010

	[8]	[9]	[10]	[11]	[12]	[13]
E[growth]	-	-	-	-	-0.025 (0.005)	-0.025 (0.009)
V[growth]	-	-	-	-	-0.005 (0.005)	-
S[growth]	-	-	-	-	0.008 (0.011)	-
$S[\cdot]^{1/3} \cdot V[\cdot]^{1/2}$	-0.024 (0.005)	-0.023 (0.005)	-0.023 (0.006)	-0.021 (0.006)	-0.035 (0.011)	-0.023 (0.007)
cay	0.017 (0.006)	-	-	-	-	0.009 (0.010)
default	-	0.010 (0.008)	-	-	-	-0.002 (0.008)
term pr.	-	-	0.014 (0.008)	-	-	0.022 (0.011)
DP	-	-	-	0.012 (0.011)	-	0.10 (0.012)
Adj. R^2	0.066	0.048	0.060	0.048	0.079	0.125

Predicting returns

Panel B: Livingston (Up to 98) + Blue Chip, 1990-2010

	[1]	[2]	[3]	[4]	[5]	[6]	[7]
E[growth]	-0.011 (0.008)	-	-	-	-0.019 (0.003)	-	-
V[growth]		-0.006 (0.018)	-	-	-	-0.019 (0.016)	
S[growth]	-	-	-0.021 (0.005)	-	-	-	0.029 (0.016)
$S[\cdot]^{1/3} \cdot V[\cdot]^{1/2}$	-	-	-	-0.030 (0.006)	-0.034 (0.004)	-0.030 (0.009)	-0.056 (0.013)
Adj. R^2	-0.007	-0.031	0.38	0.100	0.127	0.074	0.099

Predicting returns

Panel B: Livingston (Up to 98) + Blue Chip, 1990-2010

	[8]	[9]	[10]	[11]	[12]	[13]
E[growth]	-	-	-	-	-0.028 (0.015)	-0.013 (0.015)
V[growth]	-	-	-	-	-0.018 (0.018)	-
S[growth]	-	-	-	-	0.021 (0.030)	-
$S[\cdot]^{1/3} \cdot V[\cdot]^{1/2}$	-0.031 (0.005)	-0.028 (0.005)	-0.032 (0.006)	-0.029 (0.006)	-0.052 (0.024)	-0.032 (0.008)
VIX^2	0.017 (0.007)	-	-	-	-	- -
VRP	-	0.012 (0.006)	-	-	-	0.010 (0.005)
RV	-	-	0.013 (0.008)	-	-	0.006 (0.016)
fear	-	-	-	0.017 (0.017)	-	- -
Adj. R^2	0.121	0.097	0.100	0.040	0.118	0.094

A Model with Time-Varying Mean and Skewness

Preference

Agents have recursive risk-sensitive preferences:

$$U_t = (1-\delta)\log C_t + \delta\theta \log E_t \exp\left\{\frac{U_{t+1}}{\theta}\right\}$$

where $\theta = 1/(1-\gamma)$.

A Model with Time-Varying Mean and Skewness

Preference

Agents have recursive risk-sensitive preferences:

$$U_t = (1-\delta)\log C_t + \delta E_t\left[U_{t+1}\right]$$

where $\theta = 1/(1-\gamma)$. If $\theta \to -\infty$: time additive case.

A Model with Time-Varying Mean and Skewness

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$$U_t = (1-\delta)\log C_t + \delta\theta \log E_t \exp\left\{\frac{U_{t+1}}{\theta}\right\}$$

where $\theta = 1/(1-\gamma)$.

A Model with Time-Varying Mean and Skewness

Preference

Agents have recursive risk-sensitive preferences:

$$U_t \approx (1-\delta)\log C_t + \delta E_t[U_{t+1}] + \frac{\delta}{2\theta} V_t[U_{t+1}] + \frac{\delta}{6\theta^2} E_t\left(U_{t+1} - E_t U_{t+1}\right)^3 + \ldots$$

where $\theta = 1/(1-\gamma)$.

- Standard Expected Utility term
- Utility variance matters ($\gamma > 1 \Rightarrow \theta < 0$: agents dislike variance)
- Higher order conditional moments are potentially important...

Dynamics of consumption growth

$$\Delta y_{t+1} = \underbrace{\mu_c + x_t}_{E_t[\Delta y_{t+1}]} + \sqrt{\sigma_t^c}\varepsilon_{t+1}^c$$

$$x_{t+1} = \rho x_t + \varphi_e \sqrt{\sigma_t^x}\varepsilon_{t+1}^x$$

where $\varepsilon_{t+1}^x \sim$ Skew-Normal with the shape parameter $\phi_{t+1} \in [-1, 1]$

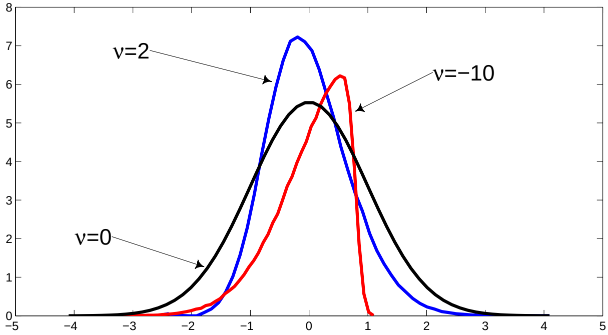
The probability distribution function of ε_{t+1}^x is:

$$2 \cdot \underbrace{\frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{(\varepsilon_{t+1}^x)^2}{2}\right\}}_{\text{Normal pdf}} \cdot \underbrace{\int_{-\infty}^{\frac{\phi_{t+1}}{\sqrt{1-\phi_{t+1}^2}}\varepsilon_{t+1}^x} \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{(t)^2}{2}\right\} dt}_{\text{Normal CDF}}.$$

- Skewness is time-varying: $\phi_{t+1} = \rho_\phi \phi_t + \sqrt{\sigma_\phi}\varepsilon_{t+1}^\phi$
- Volatility is constant: $\sigma_t^c = \sigma_t^x = \bar{\sigma}$

Skew Normal

$$\phi = -0.995(\nu = -10), \phi = 0(\nu = 0), \ \phi = 0.894(\nu = 2)$$



Financial Market and Equilibrium

In equilibrium, $\boxed{\Delta c_{t+1} = \Delta y_{t+1}.}$

P_t of any asset associated to the sequence of stochastic cash flows $\{D_j\}_{j=t}^{\infty}$ satisfies

$$P_t = E_t\left[M_{t+1}(P_{t+1} + D_{t+1})\right],$$

where

$$M_{t+1} = \frac{\partial U_t/\partial C_{t+1}}{\partial U_t/\partial C_t}.$$

SDF and Equity Return

$$m_{t+1} - E_t m_{t+1} = -\left(1 - \frac{1}{\theta}\right)\sqrt{\bar{\sigma}}\varepsilon^c_{t+1} + \frac{V_\phi \sqrt{\sigma_\phi}}{\theta}\varepsilon^\phi_{t+1} + \frac{B\varphi_e\sqrt{\bar{\sigma}}}{\theta}\varepsilon^x_{t+1}, \quad (1)$$

where

$$B = \frac{\delta}{1-\delta\rho_x}, \quad V_\phi = \frac{0.8\varphi_e B\delta\rho_\phi}{1-\delta\rho_\phi}\sqrt{\bar{\sigma}}, \quad \theta = \frac{1}{1-\gamma}.$$

High economic growth (large ε^c) | High expected growth (large ε^x)

High skewness of expected growth (large ε^ϕ)

Large Equity Premium

SDF and Equity Return

Assume

$$\Delta d_t = \mu_c + \lambda \left(x_{t-1} + \sqrt{\bar{\sigma}} \varepsilon_t^c \right) + \sqrt{\sigma_d} \sqrt{\bar{\sigma}} \varepsilon_t^d ,$$

where the innovation ε_t^d is *i.i.d* distributed as a standard normal.

The conditional equity risk premium:

$$E_t \left[r_{t+1}^d - r_t^f \right] = \bar{r} - \boxed{\alpha \frac{K_1 (\lambda - 1)}{1 - K_1 \rho_x} \varphi_e \sqrt{\bar{\sigma}} \rho_\phi} \cdot \phi_t$$

where $\alpha > 0$ is related to the slope of the normal cdf, $K_1 \in (0, 1)$ is the slope of the Campbell Shiller approximation.

A Model with Time-Varying Volatility and Skewness

A Model with Time-Varying Volatility and Skewness

Dynamics of consumption growth

$$\Delta c_{t+1} = \underbrace{\mu_c + x_t}_{E_t[\Delta c_{t+1}]} + \sqrt{\sigma_t^c}\varepsilon_{t+1}^c$$

$$x_{t+1} = \rho x_t + \varphi_e \sqrt{\sigma_t^x}\varepsilon_{t+1}^x$$

where

$\varepsilon_{t+1}^x \sim$ Skew-Normal with the shape parameter ϕ_{t+1}

- Skewness is time-varying: $\phi_{t+1} = \rho_\phi \phi_t + \sqrt{\sigma_\phi}\varepsilon_{t+1}^\phi$
- Volatility is time-varying: $\sigma_t^c = \sigma_t^x = \sigma_t = \bar{\sigma}(1-\rho_\sigma) + \rho_\sigma \sigma_{t-1} + \sqrt{\sigma_\varepsilon}\varepsilon_t^\sigma$

GMM estimation

We use GMM to estimate the parameters governing the dynamics of x_t, σ_t, and ϕ_t.

$$\widehat{E}_t^{cs}(\Delta c_{t+1}) = \frac{1}{n}\sum_{i=1}^{n} E_t^i(\Delta c_{t+1})$$

$$\widehat{V}_t^{cs}(\Delta c_{t+1}) = \frac{1}{n}\sum_{i=1}^{n}\left[E_t^i(\Delta c_{t+1}) - \widehat{E}_t^{cs}(\Delta c_{t+1})\right]^2$$

$$\widehat{S}_t^{cs}(\Delta c_{t+1}) = \frac{\frac{1}{n}\sum_{i=1}^{n}\left[E_t^i(\Delta c_{t+1}) - \widehat{E}_t^{cs}(\Delta c_{t+1})\right]^3}{\left(\widehat{V}_t^{cs}(\Delta c_{t+1})\right)^{3/2}}$$

Calibration-(Semi-annual)

Panel A: *dynamics of mean, variance, and skewness*

Parameter	Description	Model	GMM
ρ_x	AR coefficient of the expected consumption growth rate	0.690	0.540 (0.073)
$\bar{\sigma}$	Unconditional variance of the short-run shock	3.13×10^{-4}	3.85×10^{-4} (4.44×10^{-5})
$\sqrt{\sigma_\varepsilon}$	Conditional volatility of the variance of the short-run shock	1.50×10^{-4}	2.81×10^{-4} (4.89×10^{-5})
ρ_σ	AR coefficient of the variance of the short-run shock	0.464	0.407 (0.075)
$\sqrt{\sigma_\phi}$	Conditional volatility of skewness	0.425	3.644 (1.796)
ρ_ϕ	AR coefficient of skewness	0.307	0.185 (0.087)

Notes - Panel A reports the calibration of the parameters associated to the transition dynamics of x_t, σ_t, and ϕ_t (column labeled "Model"), along with the GMM estimated values (column labeled "GMM"). The numbers in parenthesis are the standard errors of the estimated coefficients. Panel B reports the calibration of all the remaining parameters of the model. The calibration is set to semi-annual frequency.

go to GMM estimation

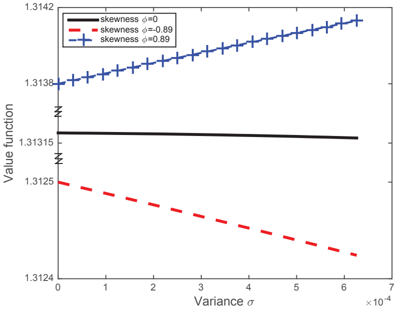
Calibration-(Semi-annual)

Panel B: *other parameters*

Parameter	Description	Model
γ	Risk aversion	10
δ	Subjective discount factor	0.993
μ_c	Average consumption growth	0.008
φ_e	Ratio of long-run shock and short-run shock volatilities	0.200
λ_2	Leverage coefficient	4.500
σ_d	Scale parameter of dividends' volatility	28

Notes - Panel A reports the calibration of the parameters associated to the transition dynamics of x_t, σ_t, and ϕ_t (column labeled "Model"), along with the GMM estimated values (column labeled "GMM"). The numbers in parenthesis are the standard errors of the estimated coefficients. Panel B reports the calibration of all the remaining parameters of the model. The calibration is set to semi-annual frequency.

Utility Function



(B)

- Time-varying skewness amplifies the uncertainty of lifetime utility
- Skewness interacts with variance:
  $\rightarrow$ high variance is welfare increasing with positive skewness
  $\rightarrow$ high variance is welfare decreasing with negative skewness

Model-implied predictive regressions at semiannual frequency

	Benchmark model simulation					Data				
	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
$(V_t)^{1/2}$	0.015	-	-	-	-	0.009	-	-	-	-
	(0.003)					(0.014)				
V_t	-	0.015	-	-	0.015	-	0.009	-	-	0.005
		(0.003)			(0.003)		(0.010)			(0.010)
S_t	-	-	-0.009	-	-	-	-	-0.019	-	-
			(0.003)					(0.009)		
$(S_t)^{1/3}$	-	-	-	-0.007	-0.007	-	-	-	-0.024	-0.023
$\cdot(V_t)^{1/2}$				(0.003)	(0.003)				(0.009)	(0.010)

Quantitative Performance

[2]Correlation of Excess Returns: Quintile Analysis

Step1 Solve the model and compute the conditional risk premium.

Step2 Plug the time series of the cross-sectional moments of the distribution of analysts' forecasts in the conditional risk premium.

Step3 Calculate the correlation between these expected returns predicted by the model and the actual subsequent excess returns in the data.

Step4 Compare with the same set of correlations for the model in which any time-variation in skewness has been shut down.

A Model with Time-Varying Volatility and Skewness

Dynamics of consumption growth

$$\Delta c_{t+1} = \underbrace{\mu_c + x_t}_{E_t[\Delta c_{t+1}]} + \sqrt{\sigma_t^c}\varepsilon_{t+1}^c$$

$$x_{t+1} = \rho x_t + \varphi_e \sqrt{\sigma_t^x}\varepsilon_{t+1}^x$$

where

$\varepsilon_{t+1}^x \sim$ Skew-Normal with the shape parameter ϕ_{t+1}

- Skewness is time-varying: $\phi_{t+1} = \rho_\phi \phi_t + \sqrt{\sigma_\phi}\varepsilon_{t+1}^\phi$
- Volatility is time-varying:
  $\sigma_t^c = \sigma_t^x = \sigma_t = \bar{\sigma}(1-\rho_\sigma) + \rho_\sigma \sigma_{t-1} + \sqrt{\sigma_\varepsilon}\varepsilon_t^\sigma$

A Model with Time-Varying Volatility and Skewness is Shutdown

Dynamics of consumption growth

$$\Delta c_{t+1} = \underbrace{\mu_c + x_t}_{E_t[\Delta c_{t+1}]} + \sqrt{\sigma_t^c}\varepsilon_{t+1}^c$$

$$x_{t+1} = \rho x_t + \varphi_e \sqrt{\sigma_t^x}\varepsilon_{t+1}^x$$

where

$$\varepsilon_{t+1}^x \sim \text{Normal}$$

- Skewness is shut down: ~~$\phi_{t+1} = \rho_\phi \phi_t + \sqrt{\sigma_\phi}\varepsilon_{t+1}^\phi$~~
- Volatility is time-varying:
  $\sigma_t^c = \sigma_t^x = \sigma_t = \bar{\sigma}(1-\rho_\sigma) + \rho_\sigma \sigma_{t-1} + \sqrt{\sigma_\varepsilon}\varepsilon_t^\sigma$

Correlation of Excess Returns: Quintile Analysis

Panel A: *Correlations by Skewness based Quintiles*

	Q1	Q2	Q3	Q4	Q5
No Skewness	0.096	0.126	0.325	0.073	0.122
Benchmark	0.219	0.155	0.327	0.092	0.141
% Change	(128.6%)	(23.3%)	(0.8%)	(26.0%)	(16.0%)

Panel B: *The Role of Volatility*

	Q1	Q2	Q3	Q4	Q5
Low σ_t	0.157	-0.319	0.086	0.131	-0.099
High σ_t	0.101	0.402	0.289	0.209	0.228

Comparison With Other Models

Comparison With Other Models

$$
\begin{aligned}
\Delta c_{t+1} &= \mu_c + x_t + \sqrt{\sigma_t}\,\boxed{\varepsilon^c_{t+1}} \\
\Delta d_{t+1} &= \lambda \Delta c_{t+1} + \sqrt{\sigma_d}\sqrt{\sigma_t}\,\boxed{\varepsilon^d_{t+1}} \\
x_{t+1} &= \rho_x x_t + \varphi_e \sqrt{\sigma^x_t}\,\boxed{\varepsilon^x_{t+1}} + J^x_{t+1} \\
\sigma_{t+1} &= (1-\rho_\sigma)\bar{\sigma} + \rho_\sigma \sigma_t + \sqrt{\sigma_\varepsilon}\,\boxed{\varepsilon^\sigma_{t+1}} + J^\sigma_{t+1} \\
\sigma^x_t &= \sigma_t \left(1 - 2(E_t[\phi_{t+1}])^2/\pi\right)^{-1}
\end{aligned}
$$

$$\varepsilon^x_{t+1} \sim SKN(0, 1, \phi_{t+1}),$$

and J^x_{t+1} and J^σ_{t+1} are modeled as jump processes

$$
\begin{aligned}
J^x_{t+1} &= \sum_{j=1}^{N^x_{t+1}} \xi^x_{j,t+1}, \quad N^x_{t+1} \sim Poisson(l^x_1 \sigma_t), \quad \xi^x_{j,t+1} \sim -exp(\mu^x) + \mu_x \\
J^\sigma_{t+1} &= \sum_{j=1}^{N^\sigma_{t+1}} \xi^\sigma_{j,t+1}, \quad N^\sigma_{t+1} \sim Poisson(l^\sigma_1 \sigma_t), \quad \xi^\sigma_{j,t+1} \sim exp(\mu^\sigma) - \mu_\sigma.
\end{aligned}
$$

Calibration

γ	Risk aversion	10
δ	Subjective discount factor	0.998
μ_c	Average consumption growth	0.001
ρ_x	Autoregressive coefficient of the expected consumption growth rate x_t	0.9619
ϕ_e	Ratio of long-run shock and short-run shock volatilities	0.05
μ_x	Location parameter of skew normal distribution of the innovations to x_t	0
$\sqrt{\sigma_\sigma}$	Conditional volatility of the variance of the short-run shock to consumption growth	3.80×10^{-6}
ρ_σ	Persistence of the variance of the short-run shock to consumption growth	0.93
$\sqrt{\sigma_\nu}$	Conditional volatility of the scale parameter ν of the skew normally distributed innovations to x_t	0.4696
ρ_ν	Persistence of the scale parameter ν of skew normally distributed innovations to x_t	0.8
λ	Leverage	3

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λ	Leverage	3

Comparison of models

	Data		[1] Benchmark	[2] No Skewness w/ Constant Vol	[3] No Skewness
$E[r_t^d - r_t^f]$	4.84	(1.90)	5.49	1.91	3.81
$\sigma[r_t^d - r_t^f]$	16.40	(1.75)	18.11	13.70	13.67
$Skew[r_t^d - r_t^f]$	0.65	(0.23)	0.03	0.00	0.01
$E[r_t^f]$	1.75	(0.50)	2.23	2.23	2.22
$\sigma[r_t^f]$	2.30	(0.3)	2.02	1.23	1.23
$Skew[r_t^f]$	−0.74	(0.35)	−0.02	0.01	−0.03
$E[p/d]$	3.43	(0.26)	3.02	4.40	3.47
$\sigma[p/d]$	0.43	(0.12)	0.16	0.09	0.09
$Skew[p/d]$	0.36	(0.89)	0.03	−0.03	−0.04
$AC_1[p/d]$	0.97	(0.23)	0.41	0.28	0.27

Comparison of models

	Data		[1] Benchmark	[4] Adjusted Mean	[5] Negative Skewness
$E[r_t^d - r_t^f]$	4.84	(1.90)	5.49	2.14	8.98
$\sigma[r_t^d - r_t^f]$	16.40	(1.75)	18.11	14.12	14.17
$Skew[r_t^d - r_t^f]$	0.65	(0.23)	0.03	0.01	0.03
$E[r_t^f]$	1.75	(0.50)	2.23	2.23	2.42
$\sigma[r_t^f]$	2.30	(0.3)	2.02	1.19	1.50
$Skew[r_t^f]$	−0.74	(0.35)	−0.02	−0.04	0.19
$E[p/d]$	3.43	(0.26)	3.02	4.19	2.53
$\sigma[p/d]$	0.43	(0.12)	0.16	0.10	0.11
$Skew[p/d]$	0.36	(0.89)	0.03	−0.04	0.04
$AC_1[p/d]$	0.97	(0.23)	0.41	0.26	0.36

Comparison of models

	Data		[1] Benchmark	[6] DY Jump Model
$E[r_t^d - r_t^f]$	4.84	(1.90)	5.49	4.49
$\sigma[r_t^d - r_t^f]$	16.40	(1.75)	18.11	18.87
$Skew[r_t^d - r_t^f]$	0.65	(0.23)	0.03	0.23
$E[r_t^f]$	1.75	(0.50)	2.23	2.02
$\sigma[r_t^f]$	2.30	(0.3)	2.02	2.27
$Skew[r_t^f]$	-0.74	(0.35)	-0.02	-1.05
$E[p/d]$	3.43	(0.26)	3.02	3.34
$\sigma[p/d]$	0.43	(0.12)	0.16	0.14
$Skew[p/d]$	0.36	(0.89)	0.03	-1.06
$AC_1[p/d]$	0.97	(0.23)	0.41	0.33

The different dynamics of the conditional skewness of expected consumption growth

In the Drechsler and Yaron (2011):

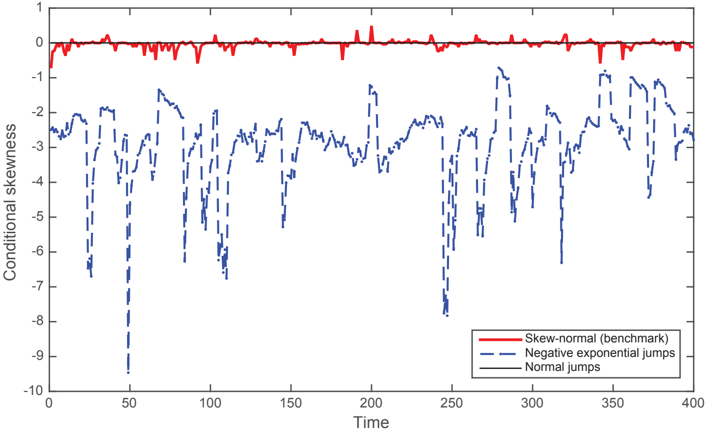
- $V_t[x_{t+1}] = (\varphi_e^2 + l_1^x \mu_x^2)\sigma_t$
- $E_t\left[(x_{t+1} - E_t[x_{t+1}])^3\right] = -2\mu_x^3 l_1^x \sigma_t$
- $Skewness_t[x_{t+1}] = -\frac{2\mu_x^3 l_1^x}{(\varphi_e^2 + l_1^x \mu_x^2)^{3/2}} \frac{1}{\sqrt{\sigma_t}}$

Equity risk premia ==> an increasing function of skewness!

We need a model that allows us to disentangle variance from skewness.

Our model with time-varying Skew-Normal innovations has this property!

Comparison of Conditional Skewness of Expected Consumption Growth



The role of skewness in predictive regressions

[1] Benchmark Model

	[1]	[2]	[3]	[4]	[5]
$(\widehat{V}_t^{cs})^{1/2}$	0.013	-	-	-	-
	(0.003)				
$\widehat{V}_t^{cs}$	-	0.013	-	-	0.014
		(0.003)			(0.003)
$\widehat{S}_t^{cs}$	-	-	-0.065	-	-
			(0.003)		
$(\widehat{S}_t^{cs})^{1/3} \cdot (\widehat{V}_t^{cs})^{1/2}$	-	-	-	-0.129	-0.138
				(0.003)	(0.003)

The role of skewness in predictive regressions

[2] Model with jumps					
	[1]	[2]	[3]	[4]	[5]
$(\widehat{V}_t^{cs})^{1/2}$	0.116	-	-	-	-
	(0.007)				
$\widehat{V}_t^{cs}$	-	0.124	-	-	0.121
		(0.009)			(0.017)
$\widehat{S}_t^{cs}$	-	-	0.037	-	-
			(0.003)		
$(\widehat{S}_t^{cs})^{1/3} \cdot (\widehat{V}_t^{cs})^{1/2}$	-	-	-	-0.108	-0.003
				(0.006)	(0.011)

Concluding Remarks

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- There is a sizeable skewness premium
- Extensions
  - Average skewness is negative: results are almost unaffected, because what matters is the volatility of the skewness and its predictive power for the mean
  - Cross-sectional implications: assets whose skewness of expected cash flows' forecasts is more volatile should command larger risk premia
    $\rightarrow$ Cross-section of US equities
    $\rightarrow$ Cross-section of int'l equities

Skewness in Expected Macro Fundamentals and the Predictability of Equity Returns: Evidence and Theory

Ric Colacito, Eric Ghysels, Jinghan Meng, and Wasin Siwasarit

THE UNIVERSITY
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at CHAPEL HILL

GMM Estimation

Model: The transition dynamics of the three state variables (x_t, σ_t, ϕ_t) is defined as follows:

$$\begin{aligned} x_t &= \mu_x(1-\rho_x) + \rho_x x_{t-1} + \varphi_e \sqrt{\sigma_{t-1}} \varepsilon_t^x \\ \sigma_t &= \bar{\sigma}(1-\rho_\sigma) + \rho_\sigma \sigma_{t-1} + \sqrt{\sigma_\varepsilon} \varepsilon_t^\sigma \\ \phi_t &= \bar{\phi}(1-\rho_\phi) + \rho_\phi \phi_{t-1} + \sqrt{\sigma_\phi} \varepsilon_t^\phi \end{aligned}$$

where $\varepsilon_t^x \sim SKN(0,1,\nu_t)$ with $\nu_t = \phi_t / \sqrt{1-\phi_t^2}$ and ε_t^σ, ε_t^ϕ are standard Normal distributed. All the shocks are i.i.d.

We are going to estimate 8 parameters using GMM:
$\rho_x, \mu_x, \bar{\sigma}, \rho_\sigma, \sigma_\varepsilon, \rho_\phi, \bar{\phi}, \sigma_\phi$.

go to Calibration

$$
\begin{aligned}
E_t(x_{t+1}) &= \rho_x x_t + \left(\frac{2}{4-\pi}\right)^{1/3} V_t(x_{t+1})^{1/2} |S_t(x_{t+1})|^{1/3} \mathrm{sign}(S_t(x_{t+1})). \\
V_t(x_{t+1}) &= \varphi_e^2 \sigma_t^c. \\
S_t(x_{t+1}) &= \frac{4-\pi}{2} \frac{\left(\sqrt{2/\pi}\, E_t \phi_{t+1}\right)^3}{\left(1 - 2\left(E_t \phi_{t+1}\right)^2/\pi\right)^{3/2}}.
\end{aligned}
$$