ARC CENTRE OF EXCELLENCE IN POPULATION AGEING RESEARCH

Demographic Impacts on Life Cycle Portfolios and Financial Market Structures

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Overview of the thesis

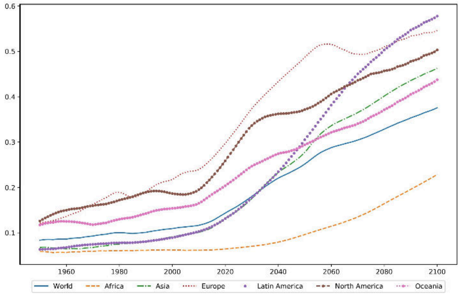
- Paper 1: Estimations of time-varying relative risk aversion and demographic characteristics *(In progress)*
- Paper 2: Risk-sensitive preferences and age-dependent risk aversion *(Working paper)*
- Paper 3: Age-dependent risk aversion: Re-evaluating fiscal policy impacts of population ageing *(In progress)*
- **Paper 4: Demographic impacts on life cycle portfolios and financial market structures** *(with Larry Liu)*

Outline

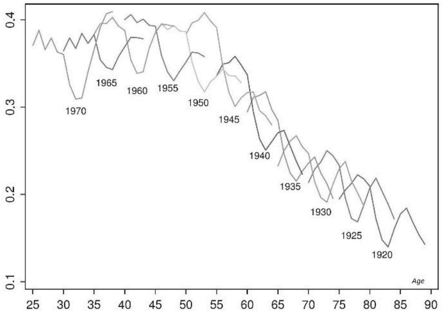
1. Motivation
2. The Model
3. Equilibrium conditions
4. Calibration
5. Computation
6. Results
7. Conclusion

1 – Motivation

- The world has been experiencing changes in demographic structure.
- This has important impacts on, among others, financial markets such as asset prices, asset returns, portfolio allocations and international capital flows.
- Empirical data suggests that portfolio choice exhibits strong life-cycle patterns.



(a) Elderly dependency ratio



(b) Stock Share in Financial Wealth

Sources: (a) United Nations World Population Prospects 2019; (b) Fagereng et al. (2017)

1 – Motivation

- Portfolio choice has been widely studied as a central topic in finance
- Samuelson (1969): Optimal portfolio shares do not change with age in a model with the absence of labor income, frictionless markets, and independent and identically distributed returns on financial assets.
- Merton (1971): With deterministic labor income, the share of risky assets tends to decline as an individual get older because of decreases in human wealth.
- Extending on Merton (1971), most models reach a broad consensus that portfolio choice has a strong life-cycle pattern consistent with the empirical findings.

1 – Challenges facing the study of portfolio choice and aging population

- However, most studies on life-cycle portfolio choice are not suitable to examine the interaction between portfolio choice and demographic changes

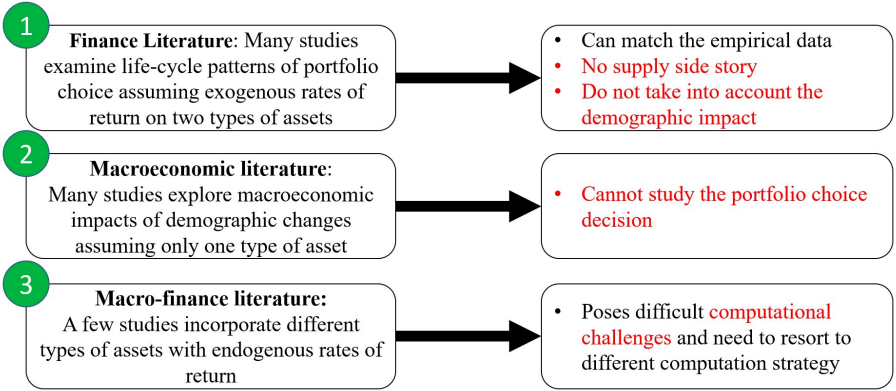
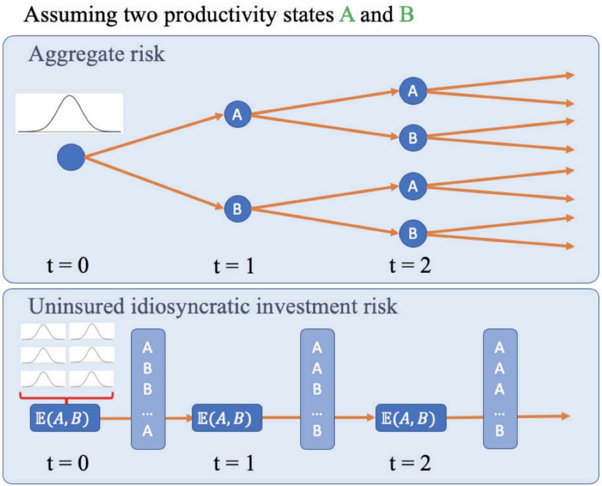


Figure 2: Existing studies and their drawbacks

1 – Aggregate risk vs. idiosyncratic risk



Arguments for uninsured idiosyncratic risk (Angeletos & Calvet (2006))

- Undiversifiable investment risks are pervasive in economic activities.
- In the US, privately-held firms account for about half of production, employment and corporate equity, and represent more than half the financial wealth of rich households (Carroll 2000)
- The "representative" investor has a very poorly diversified portfolio

1 – Research Question and Contributions

This paper examines demographic impacts on life-cycle portfolio choice and financial market structure in an overlapping generation (OLG) model with endogenous rates of return on risky and risk-free assets.

Contributions

1. Endogenize the rate of return on risky assets in a production economy by introducing idiosyncratic shocks in the spirit of Angeletos & Calvet (2006).
2. Provide a new framework to illustrate life-cycle portfolio patterns in a general equilibrium model with endogenous rates of return.
3. Enable the interaction between financial markets and macroeconomic conditions.
4. Serve as a new framework to endogenize risk premia in a production economy

2 – Outline

2 – Demographics

- Time is discrete and goes forever, i.e., $t = 0, 1, 2, \ldots$.
- In each period, there are $J = 14$ generations in the economy
- N_t^j denotes amount of generation j in period t
- $N_t = \sum_{j=1}^{J} N_t^j$
- $N_t^{j=1} = (1 + g_t) N_{t-1}^{j=1}$
- Survival probability is denoted by ξ_t^j, where $N_{t+1}^{j+1} = \xi_{t+1}^{j+1} N_t^j, j \leq J - 1$ and $\xi_t^{J+1} = 0$
- Total population growth in period t is $n_t = \frac{N_t}{N_{t-1}} - 1$

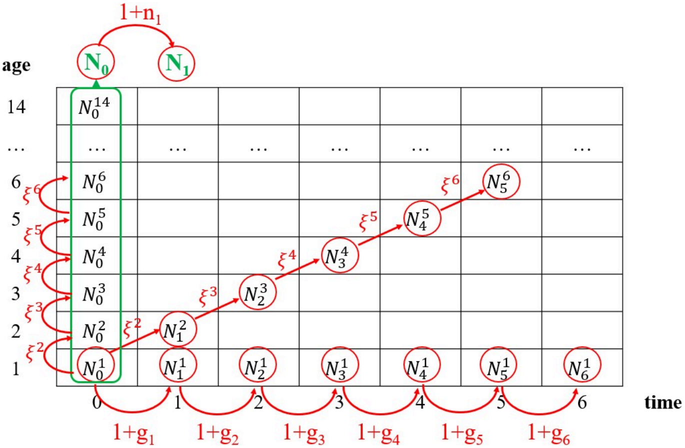


Figure 3: Demographics structure

2 – Assets

- There are two types of assets in this economy: risk-free bonds and risky capital
- Bonds serve as a financial instrument like a banking system through which households can borrow and lend from each other. There are borrowing costs in the bond market.
- Households accumulate capital through direct investment in their own firms, and there is no capital market, so households are faced with heterogeneous risky rates of return on capital, which are driven by idiosyncratic production shocks.

2 – Households: Preferences

- The period utility of a household in period t is represented by a CRRA function

$$u\left(c_t^{j,i}\right) = \frac{(c_t^{j,i})^{1-1/\gamma}}{1-1/\gamma} \tag{1}$$

- Households maximize standard time-separable preferences

$$U_t^j = u(c_t^j) + \beta \xi_{t+1}^{j+1} E_t(U_{t+1}^{j+1}) \tag{2}$$

2 – Households: Budget Constraints

- The budget constraint of a household is

$$c_t^{j,i} + a_{t+1}^{j+1,i} = R_t^{j,i} a_t^{j,i} + (1 - \tau_t) w_t^{j,i} e^j + p_t^j + q_t - \delta_B(\omega_t^{j,i}, a_t^{j,i}), \tag{3}$$

where

$$a_t^{j,i} = b_t^{j,i} + k_t^{j,i}, \quad \omega_t^{j,i} = \frac{k_t^{j,i}}{a_t^{j,i}},$$

$$R_t^{j,i} = 1 + r_t^b + \omega_t^{j,i}\left(r_t^{j,i} - r_t^b\right),$$

$$\delta_B(\omega, a) = \begin{cases} 0, & \omega \leq 1 \\ \left(\frac{\eta_1}{2}(\omega - 1)^2 + \eta_2(\omega - 1)\right) a, & \omega > 1 \end{cases}$$

2 – Households' problem

$$V(z_t^j) = \max_{\{c_t^j, a_{t+1}^{j+1}, \omega_{t+1}^{j+1}\}} u(c_t^j) + \beta \xi_{t+1} \mathbb{E}\left[V(z_{t+1}^{j+1})\right]$$

$$\text{s.t.} \quad c_t^j + a_{t+1}^{j+1} = R_t^j a_t^j + (1-\tau_t) w_t^j e^j + p_t^j + q_t - \delta_B(\omega_t^j, a_t^j)$$

$$R_t^j = 1 + r_t^b + \omega_t^j \left(r_t^k - r_t^b\right) \tag{4}$$

$$q_t \sum_{j=1}^{J} m_t^j = \sum_{j=1}^{J} \left[(1-\xi^{j+1}) a_{t+1}^{j+1} m_t^j\right]$$

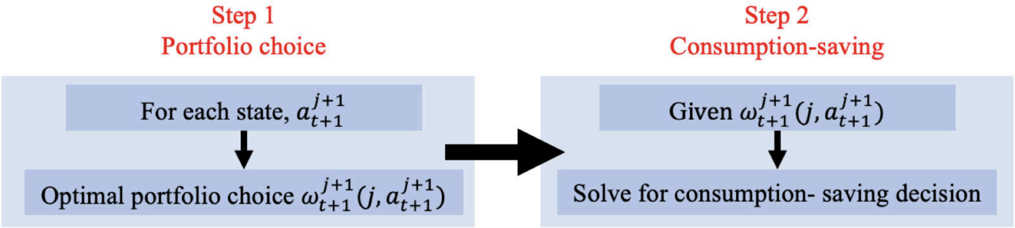
$$(\omega_{t+1}^{j+1} - 1) a_{t+1}^{j+1} \leq L = \mathbb{E}\left[\sum_{k=j+1}^{J} \frac{(1-\tau_{t+k-j}) w_{t+k-j}^k e^k + p_{t+k-j}^k}{\prod_{i=1}^{k-j}(1 + r_{t+i}^b)}\right]$$

- To simplify the notation, we sum up all available resources into a cash-on-hand variable

$$X_t^j(z_t^j) = R_t^j (a_t^j + q_t) + (1-\tau_t) w_t^j e^j + p_t^j - \delta_B(\omega_t^j, a_t^j) \tag{5}$$

2 – Portfolio Choice and Consumption-Saving Decisions

- Household's optimization decisions involve two steps:



2 – Step 1: Portfolio Choice Decision

- For every next-period asset a_{t+1}, we can determine optimal investment structure $\omega_{t+1}^{j+1}(j, a_{t+1}^{j+1})$ by solving

$$\begin{aligned} Q(j, a_{t+1}^{j+1}) &= \max_{\{\omega_{t+1}^{j+1} \geq 0\}} \mathbb{E}\left[V(j+1, X_{t+1}^{j+1})\right] \\ s.t. \quad X_{t+1}^{j+1} &= R_{t+1}^{j+1} a_{t+1}^{j+1} + (1-\tau_{t+1}) w_{t+1}^{j+1} e^{j+1} + p_{t+1}^{j+1} + q_{t+1} - \delta_B(\omega_{t+1}^{j+1}, a_{t+1}^{j+1}) \end{aligned} \tag{6}$$

  The first-order condition is

$$\mathbb{E}\left\{\left[r_{t+1}^{j+1} - r_{t+1}^{b} + \mathbb{1}\{\omega_{t+1}^{j+1} > 1\} * \left(\eta_1(\omega_{t+1}^{j+1} - 1) + \eta_2\right)\right] a_{t+1}^{j+1} \left(c_{t+1}^{j+1}\right)^{-1/\gamma}\right\} = 0 \tag{7}$$

- Given a certain level of a_{t+1}^{j+1}, as all households share the same production shock, the expectation removes the household heterogeneity, so the portfolio choice is independent of production shocks.

2 – Step 2: Consumption-Saving Decision

- Given optimal investment structure $\omega_{t+1}^{j+1}(j, a_{t+1})$, households make decisions on consumption c_t^j and savings a_{t+1}^{j+1} by maximizing

$$\begin{aligned} V(j, X_t^j) &= \max_{\{c_t^j, a_{t+1}^{j+1}\}} u(c_t^j) + \beta \xi^{j+1} Q(j, a_{t+1}^{j+1}) \\ s.t. \quad X_t^j &= c_t^j + a_{t+1}^{j+1} \end{aligned} \tag{8}$$

The optimality condition is

$$(c_t^j)^{-1/\gamma} = \beta \xi^{j+1} \, \mathbb{E} \left[R_{t+1}^{j+1} \left(c_{t+1}^{j+1} \right)^{-1/\gamma} \right] \tag{9}$$

Together with the budget constraint, we can solve a_{t+1}^{j+1} and c_t^j for all states $\{j, i, X\}$.

2 – Firms

- Each household aged j indexed i runs their own firm and produces

$$y_t^{j,i} = A_t^{j,i}(k_t^{j,i})^{\alpha}(l_t^{j,i})^{1-\alpha} \tag{10}$$

where $\ln\left(A_t^{j,i}\right) \overset{i.i.d.}{\sim} N(\mu_A, \sigma_A^2)$

- The labor demand of each firm is derived from the first-order condition as

$$l_t^{j,i} = \left(A_t^{j,i}\frac{1-\alpha}{w_t}\right)^{1/\alpha} k_t^{j,i} \tag{11}$$

- The rate of return on capital is

$$r_t^{j,i} = \frac{\pi_t^{j,i}}{k_t^{j,i}} = \frac{y_t^{j,i} - w_t l_t^{j,i}}{k_t^{j,i}}$$
$$= \left(A_t^{j,i}\right)^{1/\alpha} \alpha \left(\frac{1-\alpha}{w_t}\right)^{(1-\alpha)/\alpha}$$

2 – Government

- The government's roles are
  1. to operate a pay-as-you-go pension system by collecting labor income taxes and distributing tax revenues to retirees as pensions

$$p_t^j = \kappa w_t e^{j_R - 1}, j = J_R, ..., J$$

$$\tau_t = \frac{\sum_{j=J_R}^{J} m_t^j p_t^j}{\sum_{j=1}^{J_R - 1} m_t^j w_t e^j}$$

  2. to distribute accidental bequest to living cohorts

$$q_t \sum_{j=1}^{J} m_t^j = \sum_{j=1}^{J} \left[(1 - \xi^{j+1}) a_{t+1}^{j+1} m_t^j \right] \tag{12}$$

3 – Outline

3 – Equilibrium conditions

1. Aggregate and individual behaviors are consistent
2. Given prices (w_t, r_t^b, r_t), the policy functions $c(z_t)$, $a(z_t)$ and $\omega(z_t)$ solve the household problem (4)
3. The government balances its budget and allocates accidental bequests
4. All markets clear
   - Goods market clears: $Y_t = C_t + I_t + \Delta_t$
   - Bonds market clears: $\sum_{j=1}^{J} \int_{i=0}^{N_t^j} b_t^{j,i} = 0$
   - Labor market clears: $L_t^D = L_t^S$
5. The capital stock of each cohort evolves over time as

$$\xi_{t+1}^{j+1} K_{t+1}^{j+1} = K_t^j + I_t^j \tag{13}$$

4 – Outline

1. Motivation
2. The Model
3. Equilibrium conditions
4. **Calibration**
5. Computation
6. Results
7. Conclusion

4 – Calibration

Table 1: Parameter Values

Description	Parameter	Value
Demographics		
- Number of age cohorts	J	14
- Retirement age	J_R	10
Household preferences		
- Discount factor	β	0.815
- Risk aversion parameter	$1/\gamma$	2
Production parameters		
- Capital share in production	α	0.36
- Technology level	A	1.57
- Idiosyncratic productivity risk	σ_A^2	0.008
Borrowing cost function		
- Quadratic term	η_1	0.08
- Linear term	η_2	0.1

5 – Outline

5 – Computation

- The general solution algorithm is similar to Auerbach & Kotlikoff (1987) with simplified steps as follows:
  - Step 1: Initialize demographic composition and guess a set of equilibrium variables.
  - Step 2: Calculate price variables.
  - Step 3: Solve optimal portfolio choice for each next-period asset level.
  - Step 4: Solve individuals' consumption-saving decisions given the portfolio decisions from Step 3.
  - Step 5: Compute the distribution of individuals across states.
  - Step 6: Compute aggregate variables and verify if equilibrium conditions are satisfied. If not, update guesses of bond returns, wage rate, income tax, pension rate, bequest and return to Step 2.

6 – Outline

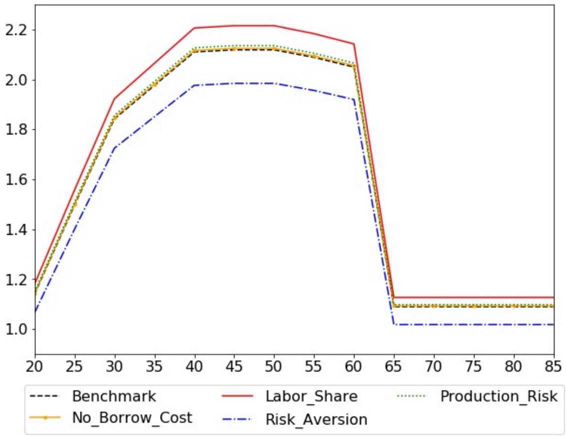
6 – Results

- Baseline scenarios
  1. (B1) includes standard borrowing costs, a standard labor income share, a standard risk aversion degree, and a standard production risk.
  2. (B2) no borrowing costs
  3. (B3) a higher labor income share where the capital share in production, α, decreases from 0.36 to 0.30
  4. (B4) a higher risk aversion degree where the relative risk aversion coefficient increases from 2 to 2.5
  5. (B5) a higher production risk with a 50 percent increase in the standard deviation of the production risk.
- Ageing scenarios
  1. A decrease in the fertility rate
  2. A decrease in the mortality rate (or an increase in the survival rate)
  3. A combination of the two cases

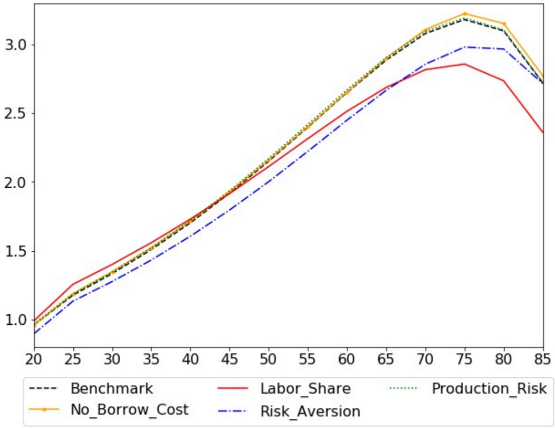
6 – Baseline Scenario: Life-Cycle Variables

Figure 4: Labor Income and Consumption

(a) Labor income

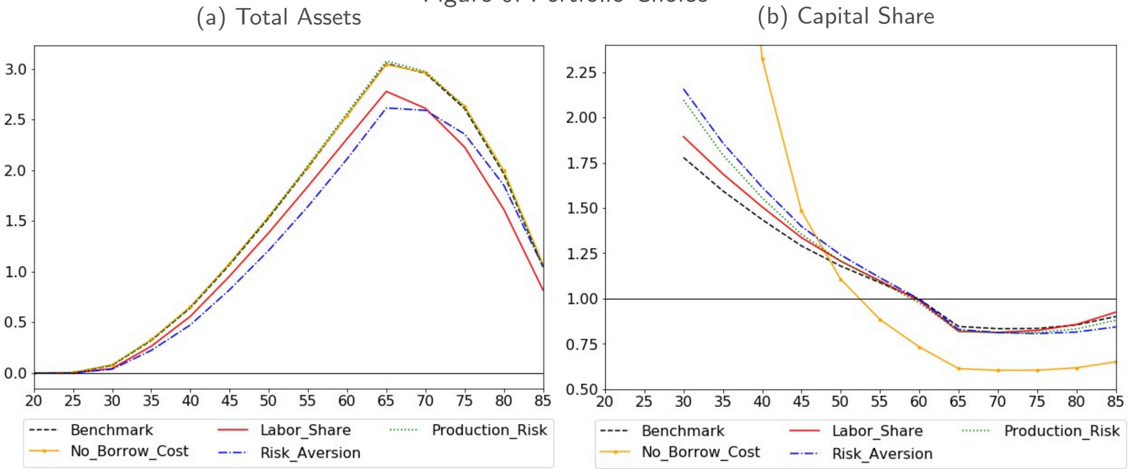


(b) Consumption



6 – Baseline Scenario: Portfolio Choice I

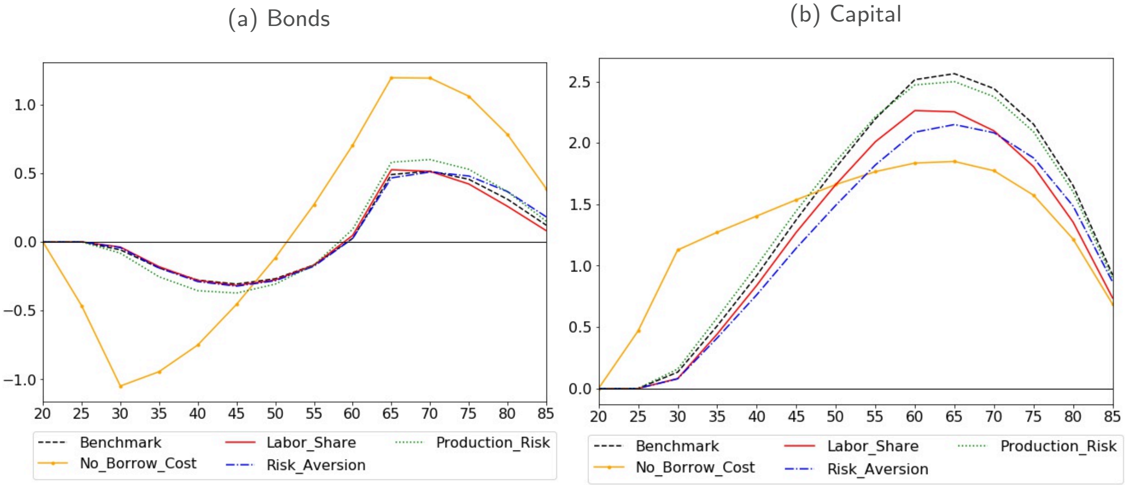
Figure 6: Portfolio Choice



Households would keep the share of capital in total wealth, $\eta = \frac{\omega * a}{a+h}$, constant over time

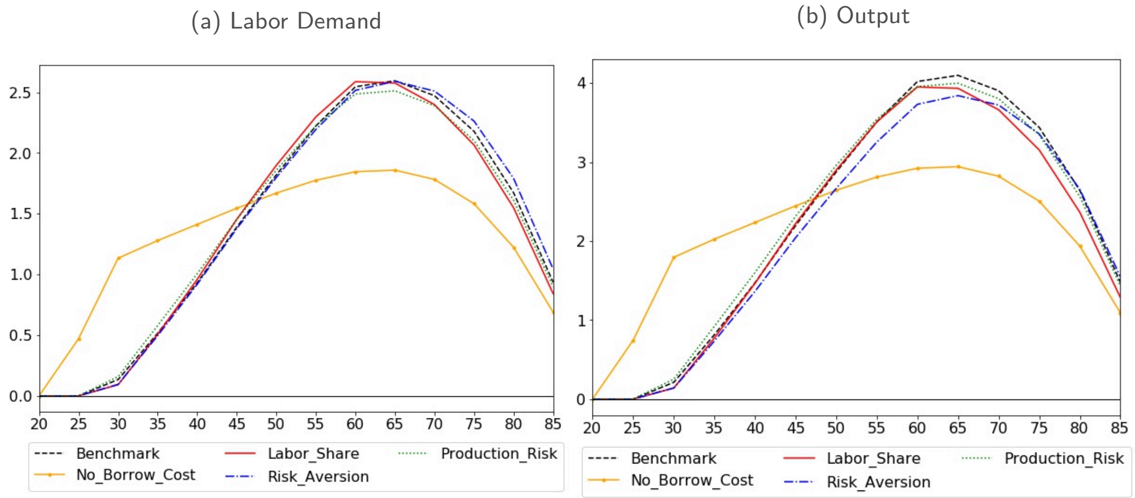
6 – Baseline Scenario: Portfolio Choice II

Figure 8: Bonds and Capital



6 – Baseline Scenario: Labor Demand and Output

Figure 10: Firm's Labor Demand and Output



6 – Baseline Scenario: Risk Premium

Table 2: Rates of Return on Assets

Rates of Return (% p.a.)	Variable	B1	B2	B3	B4	B5
Risk-Free Rate	r^b	9.23	11.26	8.17	10.47	9.13
			*(2.03)**	*(-1.06)*	*(1.24)*	*(-0.11)*
Risky Rate	$E(r)$	11.51	11.46	10.47	12.87	11.52
			(-0.05)	*(-1.03)*	*(1.36)*	*(0.01)*
Risk Premium	$E(r) - r^b$	2.27	0.20	2.30	2.40	2.40
			(-2.07)	*(0.03)*	*(0.12)*	*(0.12)*

*Values in brackets are percentage point deviations from the baseline scenario.

$$\delta_B(\omega, a) = \begin{cases} 0, & \omega \leq 1 \\ \left(\frac{\eta_1}{2}(\omega - 1)^2 + \eta_2(\omega - 1)\right) a, & \omega > 1 \end{cases}$$

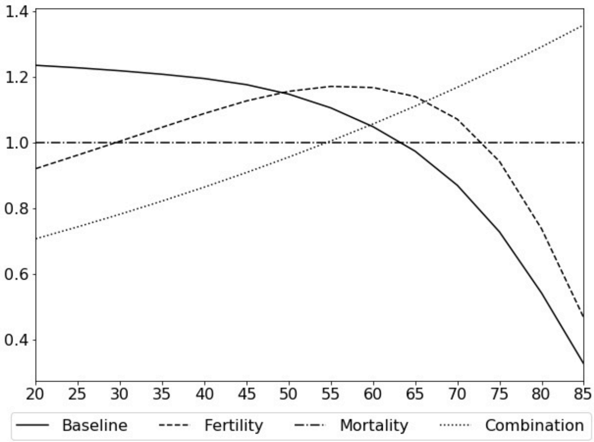
6 – Baseline Scenario: Macroeconomic Variables

Table 3: Aggregate variables

Description	Variable	B1	B2	B3	B4	B5
Output	Y	28.77	28.84	27.82	27.02	29.00
			(0.2%)	*(-3.3%)*	*(-6.1%)*	*(0.8%)*
Consumption	C	28.63	28.84	27.69	26.88	28.83
			(0.7%)	*(-3.3%)*	*(-6.1%)*	*(0.7%)*
Borrowing Costs	Δ	0.14	0.00	0.13	0.14	0.17
			(-100%)	*(-7.1%)*	*(0)*	*(21.4%)*
Capital	K	18.00	18.12	15.94	15.12	18.12
			(0.7%)	*(-11.4%)*	*(-16.0%)*	*(0.7%)*
Bond Demand (Supply)	B	1.52	4.61	1.51	1.56	1.85
			(203.3%)	*(-0.7%)*	*(2.6%)*	*(21.7%)*
Labor Demand (Supply)	L	18.23	18.23	18.23	18.23	18.23
Capital-Labor Ratio	K/L	0.99	0.99	0.87	0.83	0.99
			(0.7%)	*(-11.4%)*	*(-16%)*	*(0.7%)*
Wage	w	1.01	1.01	1.07	0.95	1.02
			(0.2%)	*(5.8%)*	*(-6.1%)*	*(0.8%)*
Pension Tax Rate	τ	0.18	0.18	0.18	0.18	0.18

6 – Aging Scenario: Demographic Composition

Figure 12: Population Structure

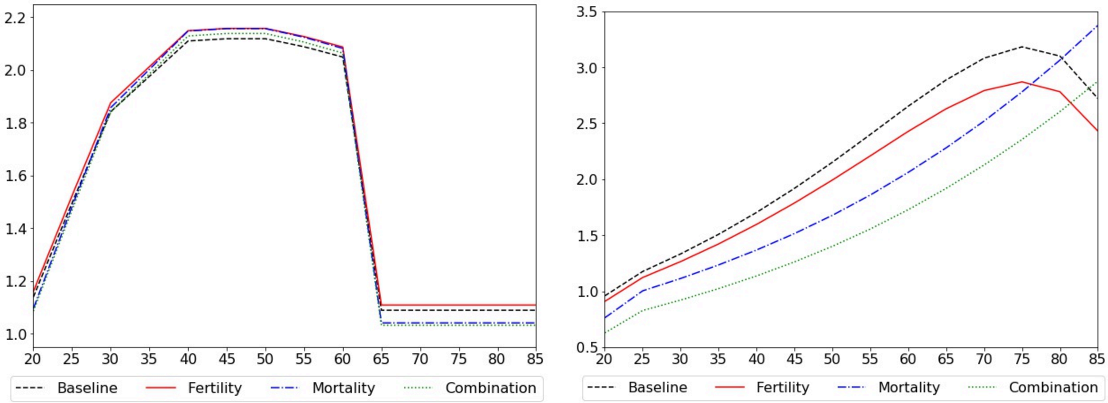


6 – Aging Scenario: Life-Cycle Variables

Figure 13: Labor Income and Consumption

(a) Labor income

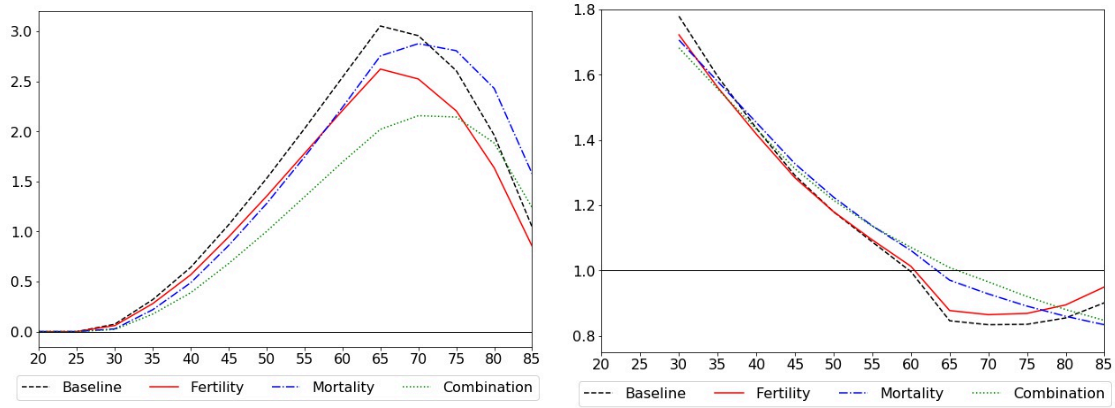
(b) Consumption



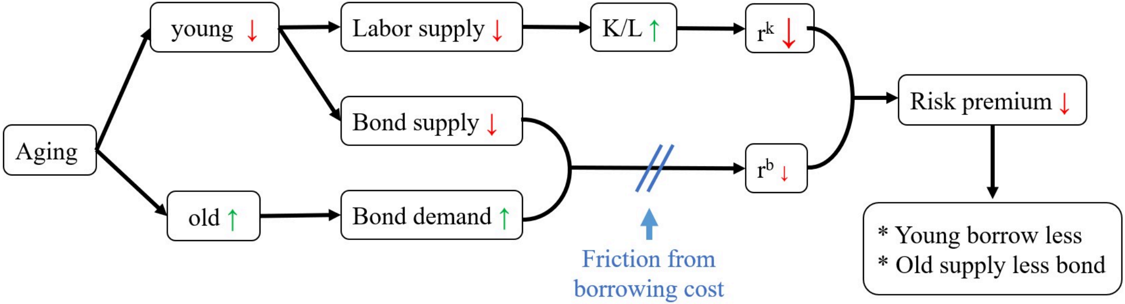
6 – Aging Scenario: Portfolio Choice I

Figure 15: Portfolio Choice

(a) Total Assets (b) Capital Share

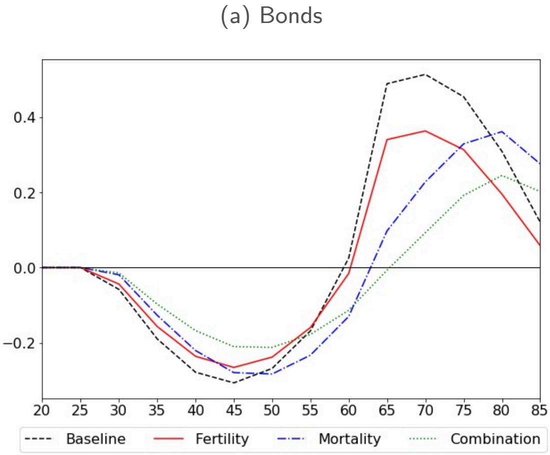


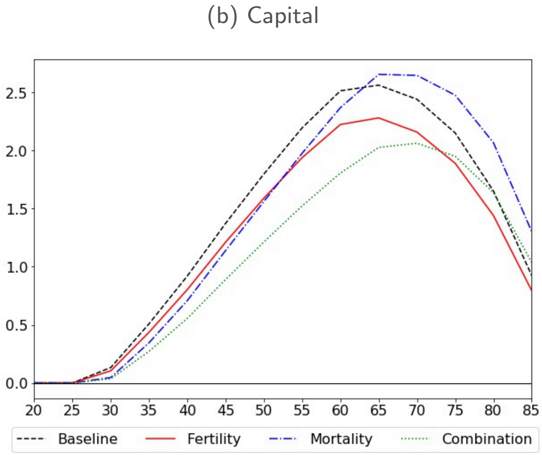
6 – Aging Scenario: Portfolio Choice II



6 – Aging Scenario: Portfolio Choice III

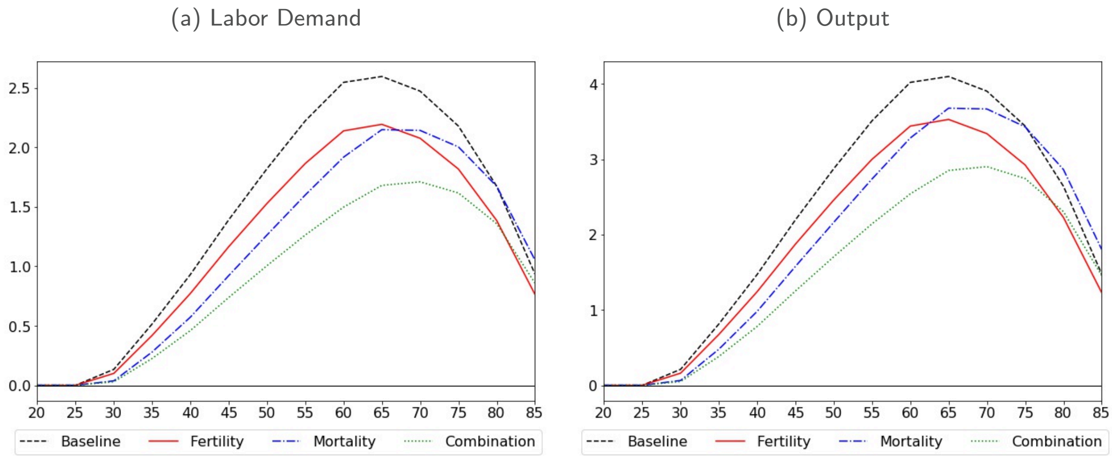
Figure 17: Bonds and Capital





6 – Aging Scenario: Labor Demand and Output

Figure 19: Firm's Labor Demand and Output



6 – Aging Scenario: Per-Capita Variables

Table 4: Per-Capita Variables in Aging Scenarios

Description	Variable	Baseline	Fertility	Mortality	Fert.& Mort.
Output	Y	2.055	1.948	1.909	1.680
			(-5.2%)	*(-7.1%)*	*(-18.3%)*
Consumption	C	2.045	2.001	1.901	1.732
			(-2.1%)	*(-7.1%)*	*(-15.3%)*
Investment	I	0.00	-0.061	0.00	-0.059
			(NA)	*(NA)*	*(NA)*
Borrowing Costs	Δ	0.010	0.008	0.009	0.006
			(-20%)	*(-10%)*	(-40%)
Capital	K	1.286	1.260	1.379	1.195
			(-2.0%)	*(7.2%)*	*(-7.1%)*
Bond Demand (Supply)	B	0.109	0.092	0.094	0.070
			(-15.1%)	*(-13.8%)*	*(-35.2%)*
Labor Demand (Supply)	L	1.302	1.211	1.115	0.990
			(-7.0%)	*(-14.4%)*	*(-24.0%)*

*Values in brackets show percentage deviations for variables in levels and absolute deviations for variables in rates from the baseline scenario.

6 – Aging Scenario: Price Variables

Table 5: Price Variables in Aging Scenarios

Description	Variable	Baseline	Fertility	Mortality	Fert.& Mort.
Capital-Labor Ratio	K/L	0.99	1.04	1.24	1.21
			(1.9%)	*(8.4%)*	*(7.5%)*
Wage	w	1.01	1.03	1.10	1.09
			(1.9%)	*(8.4%)*	*(7.5%)*
Risk-Free Rate (% p.a.)	r^b	9.23	8.88	7.74	7.90
			(-0.35)	*(-1.50)*	*(-1.33)*
Risky Rate (% p.a.)	$E(r)$	11.51	11.13	9.97	10.12
			(-0.37)	*(-1.54)*	*(-1.38)*
Risk Premium (% p.a.)	$E(r) - r^b$	2.27	2.25	2.23	2.22
			(-0.02)	*(-0.04)*	*(-0.05)*
Labor Tax Rate (%)	τ	0.18	0.24	0.30	0.42
			(0.06)	*(0.13)*	*(0.24)*

*Values in brackets show percentage deviations for variables in levels and absolute deviations for variables in rates from the baseline scenario.

7 – Outline

1. Motivation

2. The Model

3. Equilibrium conditions

4. Calibration

5. Computation

6. Results

7. **Conclusion**

7 – Conclusion

- This paper provides a novel framework to endogenize rates of return for risk-free bonds and risky capital in an OLG model.
- The results show an empirically realistic life-cycle portfolio choice pattern and suggest lower rates of interest as well as risk-premium in an aging scenario.
- Portfolio choice patterns can be affected by borrowing costs, labor income levels, production risk, and risk aversion degree.
- Population aging leads to higher capital-labor ratios, and reduces the rates of return on both assets. The bond market shrinks significantly, and capital decreases if the fertility rate declines but increases if the mortality rate declines, leading to structural change in financial markets.
- This approach enables the interaction between financial markets and macroeconomic conditions and can potentially be applied to study impacts of demographic changes on various topics, for instance, the asset price, the macroeconomy, and the fiscal policy.