Epidemics with Behavior

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Bangkok, Milan, and Washington DC

Introduction

The paper:

- studies equilibrium social distancing behavior during an epidemic
- The effect of distancing on
  - the outset
  - the peak
  - the final size of the epidemic

High level Conclusion

- How should interventions be modeled during an epidemic when individuals internalize the cost and benefit of social distancing?
- Interventions that alter contact behavior should not be modeled as a change in the transmission rate but as a change in $\underbrace{\text{the cost of social distancing}}_{\text{preference/deep parameter}}$

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Individuals optimally respond to an epidemic by social distancing

- Distancing is costly, but
- Distancing reduces the individual's probability of getting infected

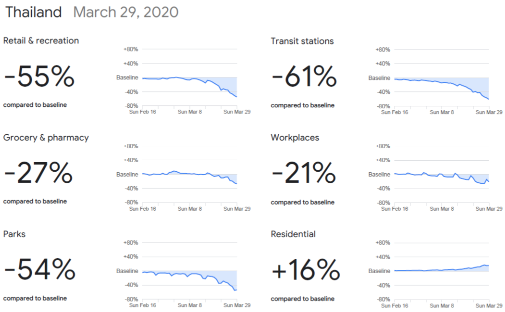

Figure: Google COVID-19 Community Mobility Report (March 29, 2020)

Washington DC

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Figure: 1918 Influenza Pandemic (https://www.nytimes.com/2020/08/03/us/mask-protests-1918.html)

Summary (1/2)

High level Conclusion (Again)

Interventions that alter contact behavior should not be modeled as a change in the transmission rate but as a change in the cost of distancing

- Transmission-suppressing policy (e.g., mask mandate) $\Rightarrow$ Transmission Rate ($\downarrow$) + $\underbrace{\text{Exposure } (\uparrow)}_{\text{Response}}$
- A decrease in the transmission rate
  - in the short-run, *may increase* the peak prevalence
  - in the long-run, decreases the total number of infected individuals
- Cost of distancing ($\downarrow$) $\Rightarrow$ Exposure ($\downarrow$) $\Rightarrow$ Peak prevalence ($\downarrow$), Total infection ($\downarrow$)

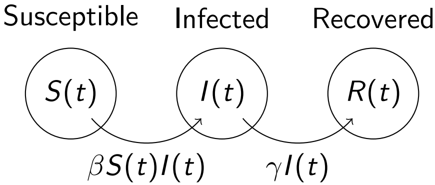
Summary (2/2)

We study equilibrium distancing:

1. Onset
   - The infection takes off only when the transmission rate is moderate
2. Peak prevalence
   - Distancing flattens the curve
   - Transmission rate ($\uparrow$) $\Rightarrow$ Peak prevalence ($\uparrow$ and then $\downarrow$)
   - Cost of distancing ($\downarrow$) $\Rightarrow$ Peak prevalence ($\downarrow$)
3. Final size
   - Distancing decreases the final size of the epidemic
   - Transmission rate ($\uparrow$) $\Rightarrow$ the final size of the epidemic ($\uparrow$)
   - Cost of distancing ($\downarrow$) $\Rightarrow$ the final size of the epidemic ($\downarrow$)

Outline

SIR Model without Behavior: Model



$$\dot{S}(t) = -\beta S(t)I(t)$$
$$\dot{I}(t) = \beta S(t)I(t) - \gamma I(t)$$
$$\dot{R}(t) = \gamma I(t)$$
$$(S(0), I(0), R(0)) = (S_0, I_0, 0) \text{ with } I_0 = 1 - S_0$$

SIR Model without Behavior: Onset, Peak and Final Size

1. Onset: Infection takes off when the transmission rate β is high enough

$$\dot{I}(0) > 0 \iff R_0 := \frac{\beta}{\gamma} S_0 > 1$$

2. Peak Prevalence: Transmission rate $\beta \uparrow \implies$ Peak $\uparrow$
3. Final Size: Transmission rate $\beta \uparrow \implies$ Final Size $S(\infty) \downarrow$

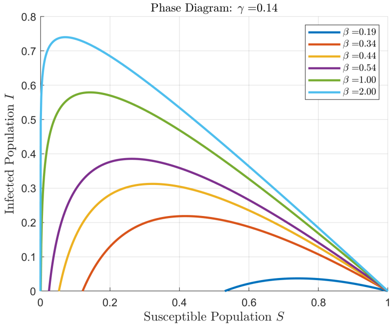


Figure: Solution Paths $(S(t), I(t))_{t \geq 0}$ for Different β

SIR Dynamics with Behavior

- We will incorporate *behavior* $\varepsilon(t) \in [0, 1]$:

$$\dot{S}(t) = -\beta\varepsilon(t)S(t)I(t)$$
$$\dot{I}(t) = \beta\varepsilon(t)S(t)I(t) - \gamma I(t)$$
$$\dot{R}(t) = \gamma I(t)$$
$$(S(0), I(0), R(0)) = (S_0, I_0, 0) \text{ with } I_0 = 1 - S_0$$

- $\varepsilon(t)$: the average exposure level of susceptible individuals at time t:

$$\varepsilon(t) := \frac{1}{S(t)} \int_{i \in S(t)} \varepsilon_i(t) di$$

Distancing

- Susceptible individual i decides exposure $\varepsilon_i(t) \in [0,1]$ at each t
- Distancing is costly but reduces the probability of getting infected:
  - Cost of distancing: $\frac{c}{2}(\underbrace{1-\varepsilon_i(t)}_{\text{distancing}})^2$
  - Rate at which i gets infected: $\beta I(t)\varepsilon_i(t)$
  - Cost of getting infected: $-\eta(>0)$

Distancing Problem

$$\max_{\varepsilon_i(t)\in[0,1]} \pi_S - \frac{c}{2}(1-\varepsilon_i(t))^2 + \beta I(t)\varepsilon_i(t)\eta$$

Distancing in Equilibrium

$$\varepsilon(t) = \max\left(1 + \frac{\eta\beta}{c} I(t), 0\right).$$

▸ Equilibrium (Definition)

Equilibrium

$$\dot{S}(t) = -\beta S(t)I(t)\underbrace{\max\left(1 + \frac{\eta\beta}{c}I(t), 0\right)}_{=\varepsilon(t)}$$

$$\dot{I}(t) = \beta S(t)I(t)\overbrace{\max\left(1 + \frac{\eta\beta}{c}I(t), 0\right)} - \gamma I(t)$$

$$\dot{R}(t) = \gamma I(t)$$

$$(S(0), I(0), R(0)) = (S_0, I_0, 0) \text{ with } I_0 = 1 - S_0$$

Proposition (Symmetric Equilibrium)

An equilibrium exists, is unique, and is symmetric.

Proposition (Single Peak)

The infection peaks (at most) once. At the peak, distancing is maximized.

Onset of an Epidemic

Questions

- When does an infection take off?
- How does behavior affect the onset of the epidemic?

1. SIR model without behavior: the infection takes off when

$$\underbrace{R_0 = \frac{\beta}{\gamma} S_0}_{\text{Basic Reproduction Number}} > 1$$

   - $\beta > \gamma / S_0$ is needed for $\dot{I}(0) > 0$

2. SIR model with behavior: the infection takes off when

$$\underbrace{R_0^b := \frac{\beta}{\gamma} S_0 \varepsilon(0)}_{\text{Behavioral Basic Reproduction Number}} > 1$$

   - A higher β is needed for $\dot{I}(0) > 0$, but *not too high* (R_0^b concave in β)
   - Effect of behavior on the estimation of R_0

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   - Effect of behavior on the estimation of R_0

Onset of the Epidemic

Proposition (Onset of the Epidemic)

1. Suppose $I_0 \leq \frac{1}{1-\frac{4\eta\gamma}{c}}$. Then, $\dot{I}(0) > 0$ iff $\beta \in (\underline{\beta}, \overline{\beta})$, with $\underline{\beta} > \frac{\gamma}{S_0}$
   - Also, $\varepsilon(\cdot) = 1 + \frac{\beta\eta}{c} I(\cdot)$
2. If $I_0 \geq \frac{1}{1-\frac{4\eta\gamma}{c}}$, then $\dot{I}(\cdot) \leq 0$

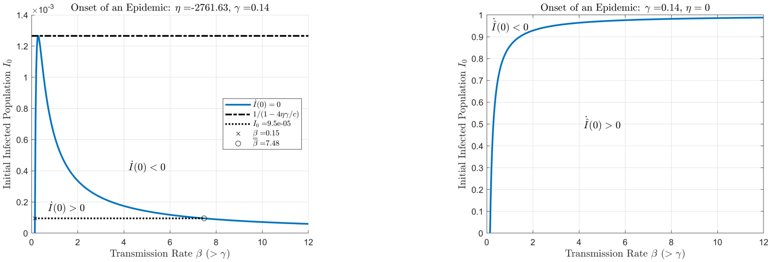


Figure: Onset of the Epidemic. Left: w/ behavior; Right: w/o behavior

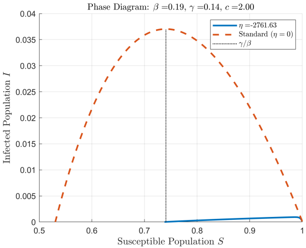
▸ Cost of Infection ▸ Parameters

Phase Diagram

- The slope of I as a function of S:

$$\frac{dI}{dS} = -1 + \frac{\gamma}{\beta}\frac{1}{S}\frac{1}{\max\left(1 + \frac{\beta\eta}{c}I, 0\right)}$$

- Implicitly solvable ▸ Proposition



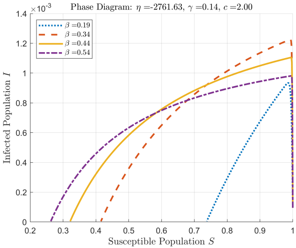


Figure: Left: Solution Paths w/ and w/o behavior; Right: Solution Paths for Different β

- Flattening of the curve
- We will study: Peak and Final Size

Peak Prevalence

Proposition (Peak Prevalence)

1. Assume $I_0 < \frac{1}{1 - \frac{4\eta\gamma}{c}}$. Then,
   the peak prevalence I^* is non-monotonic in $\beta \in (\underline{\beta}, \overline{\beta})$
2. The peak prevalence is non-decreasing in c.
   It is strictly increasing in c whenever $\dot{I}(0) > 0$

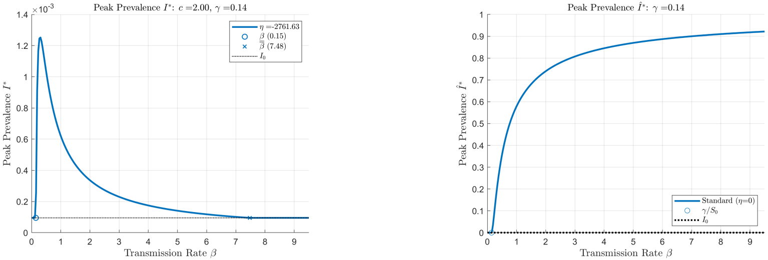


Figure: Peak Prevalence as Function of β. Left: w/ behavior; Right: w/o behavior

Peak Prevalence

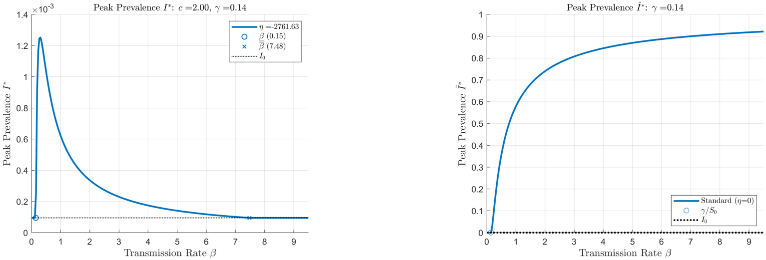


Figure: Peak Prevalence as Function of β. Left: w/ behavior; Right: w/o behavior

- More infectious versus distancing more ("High level conclusion")
- (Mask mandate and) Risk compensation

Final Size of the Epidemic

Proposition (Final Size of Susceptibles)

1. $S_\infty := \lim_{t\to\infty} S(t) \in (0, \frac{\gamma}{\beta})$
2. S_∞ is decreasing in β and c

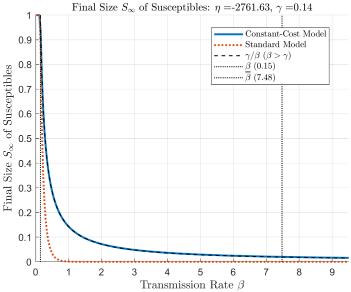


Figure: Final Size of Susceptibles as Function of β

- The effect of β in the short- and long-run

Literature Review (on Behavioral-Epidemiological Models)

- Capasso and Serio (1978) Micro-foundation
- Chen (2012), Dasaratha (2020)
- Rachel (2020a, 2020b), Toxvared (2019, 2020)
- Farboodi, Jarosch, and Shimer (2020)
- Fenichel (2013), Fenichel et al (2011), McAdams (2020), Reluga (2010)
- Survey: Funk, Sarathé, and Jansen (2010), Verelst, Willem, and Beutels (2016), McAdams (2021)

Summary

Equilibrium distancing:

1. Onset
   - The infection takes off only when the transmission rate is moderate
2. Peak prevalence
   - Distancing flattens the curve; the epidemic peaks (at most) once
   - In the short run, transmission rate ($\downarrow$) may lead to peak ($\uparrow$)
3. Final size
   - Distancing decreases the final size of the epidemic
   - In the long run, transmission rate ($\downarrow$) $\Rightarrow$ the final size ($\downarrow$)
4. The effect of transmission rate vs interventions
   - Cost of distancing ($\downarrow$) $\Rightarrow$ peak ($\downarrow$), final size ($\downarrow$)

High level Conclusion

Interventions that alter contact behavior
should not be modeled as a change in the transmission rate
but as a change in the cost of social distancing

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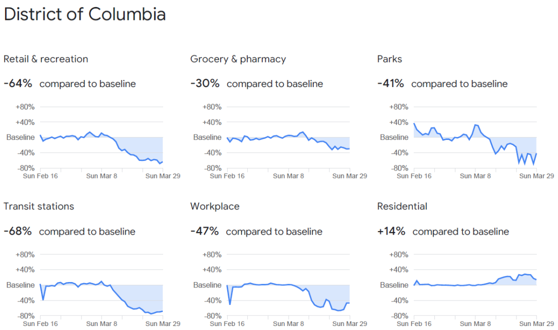

Figure: Google COVID-19 Community Mobility Report (March 29, 2020)

Back

Endogenous Cost of Infection

- The cost of infection $\eta_i(t)$ = the marginal cost of an infinitesimal increase in the (susceptible) individual's infection probability $p_i(t)$
- $p_i(t)$ follows
$$\dot{p}_i(t) = (1 - p_i(t))\beta\varepsilon_i(t)I(t)$$
- Discounting rate: ρ
- Flow payoff: π_S
- The continuation payoff once infected: V_I

Derivation

Distancing Problem

$$\max_{\varepsilon_i(\cdot)\in[0,1]} \int_0^\infty e^{-\rho t}\left\{(1 - p_i(t))[\pi_S - \frac{c}{2}(1 - \varepsilon_i(t))^2] + p_i(t)\rho V_I\right\} dt$$

$$\text{s.t. } \dot{p}_i(t) = \beta\varepsilon_i(t)I(t)(1 - p_i(t))$$

$$p_i(0) = 0$$

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$$\text{s.t. } \dot{p}_i(t) = \beta\varepsilon_i(t)I(t)(1 - p_i(t))$$
$$p_i(0) = 0$$

Endogenous Cost of Infection

Optimal Distancing

Assuming the interior solution,

$$\varepsilon_i(t) = 1 + \frac{\beta}{c}\eta_i(t) I(t)$$

The Adjoint equation for η_i

$$\dot{\eta}_i(t) = \eta_i(t)(\rho + \beta\varepsilon_i(t) I(t)) + (\pi_S - \frac{c}{2}(1 - \varepsilon_i(t))^2 - \rho V_I).$$

Lemma (Bounds for η_i)

$$-\frac{\pi_S - \rho V_I}{\rho} \leq \eta_i(t) \leq -\frac{\pi_S - \rho V_I - \frac{c}{2}}{\rho}$$

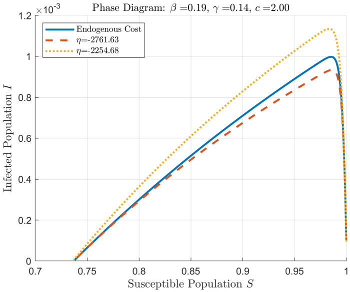
- In a symmetric equilibrium, $\varepsilon = \varepsilon_i$ for all i; let $\eta := \eta_i$

Endogenous Cost of Infection

- η_L and η_H: the lower and the upper bound on η
- $(S_j, I_j, R_j, \varepsilon_j)$ for $j \in \{L, H\}$: the equilibria of the model with the constant cost of infection corresponding to η_j

Proposition (Endogenous Cost of Infection)

In the phase space, the graph of (S_H, I_H) is above that of (S, I), which, in turn, is above that of (S_L, I_L)



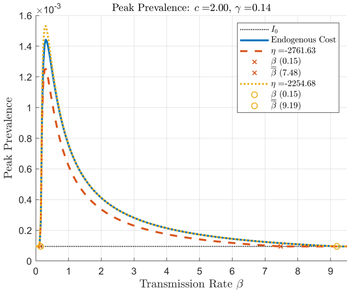


Figure: Left: Solution Path; Right: Peak Prevalence

Connection with Behavioral Epidemiology Models

- Capasso and Serio (1978):

$$\begin{aligned}\dot{S}(t) &= -g(I(t))S(t)\\ \dot{I}(t) &= g(I(t))S(t) - \gamma I(t)\\ \dot{R}(t) &= \gamma I(t)\\ (S(0), I(0), R(0)) &= (S_0, I_0, 0) \text{ with } I_0 = 1 - S_0\end{aligned}$$

- The standard SIR model: $g(I(t)) = \beta I(t)$
- Our model provides micro-foundation:
  - Cost of distancing $\Longrightarrow g(I(t)) = \beta\varepsilon(I(t))I(t)$
  - Given g, $\exists$ cost of distancing such that $g(I(t)) = \beta\varepsilon(I(t))I(t)$

Connection with Behavioral Epidemiology Models

- In particular, Capasso and Serio (1978) consider

$$g(I(t)) = \frac{\beta I(t)}{1 + \frac{I(t)}{\alpha}}$$

- The underlying cost function is

$$c(1 - \varepsilon(t)) = -\eta\beta\alpha\varepsilon(t) - \log(\varepsilon(t))$$

- When $-\eta\beta\alpha = 1$, the cost function reduces to (up to a constant $+1$) Farboodi, Jarosch, and Shimer (2020) in the macroeconomics literature

▸ Back (Literature Review)

Equilibrium $(S, I, R, (\varepsilon_i)_{i\in[0,1]})$

1. (S, I, R) follow

$$\begin{aligned}
\dot{S}(t) &= -\beta\varepsilon(t)S(t)I(t) \\
\dot{I}(t) &= \beta\varepsilon(t)S(t)I(t) - \gamma I(t) \\
\dot{R}(t) &= \gamma I(t) \\
(S(0), I(0), R(0)) &= (S_0, I_0, 0) \text{ with } I_0 = 1 - S_0 \\
\varepsilon(t) &= \frac{1}{S(t)} \int_{j\in S(t)} \varepsilon_j(t)dj
\end{aligned}$$

2. ε_i solves, given $(\varepsilon_j)_{j\neq i}$,

$$\max_{\varepsilon_i(t)\in[0,1]} \pi_S - \frac{c}{2}(1-\varepsilon_i(t))^2 + \beta\varepsilon_i(t)I(t)\eta$$

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Onset of the Epidemic

- Previous Slide: the infection does not take off if β is too high
- Current Slide: the infection does not take off if the cost of infection $-\eta$ is too high

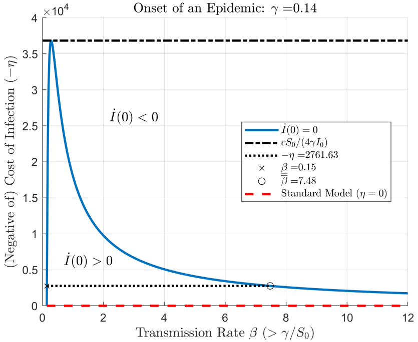

Figure: Onset of the Epidemic

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Solution Path

Proposition (Solution Path)

When $\varepsilon(0) > 0$, the solution path $(S(t), I(t))_{t\geq 0}$ is implicitly determined by

$$S = \frac{\exp\left(\frac{\beta^2\eta}{2\gamma c}\left(S + I + \frac{c}{\beta\eta}\right)^2\right)}{\exp\left(\frac{\beta^2\eta}{2\gamma c}\left(1 + \frac{c}{\beta\eta}\right)^2\right)\frac{1}{S_0} + 2\beta\sqrt{\frac{(-\eta)}{2\gamma c}}\int_{\beta\sqrt{\frac{-\eta}{2\gamma c}}\left(S+I+\frac{c}{\beta\eta}\right)}^{\beta\sqrt{\frac{-\eta}{2\gamma c}}\left(1+\frac{c}{\beta\eta}\right)} e^{-v^2}dv}.$$

The case with $\varepsilon(0) = 0$ is similar.

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Endogenous Cost of Infection

Continuation Payoff of being Infected

$$V_I = \frac{1}{\rho + \gamma}\left(\pi_I + \frac{\gamma}{\rho}\pi_R\right)$$

- Suppose an individual gets infected at time τ
- The probability of being recovered after time $\tau + t$: $1 - e^{-\gamma t}$
- Thus,

$$V_I = \int_0^{\infty} e^{-\rho t}\left(e^{-\gamma t}\pi_I + (1 - e^{-\gamma t})\pi_R\right) dt = \frac{1}{\rho + \gamma}\left(\pi_I + \frac{\gamma}{\rho}\pi_R\right)$$

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Parameters for Numerical Simulations

Table: Table of Baseline Parameters for Numerical Analysis.

Parameter	Description	Value	Source
γ	Recovery Rate	$1/7$	
β	Transmission Rate	$0.3 + \gamma$	Farboodi, Jarosch, and Shimer (2020)
I_0	Initial Seed of Infections	0.95×10^{-4}	Based on death toll in the US before March 19, 2020
$\tilde{\rho}$	Discount Rate	$0.05/365$	Farboodi, Jarosch, and Shimer (2020)
λ	Arrival Rate of Cure	$0.67/365$	Farboodi, Jarosch, and Shimer (2020)
c	Cost of Distancing	2	Normalization
π_S	Flow Payoff of Susceptibles	0	Normalization
η	Cost of Infection	$\{-2761.63, -2254.68\}$	Hall, Jones, and Klenow (2020)

Back (Constant Cost) Back (Endogenous Cost)