The Natural Rate of Interest through a Hall of Mirrors

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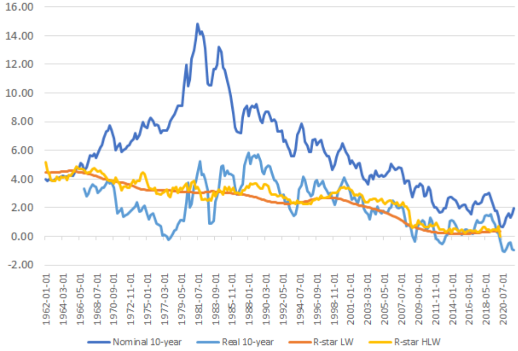
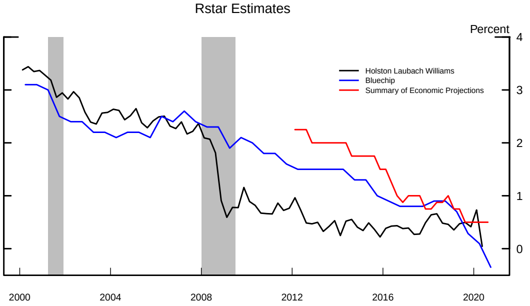
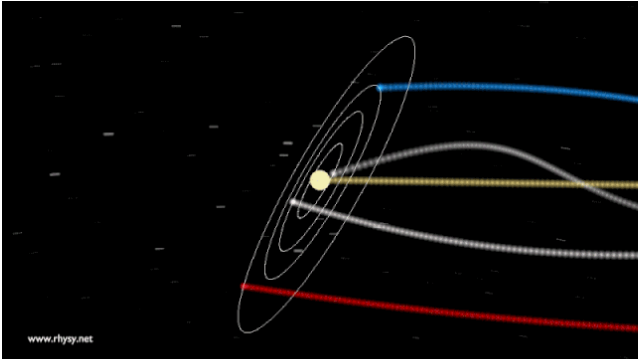
R-star and monetary policy: the typical narrative

- Secular decline in interest rates due to falling natural rate of interest (r^*)
- Linked to structural forces, e.g. slowing productivity growth, ageing population
- As CB has no control over these, it must lower rate just to be neutral
- With ELB on nominal rates, CB can fall into a liquidity trap



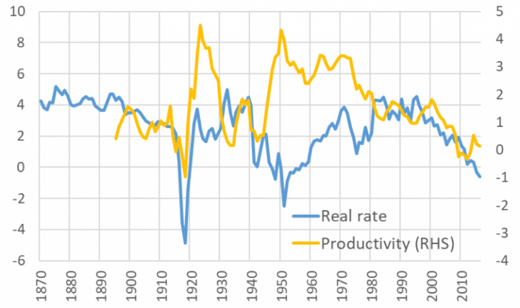


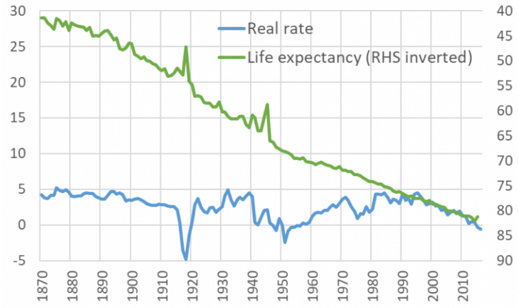
The celestial system analogue



- r^* is the 'star' around which monetary policy orbits
- Monetary policy cannot influence r^* any more than planets can affect the path of the sun

Puzzle 1: We know surprisingly little about r^* drivers...

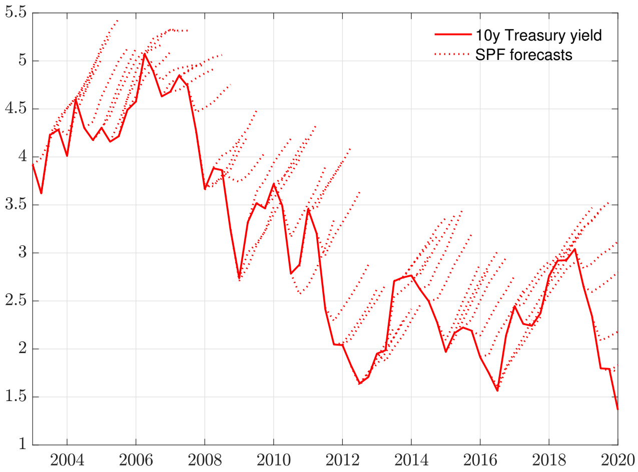




Source: Borio, Disyatat, Juselius and Rungcharoenkitkul (IJCB, forthcoming)

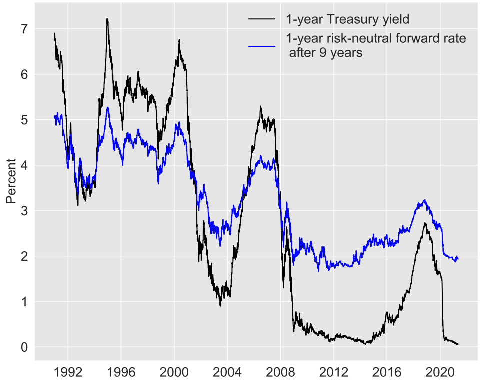
- Weak evidence that usual suspects like productivity growth, demographics, inequality, relative price of capital, MPK etc. explain real rates over long horizons

...indeed, nobody expected the secular decline in r^*

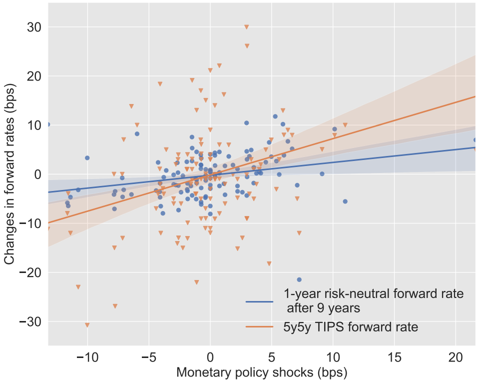


- If persistent & predictable forces like demographics were responsible for r*, why are interest rates so hard to forecast?

Puzzle 2: Monetary policy appears to affect r^*...



(a) Short and risk-neutral forward rates

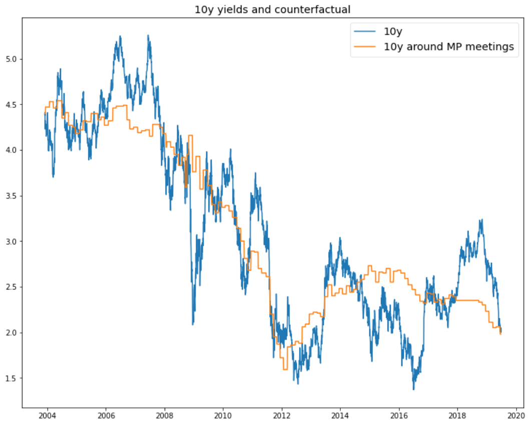


(b) Long forward rates and MP shocks

Source: Risk-neutral yields are from Adrian et al. (2013)

- Forward *risk-neutral* rate (market view of r^*) is sensitive to monetary policy
- Violation of long-run money neutrality

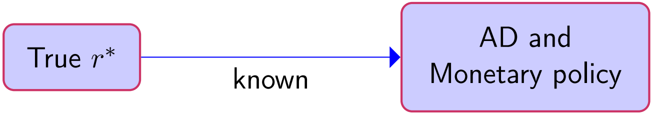
…explaining most of the trend decline in yields



Source: Replicating Hillenbrand (2022)

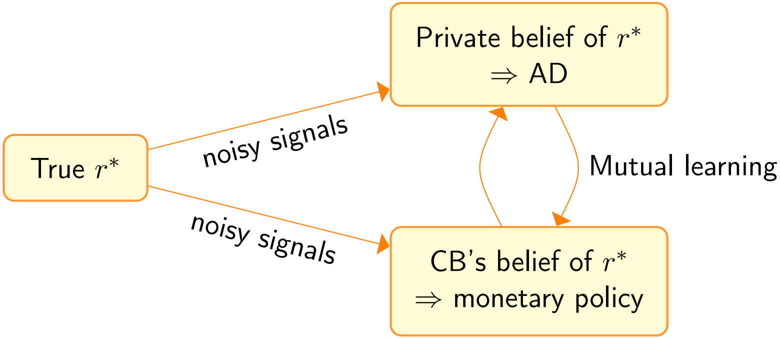
- Trend shifts in 10-year yields occurred almost exclusively around FOMC meetings
- Why should MP decisions be so informative about long-term interest rates?

Standard model and assumptions



- Everyone knows the process governing real interest rate trends ✗
- MP should not affect market expectations of r^* ✗

This paper



- r^* process is unknown and agents must learn the r^* value
  $\Rightarrow$ Beliefs matter for r^*, not just exogenous fundamentals ✓
- Agents rely on each other to learn about r^*
  $\Rightarrow$ MP and cyclical shocks could affect r^* through mutual learning ✓

New Keynesian model with learning about the natural rate

- 2-period NK model with incomplete information about r^{**} ("True" r^*)

$$\begin{aligned}\tilde{y} &= -\frac{1}{\sigma}\left(r - E^h[r^{**}] - u_h\right) \\ \pi &= \kappa \tilde{y} \\ r &= E^c[r^{**}] + \phi_\pi \pi + \phi_y \tilde{y} + u_c\end{aligned}$$

- Two natural rate concepts: $r^* \equiv E^h[r^{**}]$ and $\hat{r}^* \equiv E^c[r^{**}]$
- Each agent $i \in \{h, c\}$ observes a noisy private signal about r^{**}:

$$\begin{aligned} s_i &= r^{**} + \epsilon_i, \qquad \epsilon_i \sim \mathcal{N}\left(0, \sigma_{\epsilon i}^2\right) \\ r^{**} &\sim \mathcal{N}\left(0, \sigma_z^2\right), \qquad u_i \sim \mathcal{N}\left(0, \sigma_{ui}^2\right)\end{aligned}$$

- CB chooses r (given $E^c[r^{**}]$, π and $\tilde{y}$), HH chooses $\tilde{y}, \pi$ (given $E^h[r^{**}]$ and r)
- Each knows that the other's action reveals private signal about r^{**}

Common knowledge benchmark

- Both players know the true signal precision
- Optimal filtering produces equilibrium beliefs:

$$E^i[r^{**}] = g_{si} s_i + g_{ai}(g_{sj} s_j + u_j)$$

with the gain parameters

$$\begin{pmatrix} g_{si} \\ g_{ai} \end{pmatrix} = \frac{1}{g_{sj}^2 \left(\sigma_{\epsilon i}^2 + \sigma_{\epsilon j}^2 + \sigma_{\epsilon i}^2 \sigma_{\epsilon j}^2\right) + \sigma_{uj}^2 \left(\sigma_{\epsilon i}^2 + 1\right)} \begin{pmatrix} g_{sj}^2 \sigma_{\epsilon j}^2 + \sigma_{uj}^2 \\ g_{sj} \sigma_{\epsilon i}^2 \end{pmatrix}$$

- Nests Laubach and Williams (2003): HH has perfect info, CB tries to extract it
- Our general setup lets both learn from each other – arguably more realistic

Hall-of-mirrors equilibrium

- Agents i hold subjective beliefs $\sigma_{\epsilon j}^{|i}$ about signal precision $\sigma_{\epsilon j}$
- Equilibrium with misperception:

$$\hat{E}^i\left[r^{**}\right] = g_{si}^{|i} s_i + g_{ai}^{|i}\left(g_{sj}^{|j} s_j + u_j\right) + \frac{g_{ai}^{|i}}{1 - g_{ai}^{|i} g_{aj}^{|j}}\left[\left(g_{aj}^{|j} - g_{aj}^{|i}\right)\left(g_{si}^{|i} s_i + u_i\right) + \left(g_{ai}^{|i} - g_{ai}^{|j}\right) g_{aj}^{|j}\left(g_{sj}^{|j} s_j + u_j\right)\right]$$

- When each overestimates the other's information ($\sigma_{\epsilon j}^{|i} < \sigma_{\epsilon j}$)
  1. Each loads too much on the other's actions: $g_{ai}^{|i} > g_{ai}$
  2. Each doesn't realise the other is doing the same: $g_{aj}^{|j} > g_{aj}^{|i}$

Proposition (Hall-of-mirrors effect)

- *$r^*, \hat{r}^*$ overreact to u_c, u_h with unbounded magnitude*
- *Private sector and central bank confuse cyclical shocks for r-star movements*

General dynamic setting

- Dynamic infinite-horizon NK model

$$\tilde{y}_t = \mathbb{E}_t^h[\tilde{y}_{t+1}] - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t^h[\pi_{t+1}] - \mathbb{E}_t^h[r_t^{**}] - u_{ht}\right)$$
$$\pi_t = \beta\mathbb{E}_t^h[\pi_{t+1}] + \kappa\tilde{y}_t + u_{pt}$$
$$i_t = \rho_i i_{t-1} + (1-\rho_i)\left(\mathbb{E}_t^c[r_t^{**}] + \phi_\pi\pi_t + \phi_y\tilde{y}_t + u_{ct}\right)$$

- Signal and r_t^{**} processes

$$r_t^{**} = r_{t-1}^{**} + v_t, \qquad v_t \sim \mathcal{N}\left(0, \sigma_r^2\right)$$
$$s_{it} = r_t^{**} + e_{it} \qquad \text{(Private signals)}$$
$$x_t = r_t^{**} + f_t \qquad \text{(Public signals)}$$

- Shocks u_{ht}, u_{pt}, u_{ct}, e_{it}, $f_t \sim$ AR(1)
- Private sector observes u_{ht}, u_{pt}, sets $\tilde{y}_t, \pi_t$; central bank observes u_{ct}, sets i_t
- Agents learn from observing $\tilde{y}_t, \pi_t, i_t$, updating r_t^{**} beliefs over time

Solving the dynamic model

- Defining $Z_t \equiv (r_t^{**}, e_{ht}, e_{ct}, f_t, u_{ht}, u_{pt}, u_{ct})'$, we can write

$$Z_t = A_z Z_{t-1} + q_t, \qquad q_t \sim \mathcal{N}(0, \Sigma_q)$$

  Common knowledge: Everyone uses the correct A_z and Σ_q
  Hall-of-Mirrors: Agents hold subjective beliefs about A_z and Σ_q

- Complication: we must keep track of agent i's belief of agent j's n-order belief:

$$X_{it|t-1} \equiv E_{i|t-1}\left(E_{jt}^{(n)} Z_t\right)_{n=0}^{\infty} \sim \mathcal{N}(m_{it-1}, P_i)$$

- To solve for equilibrium, we guess m_{jt} takes a linear form

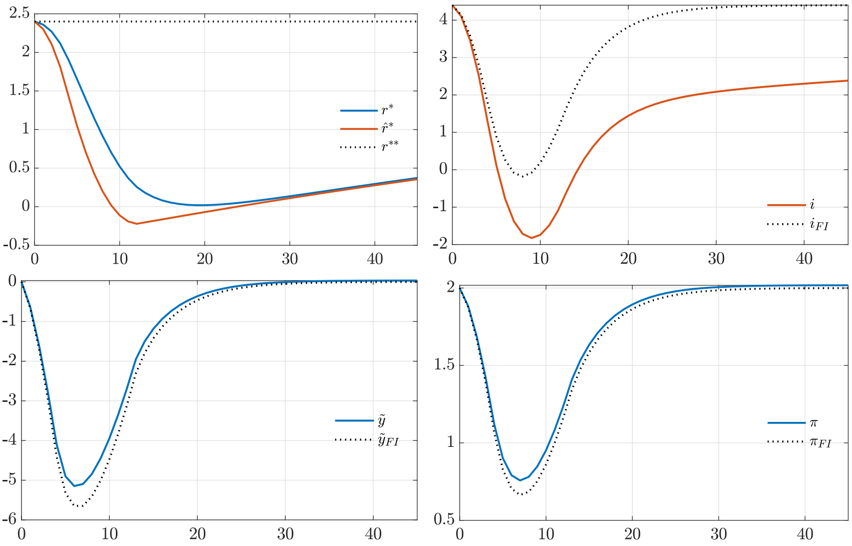
$$m_{jt} = \Phi_j m_{jt-1} + \Psi_j m_{it-1} + \Omega_j Z_t$$

  compute implied states, verify that the linear form is correct, and iterate until Φ_j, Ψ_j, Ω_j converge.

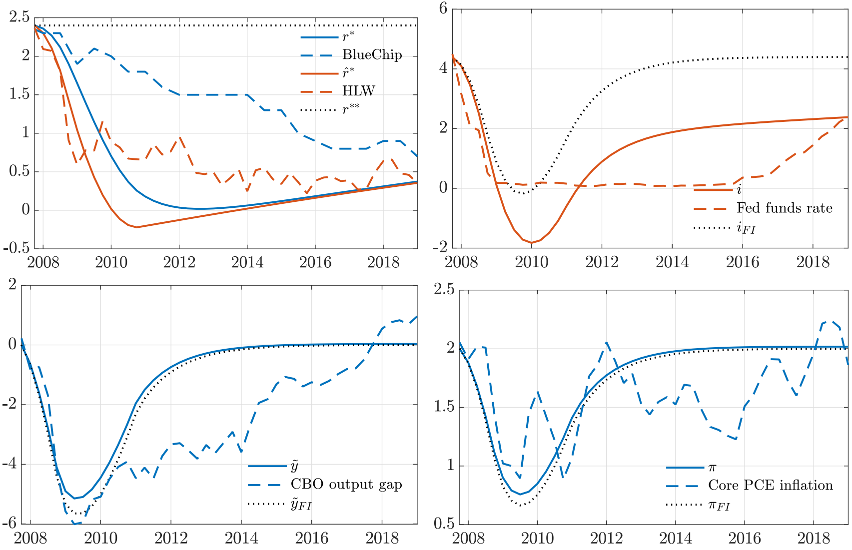
Quantitative implications

- We solve the dynamic model numerically with calibrated parameters
- Standard macro parameters Table
- Information structure
  - Initial $r_0^{**} = 2.4\%$, $\sigma_r = 0.05$ (from Holston et al. (2017))
  - Private signal noise: $\sigma_{\epsilon i} = \infty$ (actual), $\sigma_{\epsilon i}^{|j} = 0.2$ (perceived); i.e. each thinks the other knows something useful when in fact nobody does
  - Uninformative public signal noise $\sigma_\eta = 3$
- Simulation exercises
  1. Persistent demand shocks mimicking initial GFC contraction
  2. One-time expansionary MP shock
  3. Persistent negative AS shocks + positive AD shocks (current situation)

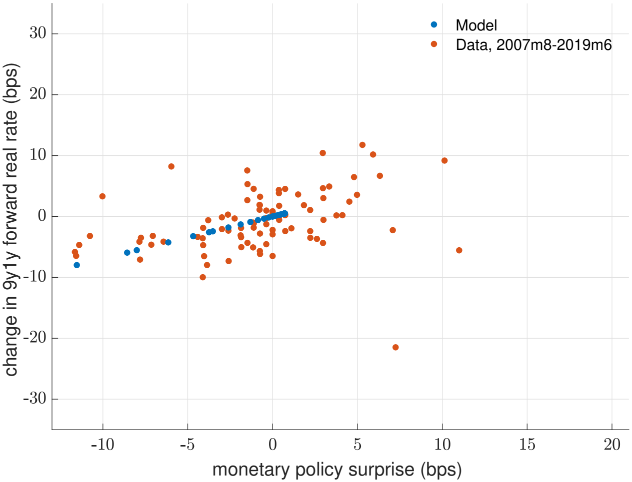
Demand shocks prompt a persistent decline in r^*



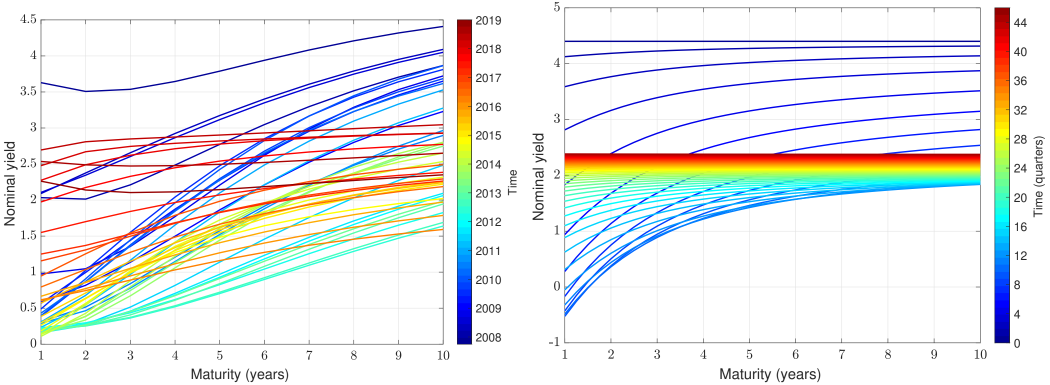
Demand shocks simulation + data



Explaining excess sensitivity of expected r^* to MP

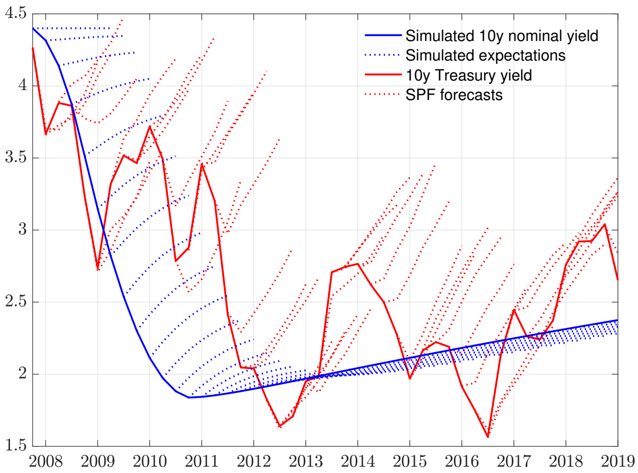


Explaining excess sensitivity of long-term yields to MP



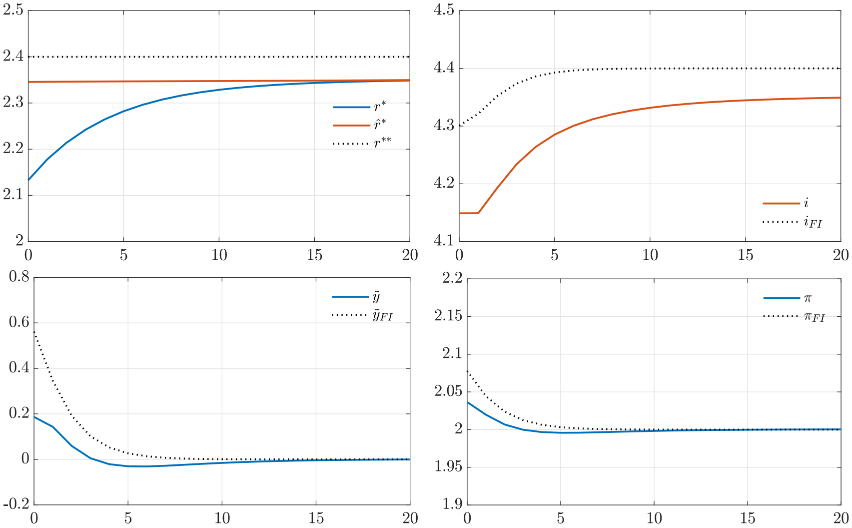
(a) Data (b) Model

Explaining failure to predict the interest rate trend



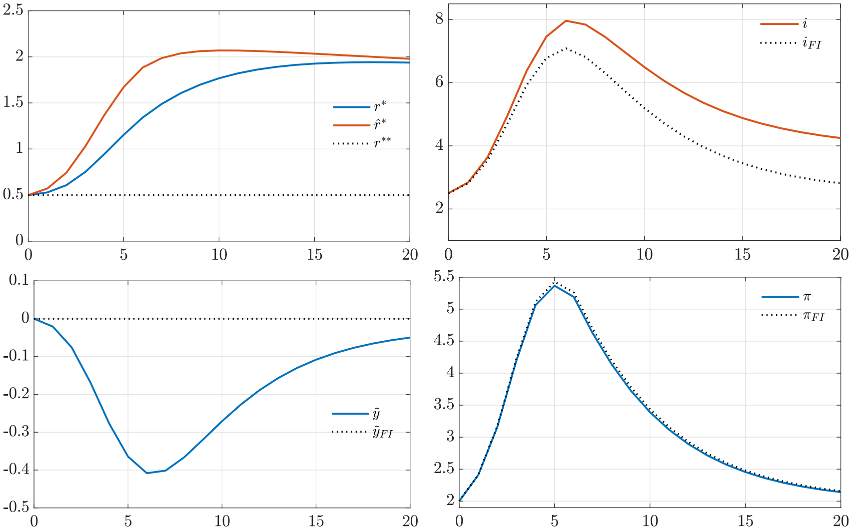
Robustness

MP shock loses force by inducing r^* changes



Analytical

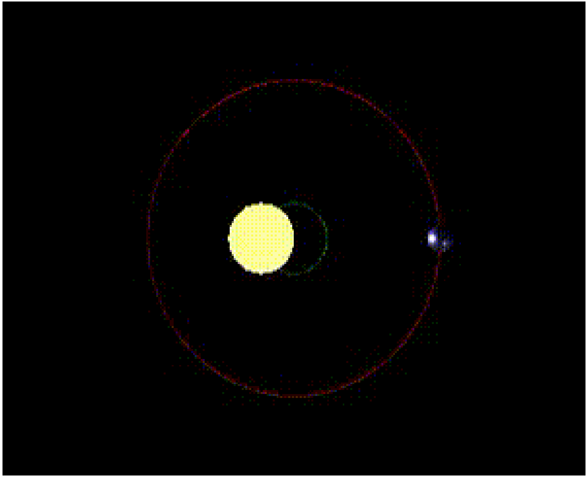
Hall-of-mirrors and endogenous stagflation



Conclusion

- r^* is endogenous to monetary policy and cyclical shocks, due to self-reinforcing two-way learning feedback
- This hall-of-mirror mechanism can quantitatively explain many post-GFC salient features: low for long interest rates, declining estimates of r^*, slow output recovery, low inflation
- Explain puzzling excess sensitivity of long-term interest rate to MP and the violation of money neutrality in the data
- Far-reaching policy implications, both when rates are falling and rising

Epilogue: more precise celestial physics



Source: nasa.gov

Extra slides

Macroeconomic implications

Full information $\tilde{y} = \lambda\left(u_h - u_c\right)$

Common knowledge $\tilde{y} = \lambda\left[(1 - g_{ac})(g_{sh}s_h + u_h) - (1 - g_{ah})(g_{sc}s_c + u_c)\right]$

Hall of mirrors $\tilde{y} = \lambda\left[b_h(g_{sh}^{|h}s_h + u_h) - b_c(g_{sc}^{|c}s_c + u_c)\right]$

where $g_{si}^{|i} < g_{si}$ and $b_i < 1 - g_{aj}$

Proposition (Muted macroeconomic impact)

In the hall-of-mirrors equilibrium:

- *Output gap and inflation under-react to shocks*
- *Monetary policy less effective*

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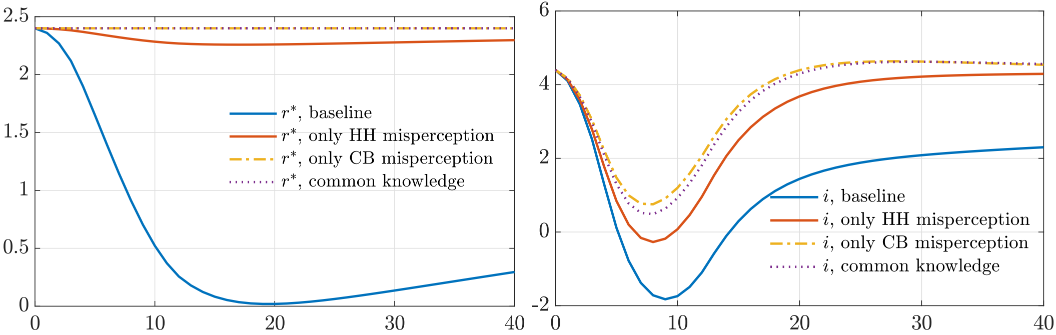
Calibration Back

Parameter	Symbol	Value	Parameter	Symbol	Value
Inverse EIS	σ	6	Initial value of r^{**}	ζ_0	2.4 %
Phillips curve slope	κ	0.015	S.d. of r^{**} shock	σ_ζ	0.05
Discount factor	β	0.9941	Steady-state inflation	π^*	2 %
Rule coefficient on inflation	ϕ_π	1.5	Autocorr. of cost-push shock	$\rho_{u\pi}$	0.8
Rule coefficient on output gap	ϕ_y	0.125	S.d. of cost-push shock	$\sigma_{u\pi}$	0.1
Rule coefficient on lagged rate	ρ_i	0.7	Autocorr. of public signal noise	ρ_f	0
Autocorr. of policy shock	ρ_{uc}	0.7	Autocorr. of private signal noise	ρ_{ei}	0
S.d. of policy shock	σ_{uc}	0.1	S.d. of public signal noise	σ_η	3
Autocorr. of demand shock	ρ_{uh}	0.8	S.d. of private signal noise	$\sigma_{\epsilon i}$	∞
S.d. of demand shock	σ_{uh}	0.2	Perceived —	$\sigma_{\epsilon i}^{\vert j}$	0.2

Robustness to alternative learning calibrations



- Less misperception: raise $\sigma_j^{|i}$ 10-fold
- Better information: lower σ_i from ∞ to 1
- More volatile fundamentals: increase σ_z to upperbound of HLW

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