Rational Bubble Theory on Cryptocurrency

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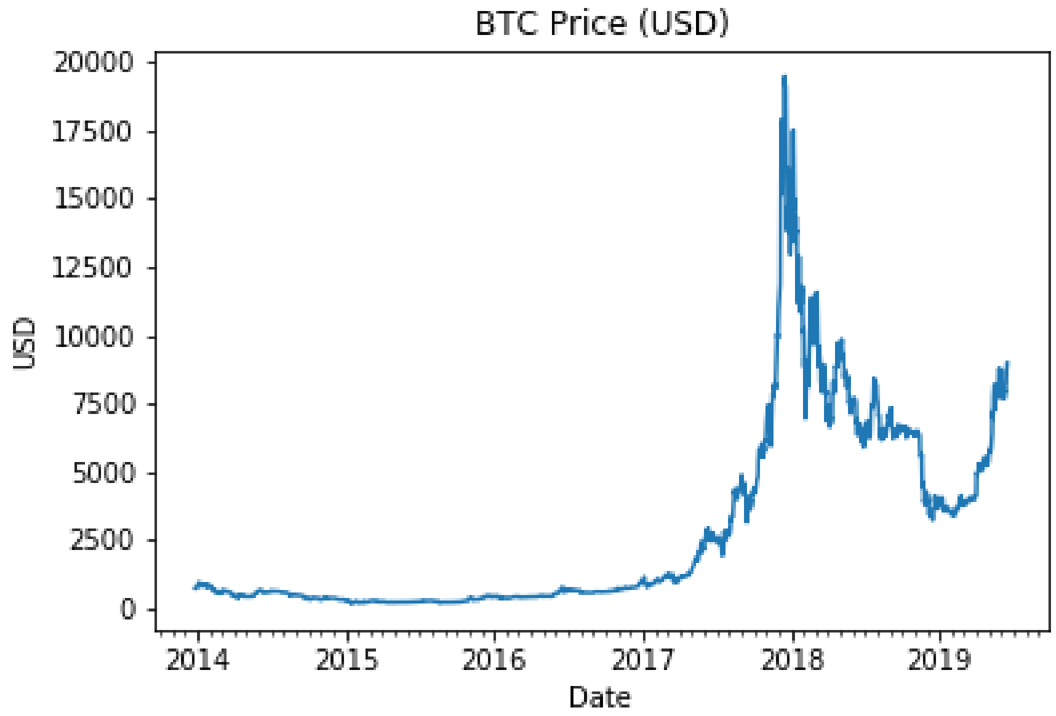
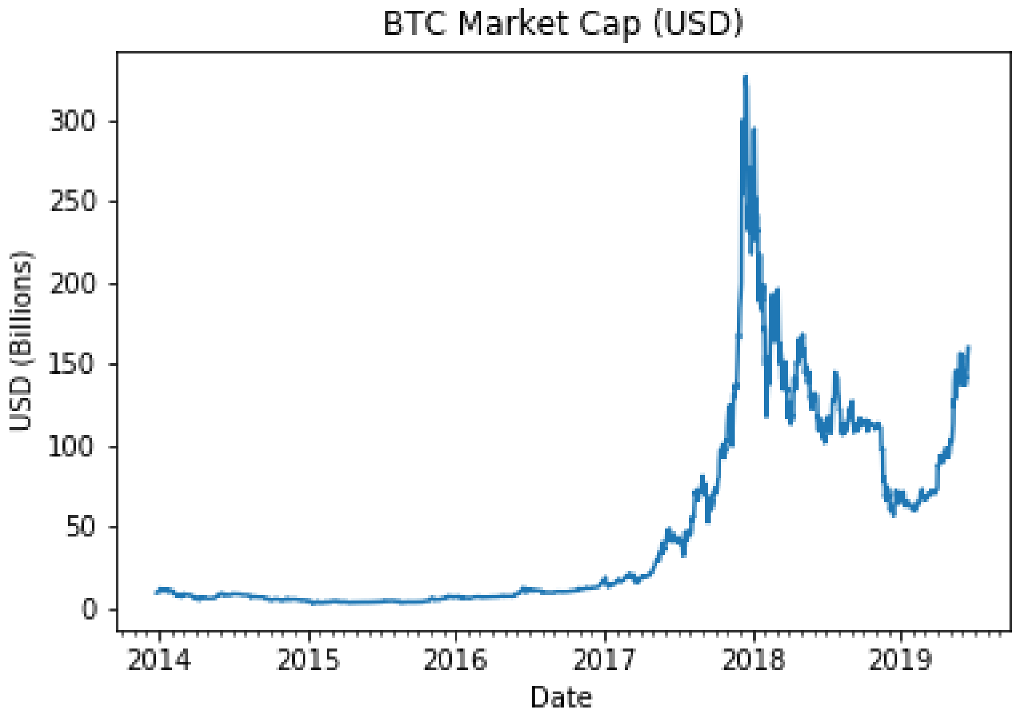
Introduction

- Purpose
  - Understand and theoretically characterize rational bubbles in cryptocurrency
- Motivation
  - A relatively new form of asset
  - Gained popularity across the world
  - Starting to be regulated by many governments

Introduction

- Price and Volatility of a Major Cryptocurrency





Introduction

- Cryptocurrencies characteristics
  - Digital Assets
  - Transparent Fixed Global Supply
  - Traded across borders
  - Negligible Transaction Cost
  - Decentralized

Literature Review

- Existence condition is usually that the no-bubble interest is lower than growth rate Weil(1987)
- Convinced that bubbles do not violate the rules of optimizing behavior and general equilibrium Tirole (1985)
- Bubbles also serve in correcting inefficiencies Martin and Ventura (2010)
- Economies can contain both efficient and inefficient investors and welfare can be increased when inefficient investors buy bubble Martin and Ventura (2010)

Literature Review

- Found that intrinsic value of bitcoin is zero and empirical results show that bitcoin price volatility comes from speculation and investment sentiment Cheah and Fry (2015)
- Found that bitcoin price surge when there are countries facing economic crisis Su, Li, Tao, & Si (2018)

Model Setup

- Two countries Overlapping Generation Model
- Endowments when young: e^1, e^2
  - $e^1 < e^2$
- Populations: N^1, N^2
  - Zero growth
  - Identical preference

Model Setup

- The probability of bubbles bursting π is governed by a Markov process

	$P_t>0$	$P_t=0$
$P_{t+1}>0$	$1-\pi$	0
$P_{t+1}=0$	π	1

Model Setup

- Agents maximize the utility function

$$U(c_{1t}) + \beta E\big(U(c_{2t+1})\big) \quad (1)$$

- Subject to

$$c_{1t} + p_t m_t^1 = e^1 \quad (2)$$

$$c_{2t+1} = p_{t+1} m_t^1 \quad (3)$$

Where m is the quantity of cryptocurrency demanded

Solving the Model

- The first order conditions are

$$U'\left(e^1 - p_t m_t^1\right)(-p_t) + \beta(1-\pi)U'(p_{t+1}m_t^1)(p_{t+1}) = 0 \qquad (4)$$

$$U'\left(e^2 - p_t m_p^2\right)(-p_t) + \beta(1-\pi)U'(p_{t+1}m_t^2)(p_{t+1}) = 0 \qquad (5)$$

Cryptocurrency Market Clearing Condition

$$N^1 m_t^1 + N^2 m_t^2 = M \qquad (6)$$

Solving the Model

- From (4)-(6), the equilibrium system can be written as

$$U'\left(e^1 - p_t m_t^1\right)(-p_t) = \beta(1-\pi)U'(p_{t+1}m_t^1)(p_{t+1}) \quad (7)$$

$$U'\left(e^2 - p_t \frac{M}{N^2} + \frac{N^1}{N^2} p_t m_t^1\right)\left(\frac{N^1}{N^2} p_t\right) = \beta(1-\pi)U'\left(p_{t+1}\frac{M}{N^2} - \frac{N^1}{N^2} p_{t+1} m_t^1\right)(p_{t+1}) \quad (8)$$

Solving the Model

- Proposition 1: There exist steady state $\bar{p} > 0$ and $\overline{m}^1 \in (0, M)$.

$$U'(e_1^1 - \bar{p}\overline{m}^1) = \beta(1-\pi)U'(\bar{p}\overline{m}^1) \quad (9)$$

$$U'\left(e_1^2 - \bar{p}\frac{M}{N^2} + \frac{N^1}{N^2}\bar{p}\overline{m}^1\right) = \beta(1-\pi)U'\left(\frac{M}{N^2}\bar{p} - \frac{N^1}{N^2}\bar{p}\overline{m}^1\right) \quad (10)$$

- To study the characteristics of the steady state, linearize around the steady state

$$(p_{t+1} - \bar{p}) = x(p_t - \bar{p}) \quad (11)$$

The Slope

- $x = (a) - (b)\left[\frac{(c)-(d)}{(f)+(g)}\right]$, where

$$a = \frac{U'\left(e^1 - \bar{p}\bar{m}^1\right) - \bar{p}\bar{m}^1 U''\left(e^1 - \bar{p}m^1\right)}{\beta(1-\pi)[U'(\bar{p}\bar{m}^1) + \bar{p}\bar{m}^1(U''(\bar{p}\bar{m}^1)]}$$

$$b = \frac{U''\left(e^1 - \bar{p}m^1\right) + \beta(1-\pi)U''(\bar{p}\bar{m}^1)}{\beta(1-\pi)[U'(\bar{p}\bar{m}^1) + \bar{p}\bar{m}^1(U''(\bar{p}\bar{m}^1)]}$$

$$c = \frac{U'\left(e^1 - \bar{p}\bar{m}^1\right) - \bar{p}\bar{m}^1 U''\left(e^1 - \bar{p}m^1\right)}{U'(\bar{p}\bar{m}^1) + \bar{p}\bar{m}^1 U''(\bar{p}\bar{m}^1)}$$

$$d = \frac{U'\left(e^2 - \frac{M}{N^2}\bar{p} + \frac{N^1}{N^2}\bar{p}\bar{m}^1\right) - (\frac{M}{N^2}\bar{p} - \frac{N^1}{N^2}\bar{p}\bar{m}^1)U''\left(e^2 - \frac{M}{N^2}\bar{p} + \frac{N^1}{N^2}\bar{p}\bar{m}^1\right)}{[U'\left(\frac{M}{N^2}\bar{p} - \frac{N^1}{N^2}\bar{p}\bar{m}^1\right) + (\frac{M}{N^2}\bar{p} - \frac{N^1}{N^2}\bar{p}\bar{m}^1)\left(U''\left(\frac{M}{N^2}\bar{p} - \frac{N^1}{N^2}\bar{p}\bar{m}^1\right)\right]}$$

$$f = \frac{U''\left(e^2 - \frac{M}{N^2}\bar{p} + \frac{N^1}{N^2}\bar{p}\bar{m}^1\right) + \beta(1-\pi)U''\left(\frac{M}{N^2}\bar{p} - \frac{N^1}{N^2}\bar{p}\bar{m}^1\right)}{U'\left(\frac{M}{N^2}\bar{p} - \frac{N^1}{N^2}\bar{p}\bar{m}^1\right) + \left(\frac{M}{N^2}\bar{p} - \frac{N^1}{N^2}\bar{p}\bar{m}^1\right)U''\left(\frac{M}{N^2}\bar{p} - \frac{N^1}{N^2}\bar{p}\bar{m}^1\right)}$$

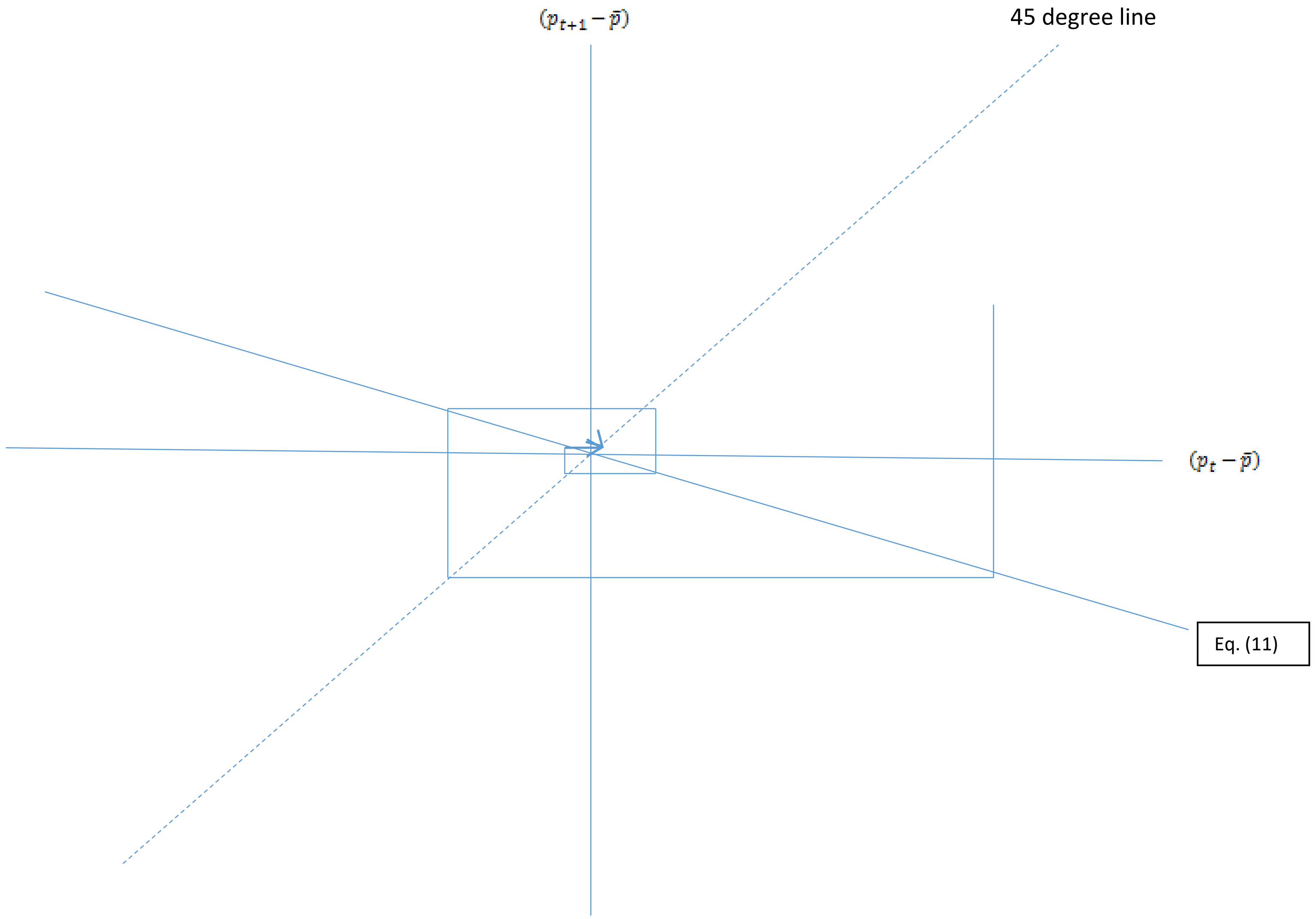
$$g = \frac{U''\left(e^1 - \bar{p}m^1\right) + \beta(1-\pi)U''(\bar{p}\bar{m}^1)}{U'(\bar{p}\bar{m}^1) + \bar{p}\bar{m}^1 U''(\bar{p}\bar{m}^1)}$$

Characterization

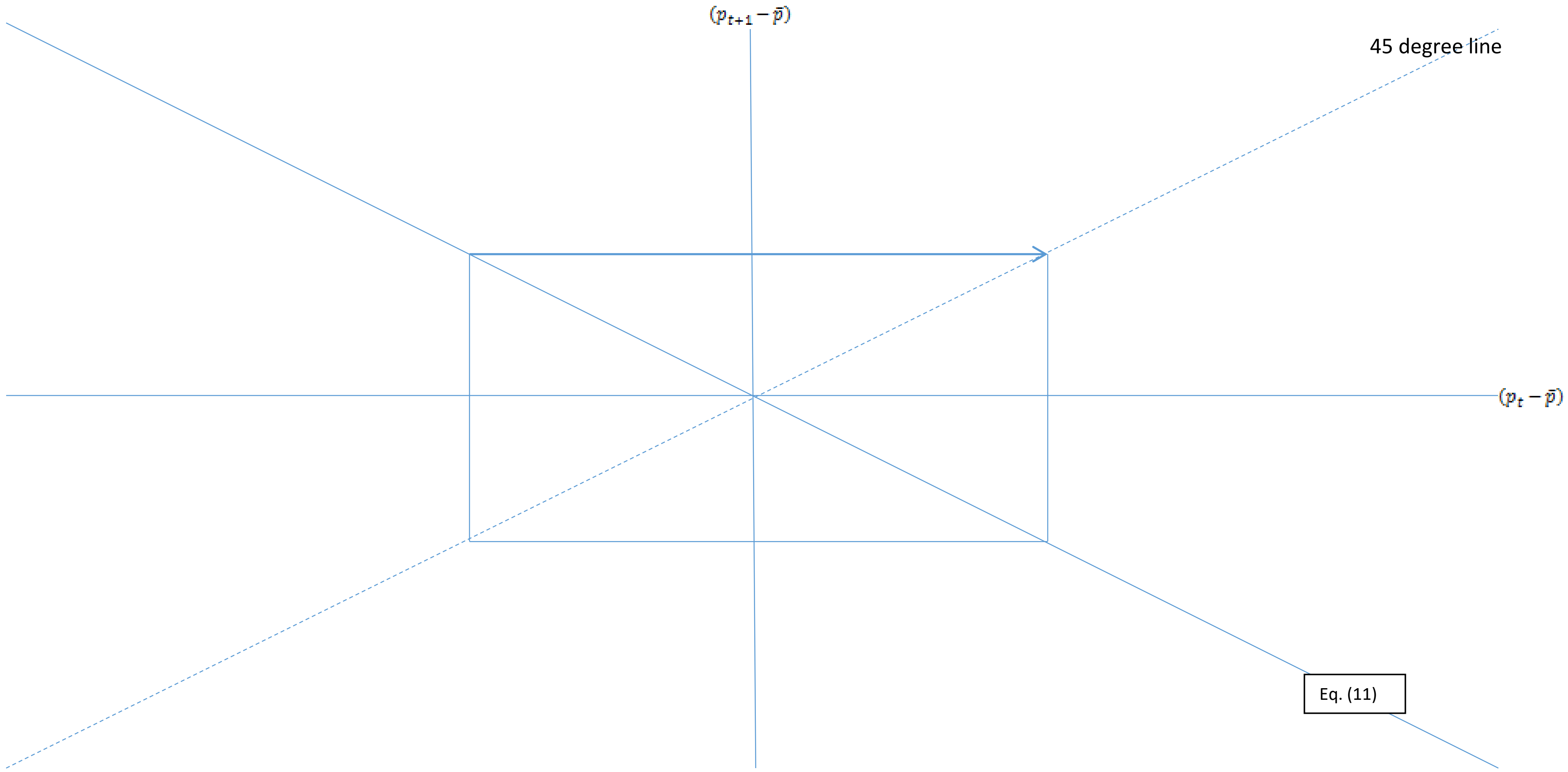
- To decode the slope, we need to make assumptions
- Assumption 1: $\frac{-cU''(c)}{U'(c)} > 1$
- Assumption 2: $\frac{-cU'''(c)}{U''(c)} > 1$
- Proposition 2: If assumptions 1 and 2 holds, then equation (11) will always have a negative slope.

$$x < 0$$

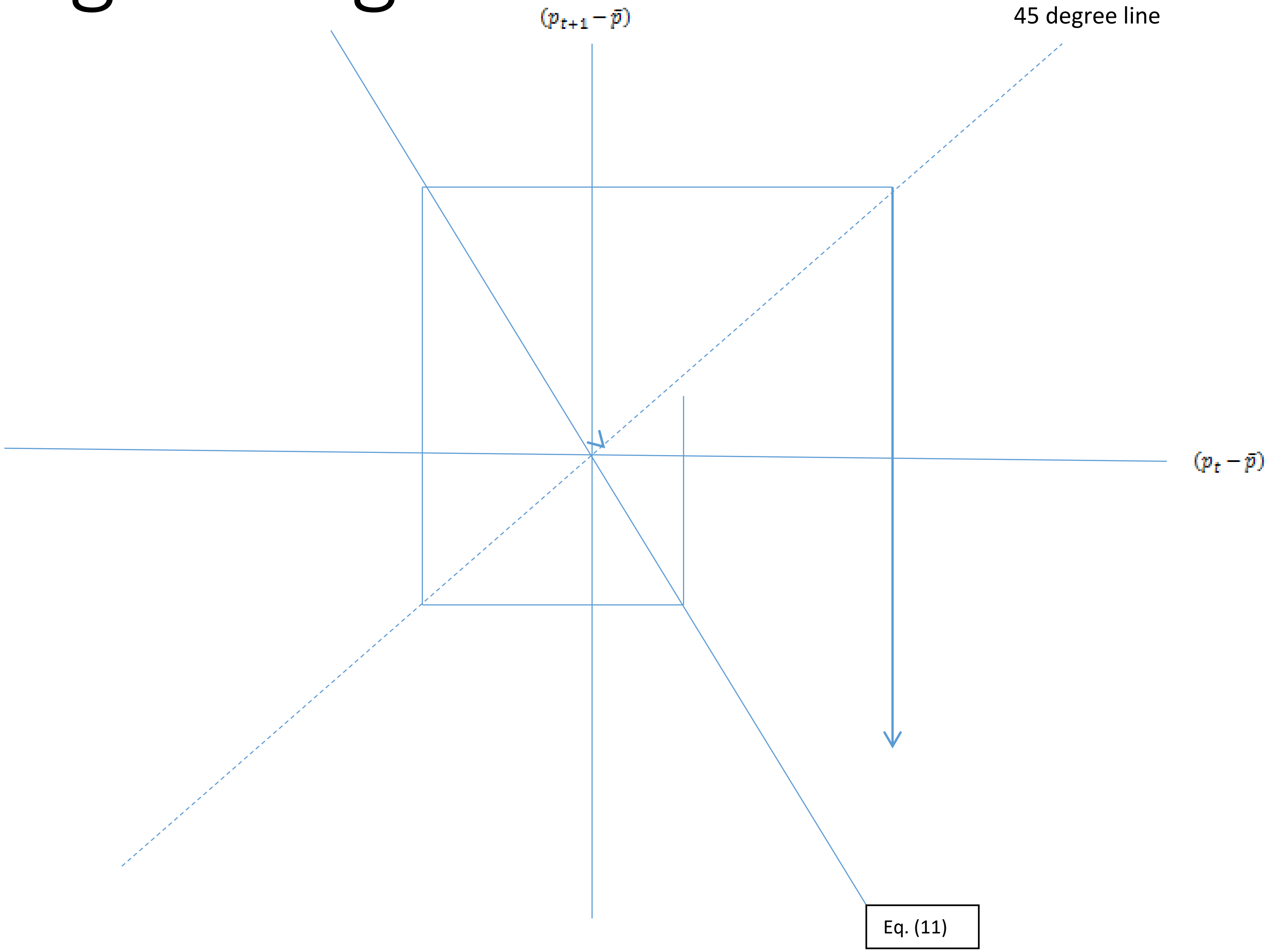
Oscillatory Convergence: $x \in (-1,0)$



Two Period Cycle: $x = -1$



Oscillating Divergence: $x < -1$



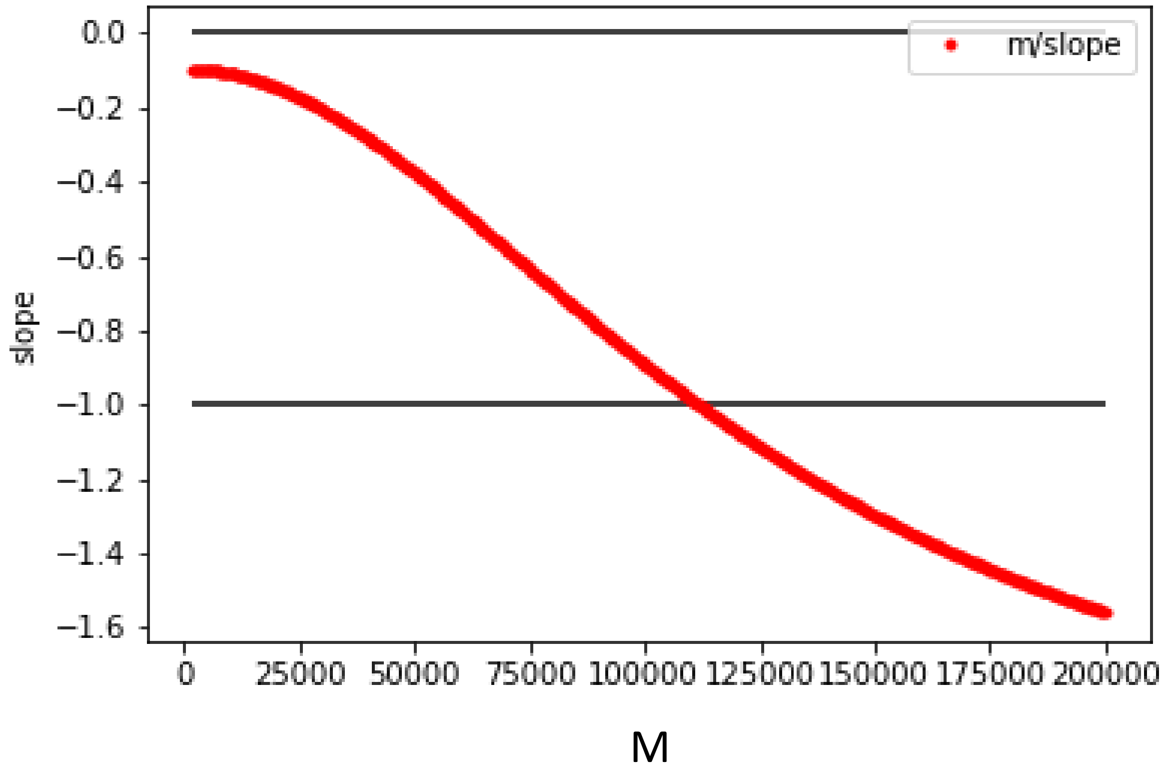
Numerical Parameterization

- The global parameters are defined as follows
  - constant relative risk aversion (θ) = 2.5
  - β = 0.9
  - π = 0.2

- Country Specific parameters
  - $e^1 = 20$
  - $N^1 = 2000$
  - $e^2 = 400$
  - N^2= 2000

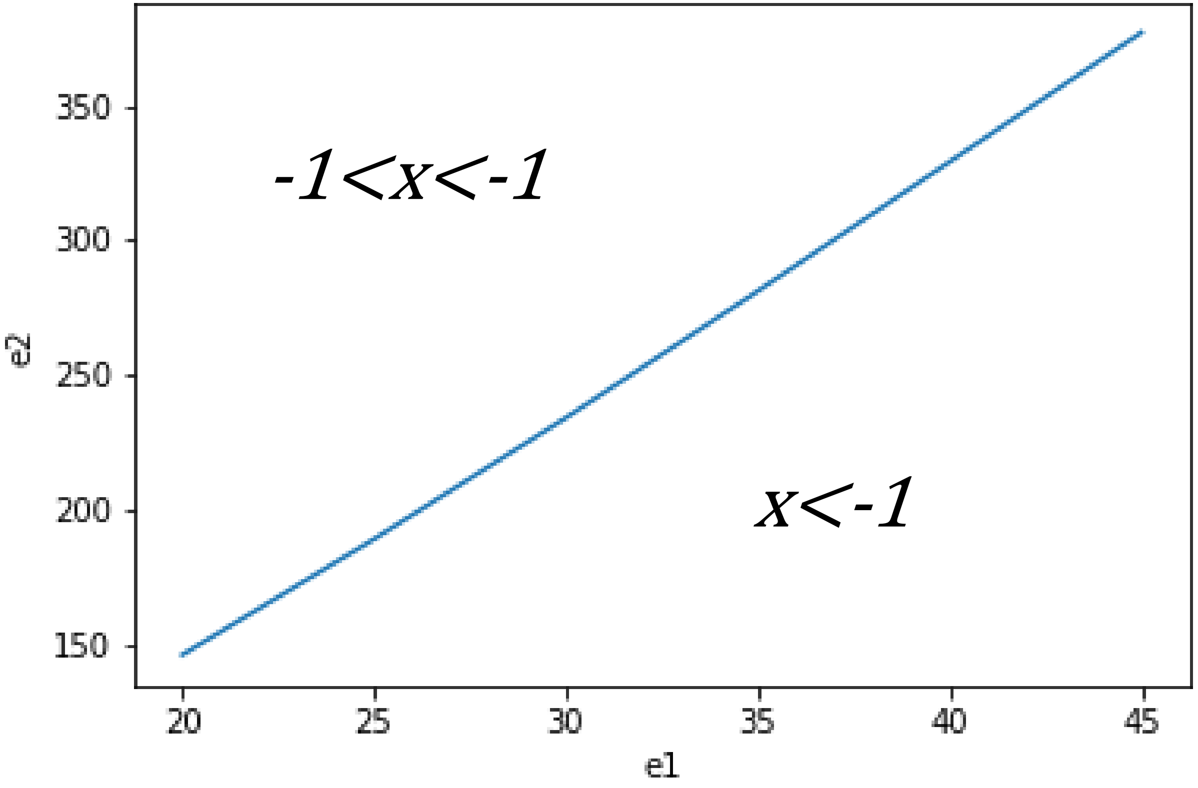
What is Required for Oscillation?

- Sufficiently low supply of cryptocurrency



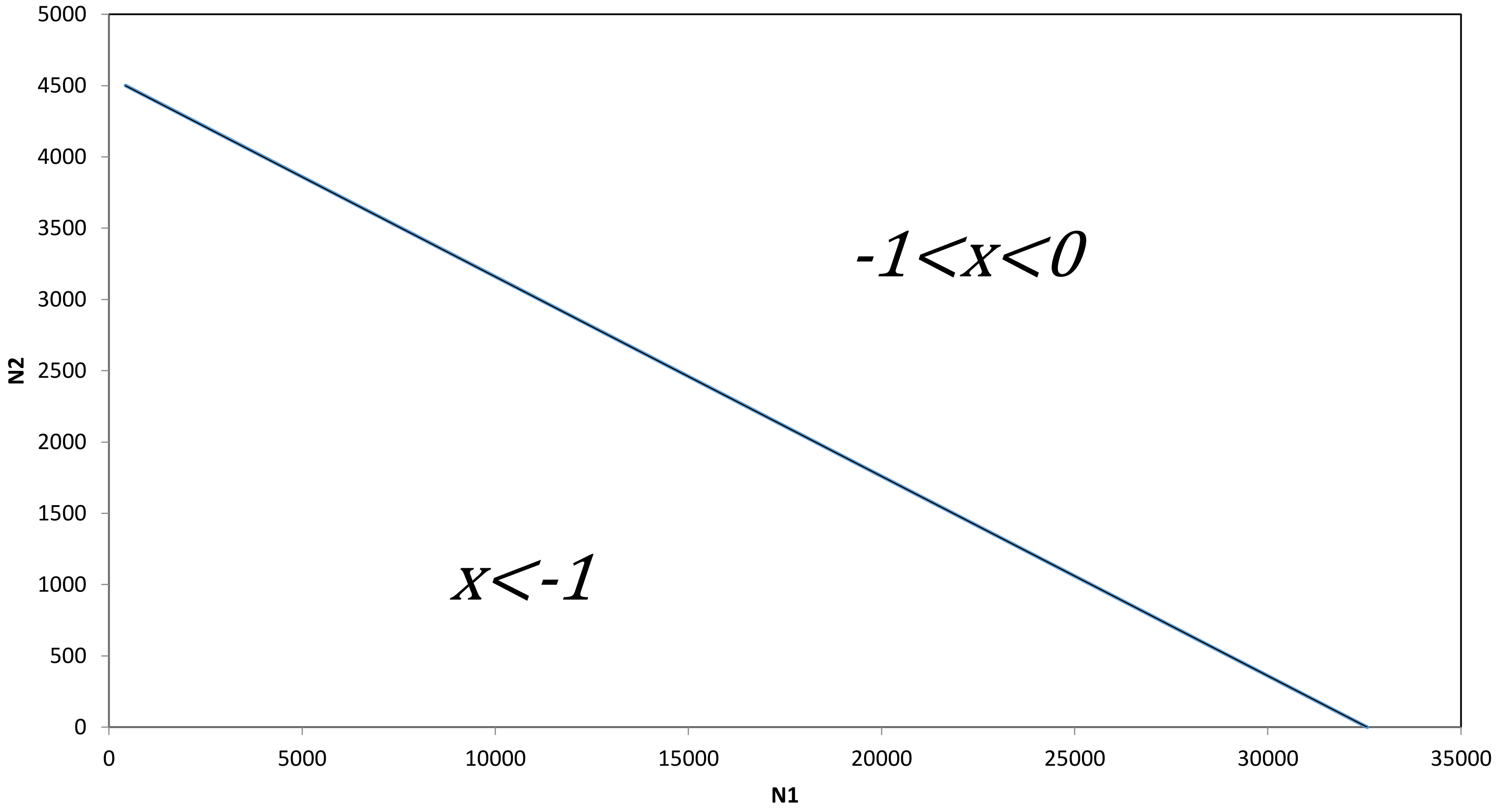
What is Required for Oscillation?

- Sufficiently income inequality across countries

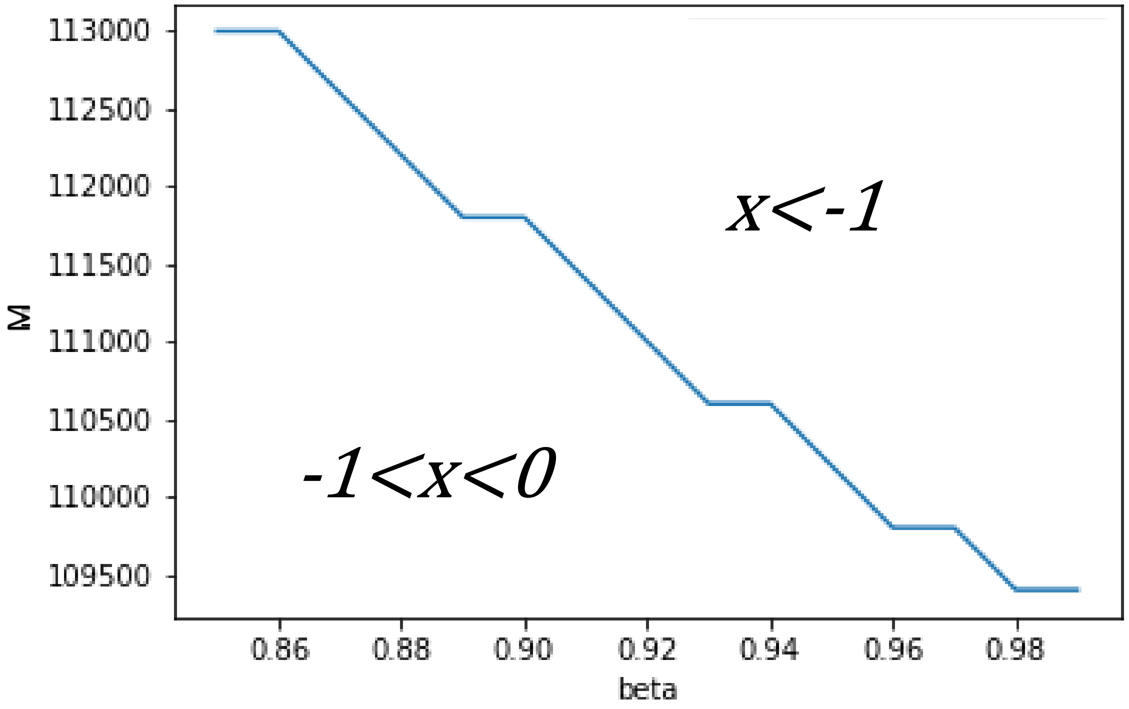


What is required for oscillation?

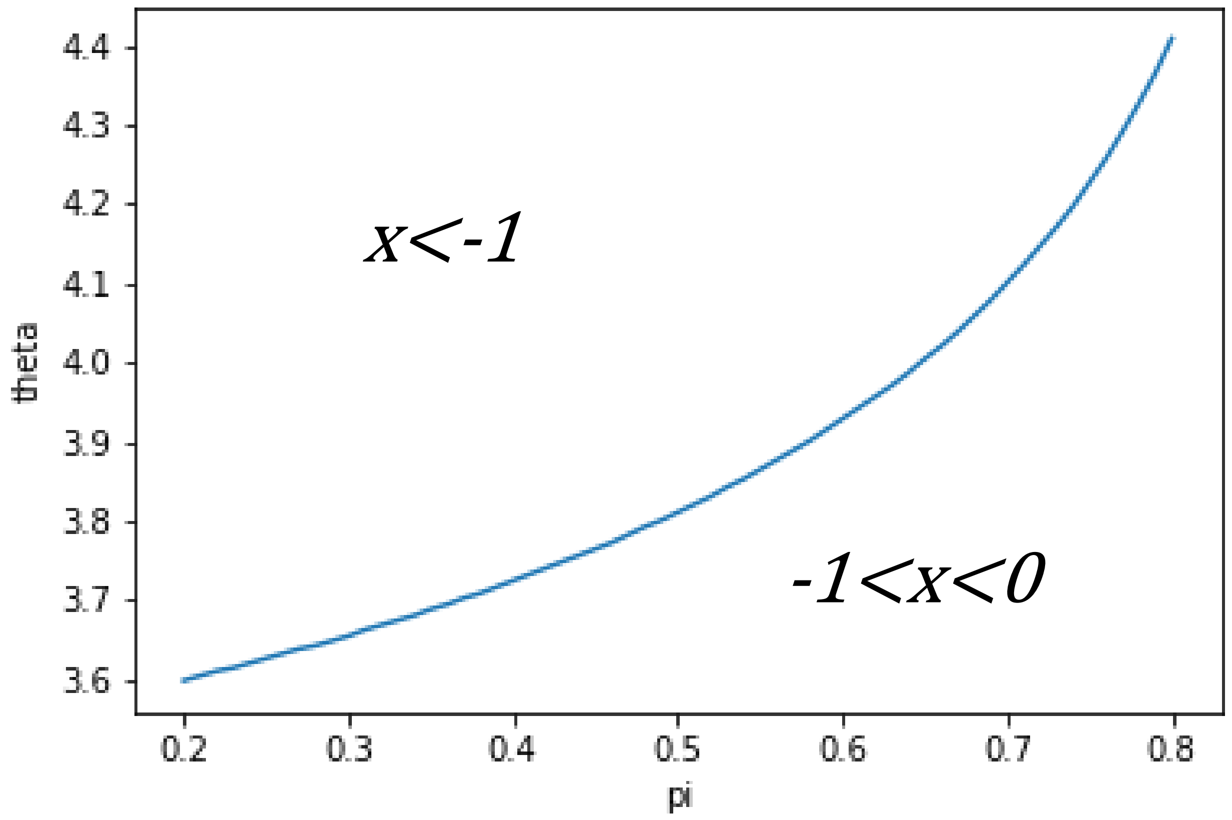
Sufficiently large world population



Discount Factor VS. Asset Supply



Risk Aversion and Crash Probability



Empirical Evidence

	CV	Supply
BTC	0.44	21m
LTC	0.65	84m
EOS	0.71	1000m
XRP	0.79	100,000m

Conclusion

- There is a vast range of parameterization such that the equilibrium is oscillatory.
- That is, high fluctuation of cryptocurrency is endogenously natural.
- Low supply of cryptocurrency, high global income inequality, less willing to save, and high probability to crash support oscillation convergence equilibrium.