Ambiguity Attitudes for Real-World Sources:
Field Evidence from a Large Sample of Investors

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29 June 2023

Ambiguity

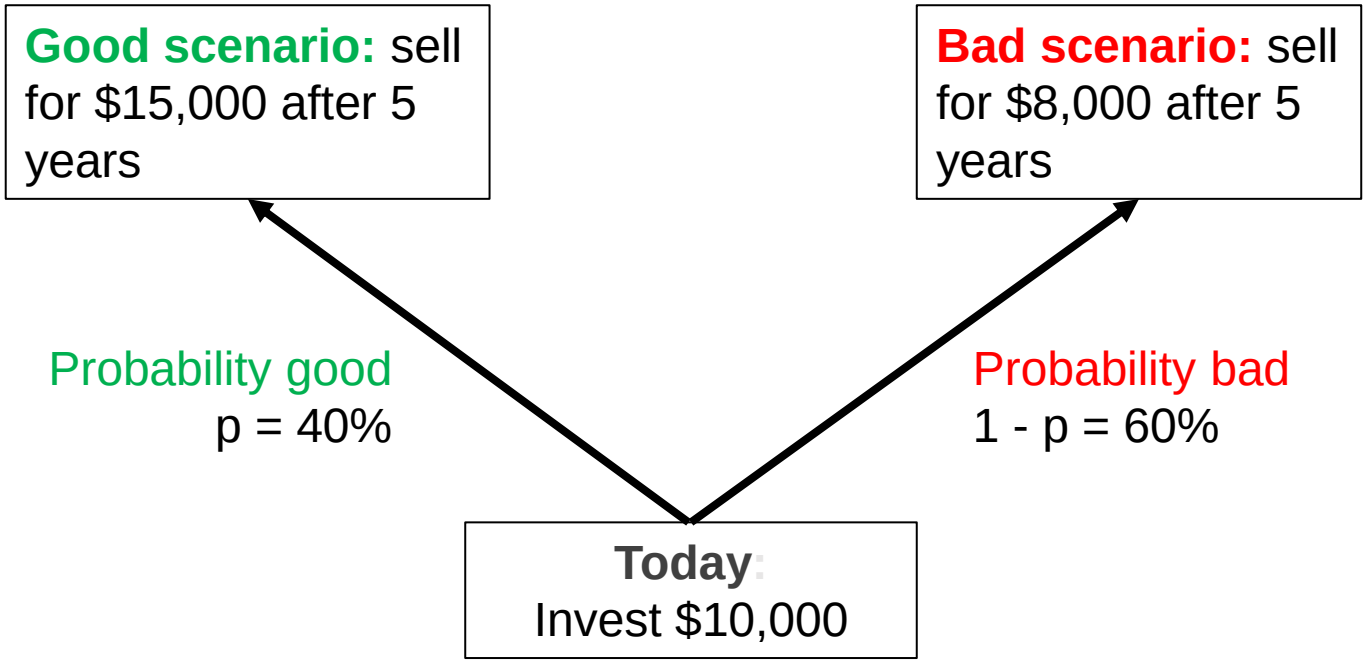
> Unlike ‘Risk’ in which the distribution of the outcomes is known, ‘ambiguity’ is an uncertainty whose **distribution is unknown.**

In practice, decision makings are often under ambiguity.

E.g. Investment, retirement saving, choosing a brand of shampoo.

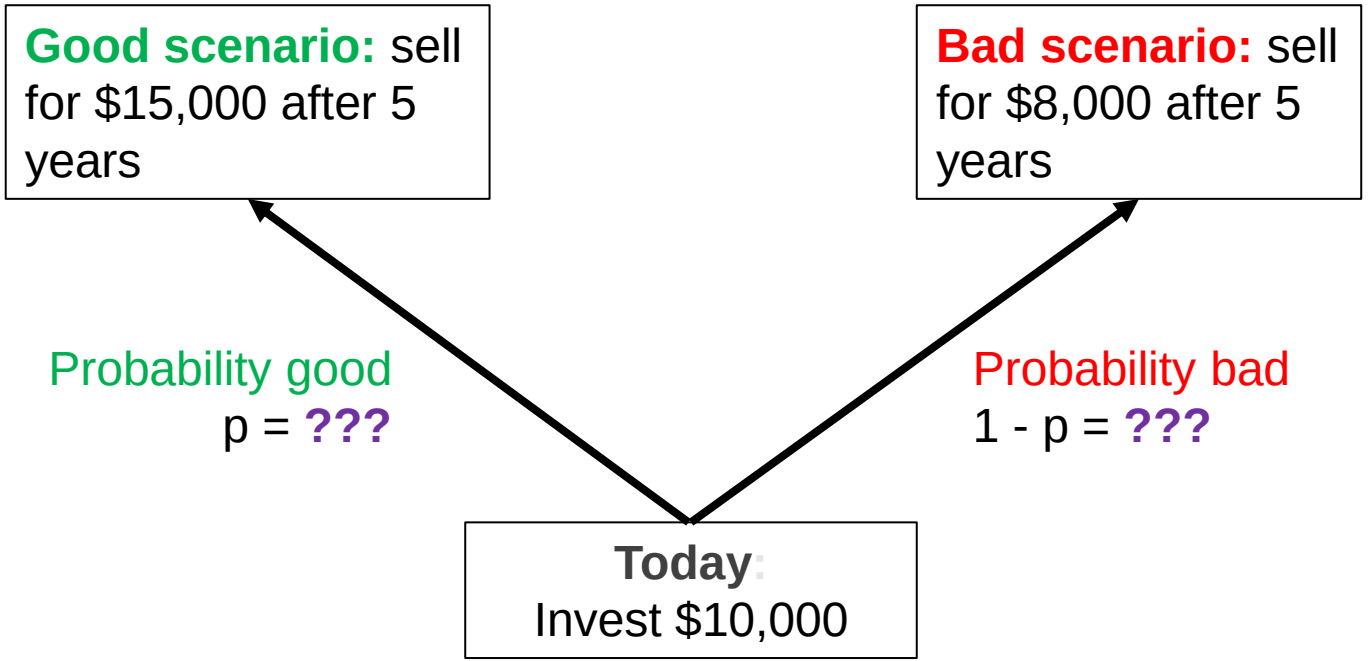
Ambiguity

Decision under risk



Ambiguity

Decision under ambiguity



Ambiguity

> *Since Keynes (1921) and Knight (1921) it has been understood that economists should look into ambiguity (unknown probabilities) more than risk (known probabilities). Yet it took till the end of the 1980s before people (Gilboa and Schmeidler) came clever enough to invent such models, because this invention took exceptional creativity. As things go, it then takes decades before the ideas get generally understood and applied, which is happening today, as an important part of the behavioral approach.*

Peter Wakker

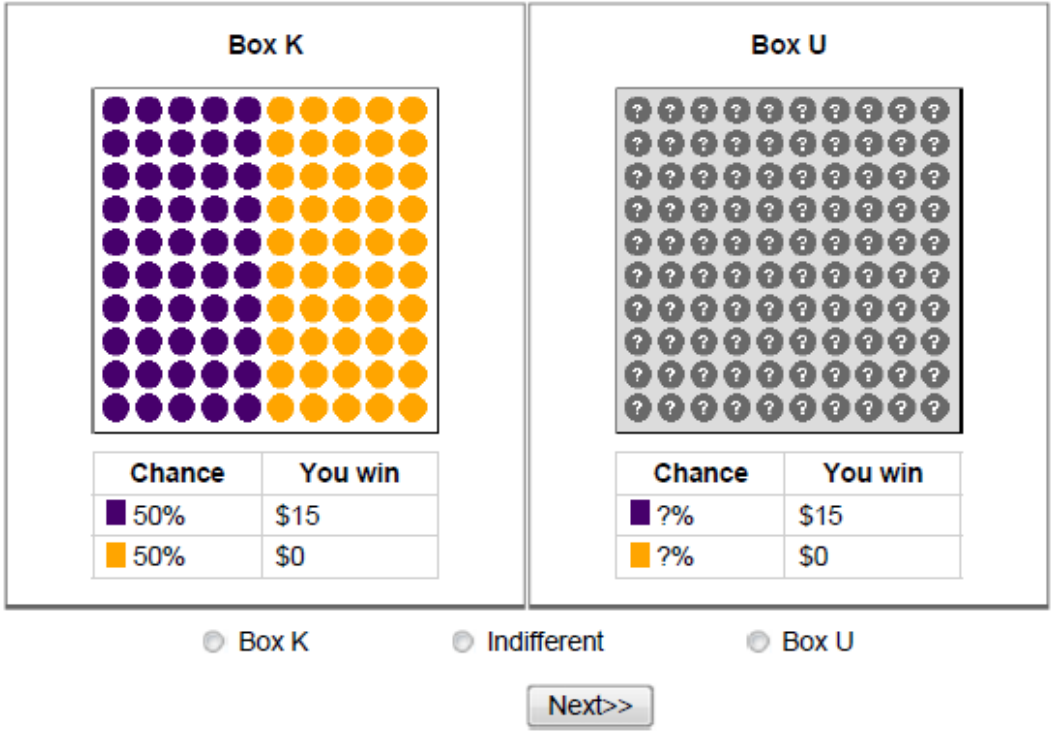
Ambiguity

Things that **cannot be explained** by traditional risk models.

- Under diversification
- Home bias
- Familiarity bias
- Low/no stock market participation
- etc…

Ambiguity

- Most famous example – **Ellsberg (1961) paradox**



Ambiguity

- When purple is the winning color, people prefer Box K (known) over Box U (unknown)

- When orange is the winning color, people prefer Box K (known) over Box U (unknown)

People act as if there are less than 50 purple balls in U, and also less than 50 orange balls in Box U.

In an expected utility model, this implies:

- P(Purple from U) + P(Orange from U) < 1
- Inconsistent beliefs!

Ambiguity

- Most people prefer making a decision **under risk** (with known probabilities) compared to **under ambiguity** (with *un*known probabilities)

- Ambiguity aversion tends to arise when people feel relatively **unknowledgeable** or **incompetent** about the source of ambiguity.

Our Research

> In 1992, Camerer and Weber already wrote that there were "diminishing returns to studying urns!" Three decades later, most ambiguity experiments still use balls and urns.

Measuring ambiguity attitudes for natural sources has been difficult.

Existing evidence about ambiguity attitudes uses **artificial Ellsberg urns** or **student samples**.

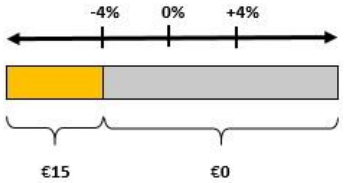
Our Research:

- Measures ambiguity attitudes for relevant **natural sources**:
  - *Investment Assets: Familiar stock, domestic index (AEX), foreign stock index (MSCI World), Bitcoin*

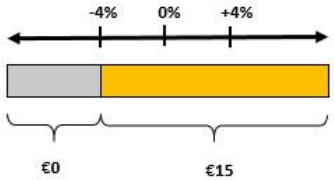
- Controls for **subjective beliefs**, using **Baillon et al. (2018)**

- Uses a representative sample of **Dutch Investors**
  - *DHS panel by **CentERdata of Tilburg University***
  - *Real incentives of 15 euro for each participant*
  - *Fielded: **May 2018**. Sample: n = **289 investors***

Measurement: Choice List

Option A Win €15 if the AEX *decreases by 4% or more* in one month	**A**	**B**	**Option B** Win €15 in one month time with the following chance
A: Win €15 if the AEX index *decreases by 4% or more* in 1 month time  	X		B: Win €15 with 0%
	X		B: Win €15 with 2.5%
	X		B: Win €15 with 5%
	X		B: Win €15 with 10%
	X		B: Win €15 with 20%
		X	B: Win €15 with 30%
		X	B: Win €15 with 40%
		X	B: Win €15 with 50%
		X	B: Win €15 with 60%
		X	B: Win €15 with 70%
		X	B: Win €15 with 80%
		X	B: Win €15 with 90%
		X	B: Win €15 with 95%
		X	B: Win €15 with 97.5%
		X	B: Win €15 with 100%

Measurement: Choice List

Option A Win €15 if the AEX does not *decrease by 4% or more*	**A**	**B**	**Option B** Win €15 in one month time with the following chance
A: Win €15 if the AEX does not *decrease by 4% or more* in 1 month time  	X		B: Win €15 with 0%
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	X		B: Win €15 with 40%
	X		B: Win €15 with 50%
		X	B: Win €15 with 60%
		X	B: Win €15 with 70%
		X	B: Win €15 with 80%
		X	B: Win €15 with 90%
		X	B: Win €15 with 95%
		X	B: Win €15 with 97.5%
		X	B: Win €15 with 100%

Measurement: Principles

Matching Probabilities

The matching probability is the known probability of winning at which the respondent is ***indifferent*** between Option A (winning €15 if the event happens) and Option B (winning €15 with known chance).

Control for Subjective Beliefs

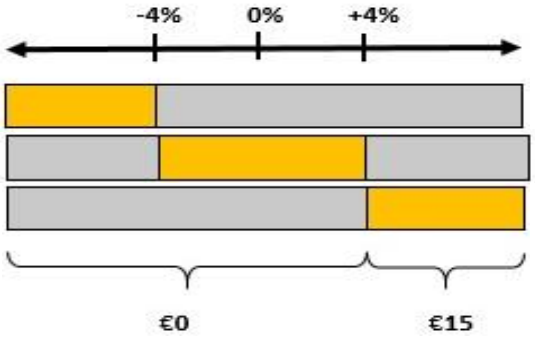
The matching probabilities of an event and its complement depends in exactly ***opposite ways*** on the unknown subjective probability P(E1).

- Subjective beliefs will be cancelled out in the process

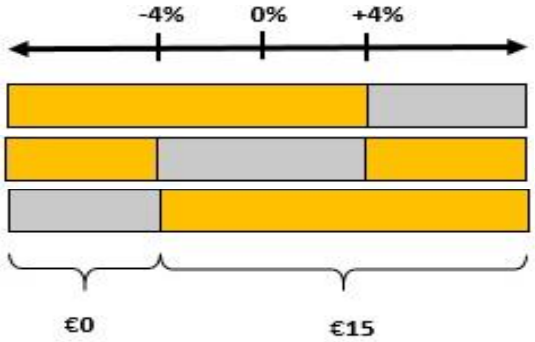
Measurement: Events

Baillon et al. (2018)

- **Three single events**
  - E_1 = [−∞ , −4%] : AEX decreases by 4% or more
  - E_2 = [−4%, +4%] : AEX changes by less than 4%
  - E_3 = [+4% , +∞] : AEX increases by 4% or more
  - Avg. matching prob.: $\bar{m}_s = (m_1 + m_2 + m_3)/3$



- Their complements, **three composite events**
  - E_{23} = [−4% , +∞]
  - E_{13} = [−∞ , −4%] U [+4% , +∞]
  - E_{12} = [−∞ , +4%]
  - Avg. matching prob.: $\bar{m}_c = (m_{12} + m_{13} + m_{23})/3$



Ambiguity Aversion, Index *b*

Baillon et al. (2018)

Ambiguity Aversion, Index (*b*)

Index $b = 1 - \bar{m}_c - \bar{m}_s$

b = 0	Ambiguity Neutral
b > 0	Ambiguity Averse
b < 0	Ambiguity Seeking

A-Insensitivity, Index *a*

Baillon et al. (2018)

- The tendency to treat all uncertain events as **50-50%**, ignoring the likelihood of events.
- **Insensitivity, Index *a***

Index $a = 3 \times (1/3 - (\bar{m}_c - \bar{m}_s))$

a > 0 A Insensitive

a < 0 A Oversensitive

a = 0 and b = 0: Ambiguity neutral (expected utility model)

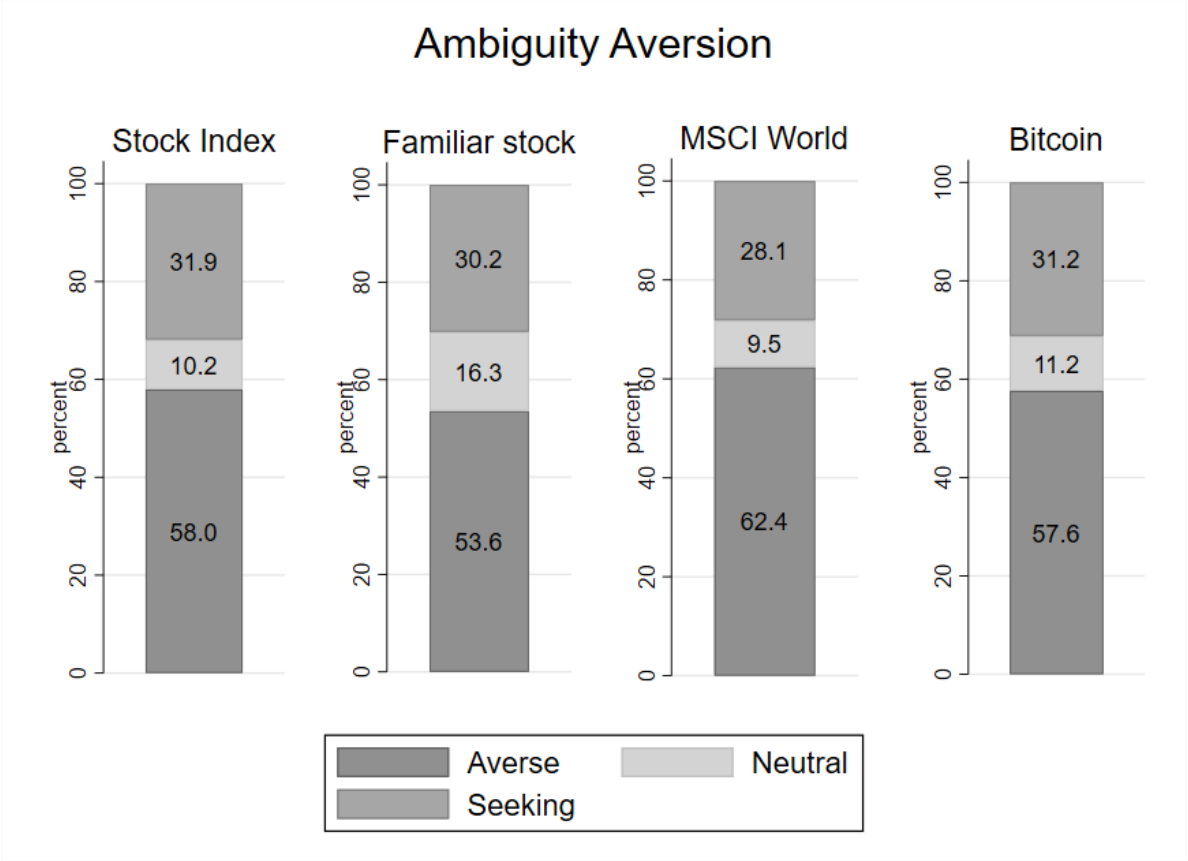
Index $a = \delta \geq 0$, perceived level of ambiguity in α-MaxMin model with prior probability set of **Chateauneauf et al. (2007)**

Findings

> **Majority of investors is ambiguity averse for investments, but aversion is not universal.**

Some people are ambiguity averse, some are seeking.

Findings: Ambiguity Aversion, Index *b*



Findings: Ambiguity Aversion for Investments

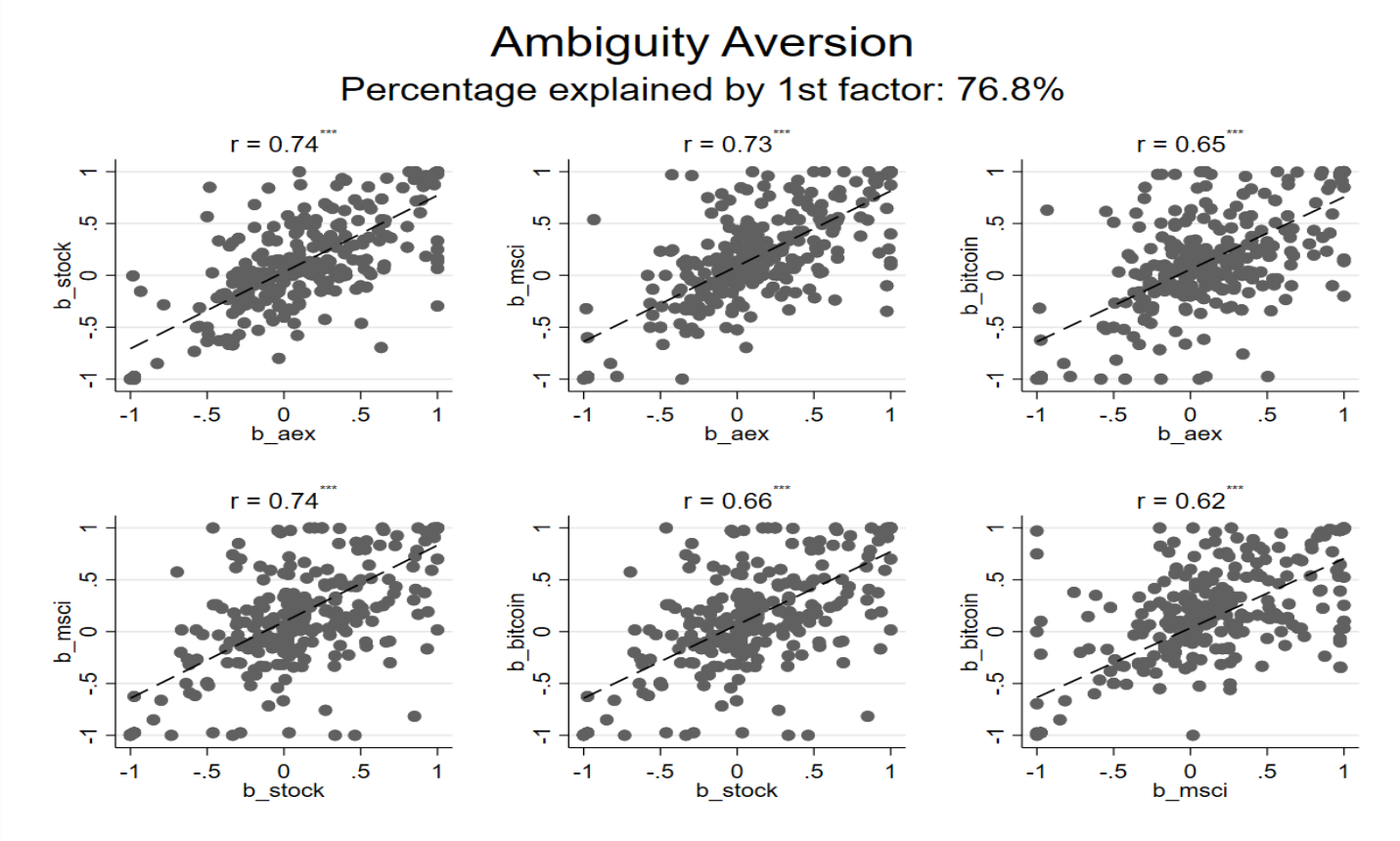
- Majority of investors is **ambiguity averse** for investments, but aversion is **not universal.**

- Lots of **heterogeneity between subjects**.

On average

- 58% Ambiguity Averse
- 30% Ambiguity Seeking
- 12% Ambiguity Neutral

Findings: Ambiguity Aversion for Investments

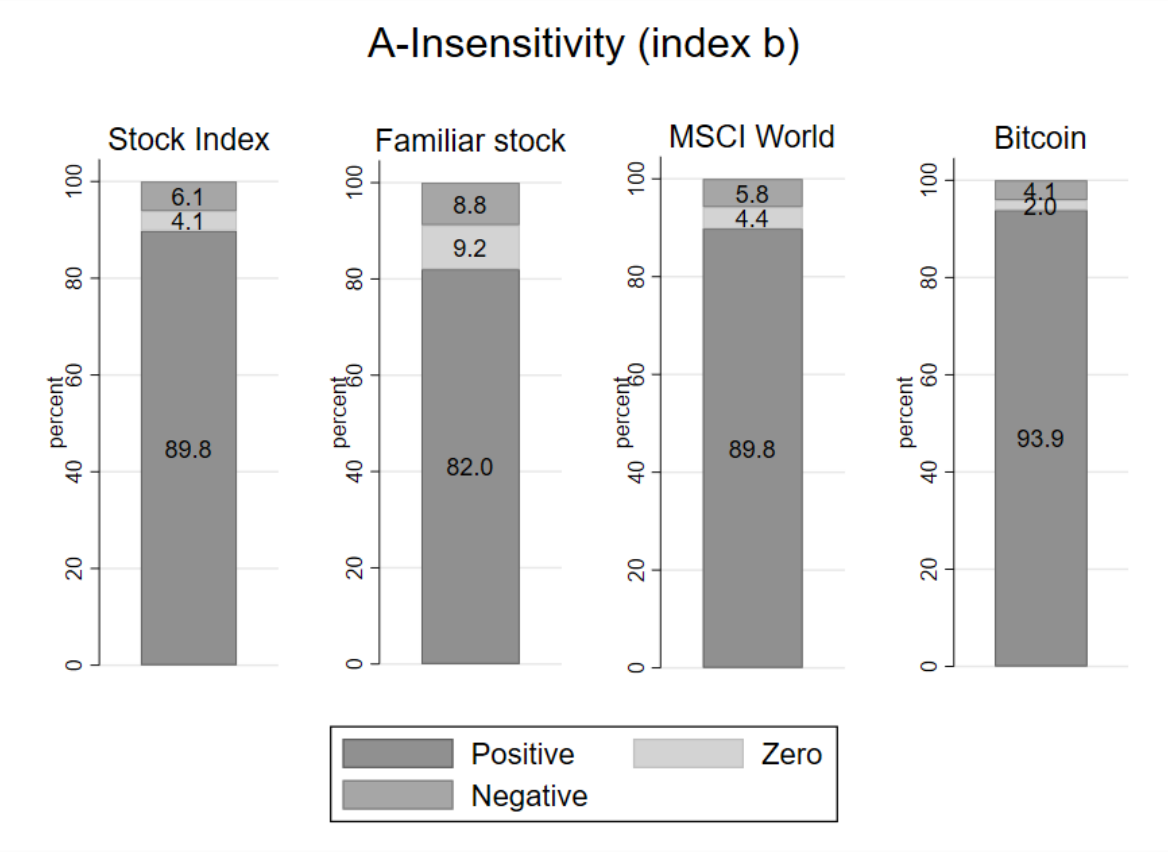


Findings: Ambiguity Aversion for Investments

- Ambiguity aversion is **highly correlated within subjects** and driven by **one factor**.

- Supports theoretical models where the investor's ambiguity aversion is modeled by **one preference parameter** (e.g., alpha, index b)

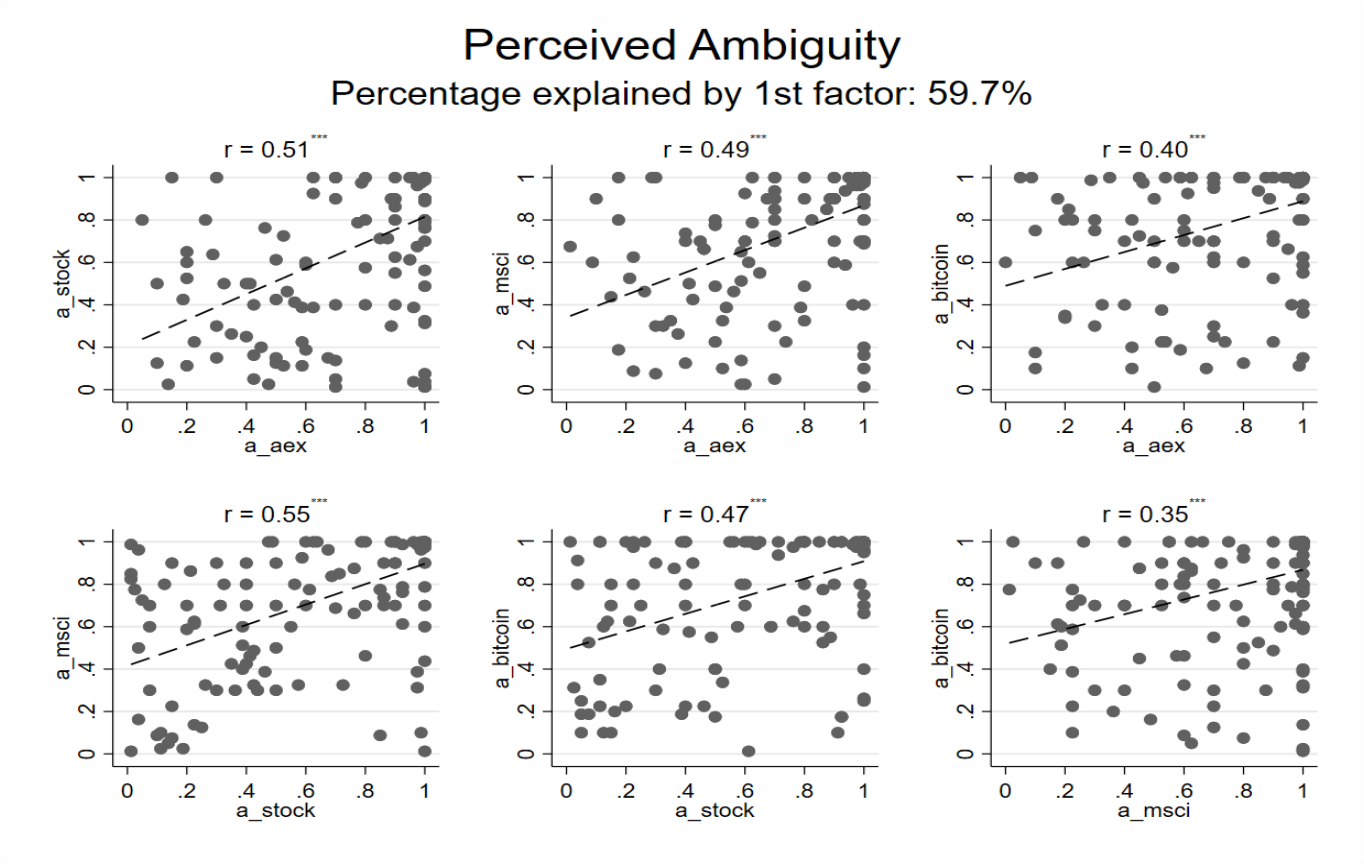
Findings: A-Insensitivity, Index a



Findings: A-Insensitivity, Index a

- **Most investors** (≥ 80%) are **insensitive** to event likelihoods, with Index a > 0.
- About 66% of the investors have index a between 0 and 1.This supports an interpretation of index a as **perceived ambiguity**.

Findings: A-Insensitivity, Index a



Findings: Perceived Ambiguity about Investments

- Perceived Ambiguity about the 4 investments tends to **differ** more **depending on the specific source** considered (not driven by 1 factor).

- Supports theoretical models where the investor's perceived level of ambiguity is **different for each investment**
  (e.g. familiar stock, index, Bitcoin, etc.)

Mean of *b* and *a* index

Ambiguity Aversion	Mean
b_aex	0.17
b_stock	0.16
b_msci	0.21
b_bitcoin	0.17
b_avg	0.18

Perceived Ambiguity	Mean
a_aex	0.83
a_stock	0.69
a_msci	0.78
a_bitcoin	0.84
a_avg	0.79

DKW (2016) for Ellsberg urns: $b = 0.14$, $a = 0.35$

Panel regression of *b* and *a* index on other variables

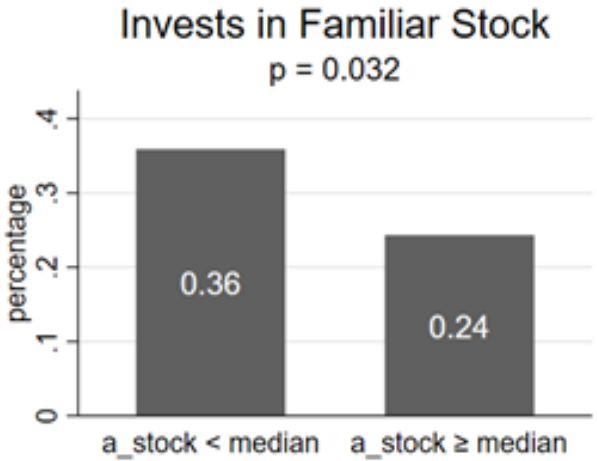
	Ambiguity Aversion *b*	Perceived Ambiguity *a*
Financial Literacy	-0.02	-0.02**
Education	-0.02	-0.03***
Risk Aversion	0.47***	0.04
Inverse-S Prob. Weighting	-0.08	0.09***

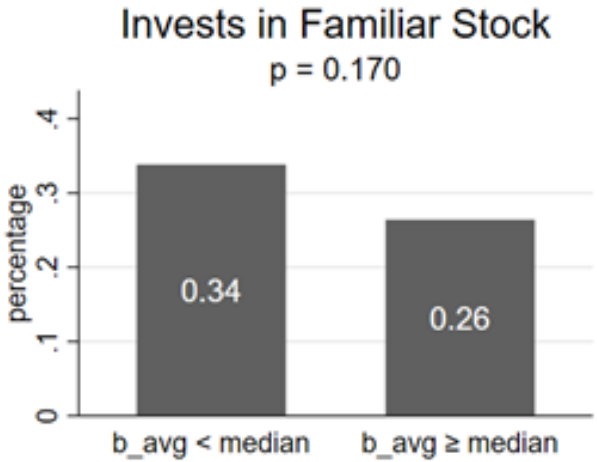
- **Ambiguity Aversion** positively related to Risk Aversion (r = 0.49), but not with education, suggesting it is a **preference component.**

- **Perceived Ambiguity** is lower for people with higher education and better financial literacy, suggesting it is a **cognitive component.**

Ambiguity Attitudes & Investment Choices: Fam. Stock

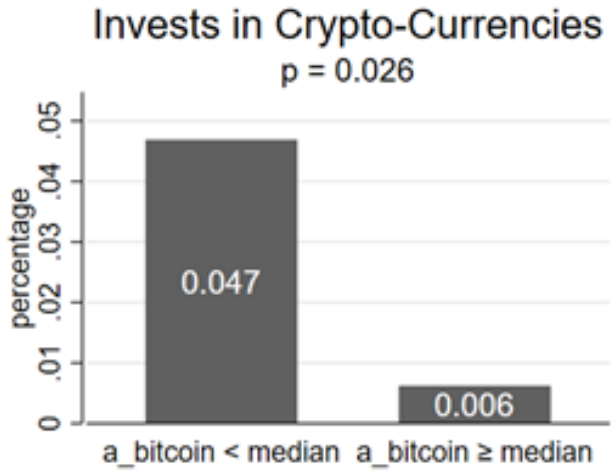
- **Expected:** Higher perceived ambiguity about the familiar stock is associated with lower likelihood of investing in it.

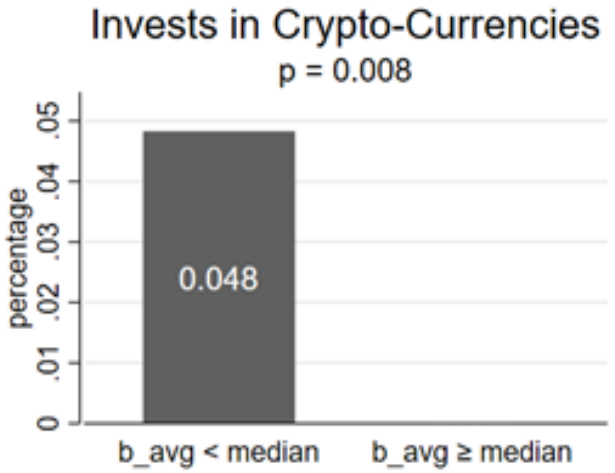




Ambiguity Attitudes & Investment Choices: Bitcoin

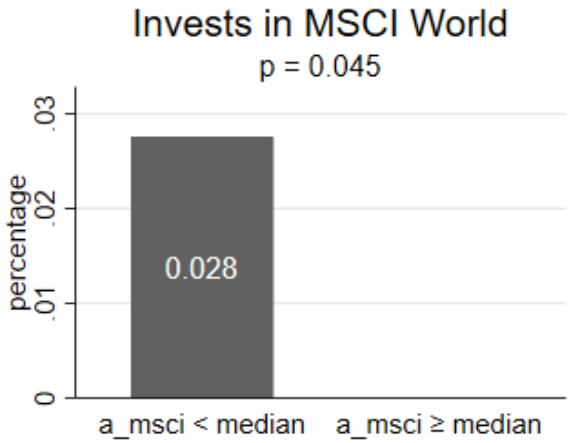
- **Expected:** Higher perceived ambiguity about Bitcoin and higher a mbiguity aversion are associated with lower likelihood of investing in crypto-currencies

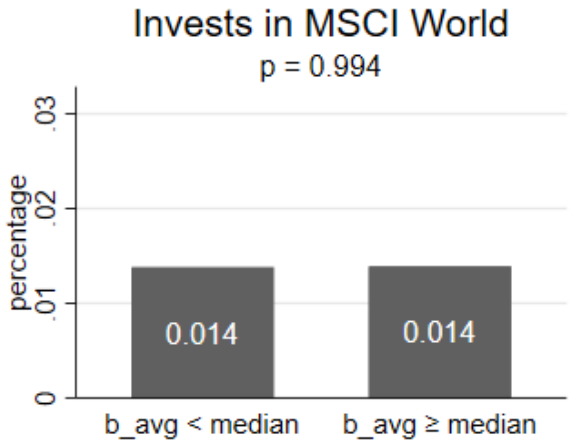




Ambiguity Attitudes & Investment Choices: MSCI World

- **Expected:** higher perceived ambiguity about MSCI World is associated with lower likelihood of investing in it.





Conclusions: Ambiguity Aversion

- **Within subjects, ambiguity aversion to the 4 investments can be described by 1 variable.**
  - Thus, no need to measure separately for each source.
  - Supports theoretical models like alpha-MaxMin

- **Between subjects, ambiguity aversion varies and is not universal: 58% are averse, but 30% are seeking.**
  - Now confirmed for relevant natural sources
  - Presence of ambiguity seeking agents matters for asset pricing.

- **Ambiguity aversion is related to risk aversion, but not to education or financial literacy, suggesting it is a preference component.**

Conclusions: Perceived Ambiguity

- **Large majority of investors are insensitive to the likelihood of ambiguous events (a-insensitivity).**
  - Supports interpretation of index a as the perceived level of ambiguity (δ in α-MaxMin model , Chateauneuf et al. 2007)
- **Respondents perceive different levels of ambiguity about the 4 investments (not 1 factor).**
  - Requires separate measurement for each source
- **Perceived ambiguity is negatively related to education and financial literacy, suggesting it is a cognitive component**

Future Research

Perceiving less ambiguity towards investment may lead to an increase in stock holdings and better portfolio diversification by households.

Questions

- Can we reduce perceived ambiguity about investments by **interventions** such as training?
- Can we reduce perceived ambiguity about investments by **learning through experience**?


Q
A

Model of Ambiguity Aversion

- **Mulitple Priors Model: MaxMin**

$\text{Max}_a \text{Min}_{P \in Q} E_P[U(W(a);a)]$ => *Ambiguity averse*

- Q is a set of prior probability distributions
- U is a utility function over wealth, depending on action a

- Maximize expected utility over all actions a,
  but using the worst-case probability distribution.
  - Gilboa and Schmeidler (1989)

Ellsberg Paradox Explained

- Suppose the decision maker beliefs that the number of purple balls in Box U (Unknown) is
  in between 30 and 70 $(30\% \leq p \leq 70\%)$
- And he makes decisions with MaxMin utility

  - When purple is the winning colour, he prefers Box K (known) because he evaluates Box U (unknown)
    with $p = 30\%$ (worst case)

  - When orange is the winning colour, he prefers Box K because he evaluates Box U (unknown)
    with $1 - p = 1 - 70\% = 30\%$ (worst case)

Ambiguity Aversion is a Bias

Ambiguity aversion (or seeking) behaviour is irrational, similar to having biased expectations.

The rational way to deal with ambiguity is to make a best estimate of the unknown probability p,
and then act upon it.

- Making decisions as if the worst case will materialize is costly in the long run, unless the stakes are high (i.e., for matters of survival, or catastrophic loss)
- But most people also display ambiguity aversion when the stake are low (peanuts); and this is a mistake

Alpha-MaxMin Model for Ambiguity Attitudes

- **Alpha-MaxMin Model:**

$$\text{Max}_a \{ \alpha \text{Min}_{P \epsilon Q} E_P[U(W;a)] + (1-\alpha)\text{Max}_{P \epsilon Q} E_P[U(W;a)] \}$$

  - Q is a set of prior probability distributions
  - $\alpha \epsilon [0, 1]$ is the ambiguity aversion parameter
    - **$\alpha > 0.5$: ambiguity averse**
    - **$\alpha = 0.5$: ambiguity neutral**
    - **$\alpha < 0.5$: ambiguity seeking**
  - Maximize expected utility over all actions a, using a mixture of the best and worst probability distribution
    - Ghirardato, Maccheroni and Marinacci (2004)

Prior Set

Decision maker has degree of confidence $(1-\delta)$ in his reference probability distribution π

$$C_\delta = \{P \in \mathcal{P}: P(E) \geq (1-\delta)\pi(E), \quad \text{for all } E \in \mathcal{E}\}$$

- **C_δ is a set of prior probability distributions**
- π is the reference probability distribution
- $(1-\delta)$ is confidence level in the reference distribution, while **$0 \leq \delta \leq 1$ is the perceived level of ambiguity**.

Proposed in: Chateauneuf, Eichberger and Grant (2007)

Related Literature

- Dimmock, Kouwenberg & Wakker (MS, 2016)
  - Measure ambiguity aversion with matching probs.
  - Ellsberg urns, Dutch general population
- Dimmock, Kouwenberg, Mitchell & Peijnenburg (JRU, 2015)
  - Estimate α-MaxMin model and perceived ambig. (δ)
  - Ellsberg urns, US general population
- Dimmock, Kouwenberg, Mitchell & Peijnenburg (JFE, 2016)
  - Show that ambiguity aversion helps explain low household stock market participation, low equity fractions and low foreign stock ownership
  - Ellsberg urns, US general population

Related Literature

- Abdellaoui, Baillon, Placido & Wakker (AER, 2011)
  - Measure weighting functions for natural sources
  - Introduce index b and index a
  - Method requires measuring utility and probability weighting functions to isolate ambiguity attitudes
  - Student sample

- Baillon, Huang, Selim & Wakker (Econometrica, 2018)
  - Measure ambiguity aversion for natural sources
  - New simple method that controls for beliefs
  - Student sample