CORPORATE GOVERNANCE, FIRM DYNAMICS AND SECURITY DESIGN

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Abstract

This paper studies the role of corporate governance in dynamic context of the firm using continuous-time dynamic contract model. It shows theoretically that value of corporate governance is a part of firm's security price and distinguished from firm's profit and executive compensation. The corporate governance is attributed by both law of a country and internal governance of a firm and determine the degree of firm's investor protection. In this paper, the role of corporate governance is to limit the amount of capital diversion, but the manager's decisions on effort and capital diversion are unobservable. We characterize the optimal dynamic contract with corporate governance. An implementation of optimal contract using dynamic capital structure shows the value of corporate governance in security price as a separate contribution from operational profit and executive compensation. The paper further analyzes dynamic value of corporate governance, as well as the firm dynamics when a renegotiation on corporate governance occurs. This value has dynamic context because it changes over time due to the expected longevity of the firm. An increase in corporate governance intensifies the dynamic incentive alignment between the manager's compensation and firm's performance. A governance change is in essence a reallocation of the weight between instantaneous and intertemporal incentive of the manager. We also show that investor's benefit from better governance changes over time. Our results provide a new framework of corporate governance, give an insight on time-inconsistency in corporate governance. The results also explain the empirical puzzle of corporate governance and suggest its remedy for the empirical study on corporate governance and security price.

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I Introduction

This paper studies the effects of corporate governance of a firm in dynamic context by using continuous-time dynamic agency model. It investigates the effects of corporate governance to limit the scope of capital diversion in dynamic agency situation when investors cannot observe manager's decisions on effort provision and capital diversion. The study of internal governance in dynamic context raises many important issues which cannot be investigated in the static framework. How does the level of governance affect the manager's benefit and investor's profit along their dynamic relationship? How does the corporate governance influence the dynamic incentive on effort provision and capital diversion of the manager? Is the value of governance mechanism a part of firm's security price? Does the value of corporate governance change along the firm's dynamics? How does a governance change affect the firm's growth and investor's profit at different stage of the firm? Why does not the security price of the firm increase when the governance mechanisms improve? This paper answers these questions.

Most of the studies of corporate governance hinge on the static framework, rooted in the *free cash flow theory* from Jensen (1986). It provides the static perspective of corporate governance on a firm. Its main thesis is following. The corporate governance limits agency conflict between investor and manager through various mechanism on managerial decision, investors' rights and available free cash flow. The higher governance level from better governance mechanism limits agency rent more efficiently and subsequently increases investor's profit and equity price. This insight provides the logical foundation for most of theoretical and empirical studies on governance mechanism. However, this static perspective neglects the effect of governance mechanism on the evolution of the firm. It does not provide an understanding on how a governance mechanism affects firm's growth, agent's incentive and investor's profit over the dynamics of the firm. It also ignores the consequence of governance change, either by renegotiation of contractual parties or governmental intervention, on the firm dynamics. As a consequence, it cannot explain the recent important issues, including the time inconsistency in corporate governance and its recent empirical puzzle on equity price. A new perspective of corporate governance is necessary for a better understanding. This paper provides it.

We propose a dynamic framework of corporate governance by using the continuous-time dynamic agency model. We proceed in three steps. First, we characterize the firm dynamics under optimal contract with effects of corporate governance. It provides understandings on how corporate governance affects manager's dynamic incentive structure, investor's profit, effort provision, investment dynamics and executive compensation along the entire firm dynamics and also termination decision of the firm. Second, we show the contribution of corporate governance, *from both law and firm's internal governance*, on the security price under optimal dynamic contract. Third, we investigate the consequence of governance change on the firm dynamics, including manager's incentive, investor's profit and entire dynamic relationship of the two. The results of three steps establish a more complete framework of how corporate governance affects a firm, particularly in the dynamic perspective. After presenting the main results, we explain the situation of time-inconsistency in corporate governance, propose the possible source of empirical puzzle and suggest the methodological remedy based on the results and insight of the dynamic framework.

In our model, the firm's corporate governance, providing a degree of investor protection, is determined by law and internal governance. The law gives the identical fundamental investor protection to all firms in the the economy, as in the *Law and Finance* literature (La Porta, Lopez-de-Silanes, Shleifer and Vishny, hereafter LLSV (1998)). The internal governance is the corporate rules and restrictions imposed on managerial discretion to protect firm's investors beyond the law level. It is the agreement on the investor's right and extent of managerial discretion between the investor and the manager[1], not a unilateral decision of either of them. The better internal governance mechanism provides better investor protection of the firm in addition to the law. Because it is not required by the law, different firms could have distinct level of internal governance. It depends on the agreement between the investor and the manager of the firm. The firm's *corporate governance* restraints the potential capital diversion for manager's private benefit. The better governance implies higher investor protection level and lesser potential capital diversion and private benefit.

The optimal dynamic contract composes of four elements; firstly agreement on corporate governance between the investor and the manager, secondly, investment rule, thirdly compensation processes and finally the terminal time of the contract. Given the mutual agreement on governance level, the investor designs the investment and compensation rules by taking into account two types of incentive constraints. The first is an instantaneous incentive constraint. It motivates the manager away from committing capital diversion. As a result, the periodic compensation includes the benefit of control. The second is the intertemporal incentive. The investor aligns his benefit to the manager's continuation value, proposed by Spear and Srivastava (1987). Using martingale method on dynamic continuation value, pioneered by DeMarzo and Sannikov (2006) and Sannikov (2008), the manager's incentives are aligned with the investor's profit and the growth of the firm though continuation value and optimal compensation processes. Under the optimal dynamic contract, manager does not divert capital, but enjoy the same amount as a part of compensation. The effort provision is determined by the equality of the proportional profit sharing and marginal effort cost. The optimal investment follows the marginal-q. The contract terminates when the manager expects future benefit to be equal to his outside option, which is normalized to zero, and when the investor cannot gain benefit from the manager's contribution because his compensation is unprofitably high.

The analysis of governance mechanism in dynamic incentive framework indicates that there is no free cashflow problem under optimal contract because the manager would not have a motivation to hoard free cashflow in order to gain private benefit. The optimal dynamic contract incorporates the private benefit of control into the his compensation, according to instantaneous incentive constraint and revelation principle. However, the benefit

[1] The internal governance considered in this paper composes of the rules and restrictions on the organization of the corporation and the rights of investors. These rules are delineated in the corporate charter and legally enforceable. The internal governance here is de facto the agreed contract between the investor and the manager which limits the set of managerial decisions within the firm. As a result, the internal governance gives more protection to investors, while puts more restriction on the managerial decision set.

of control portion in the compensation is limited by corporate governance. The internal governance, as a part of firm's total corporate governance, is negotiable between the investor and the manager. If both parties agree to lessen the benefit of control by raising the internal governance level, the manager will be compensated by higher pay from future growth of the firm. Consequently, the optimal contract motivates the manager to invest all the available capital for future incoming cashflow of the firm in order to gain higher future compensation. There is no agency cost of free cashflow under the optimal dynamic contract.

An implementation of optimal contract illustrates the dynamic value of corporate governance as a part of the security price. We show that the security price composes of four parts; investor's profit, manager's normalized compensation, country's discount term and firm's governance premium. The country's discount term captures the inefficiency of the laws on investor protection of a country. The firm's governance premium reflects the value of internal governance of a specific firm. An important result is the dynamic property of the discount term and governance premium. They are different overtime, even though the law and internal governance mechanism remain unchanged. Intuitively, the dynamic valuation of the corporate governance elements in security price depends on the longevity of the firm, which is the distance between the current time and the expected termination period[2]. Considering governance mechanism in the dynamic contract provides a method to evaluate the dynamic price of corporate governance from both the country's law and firm's internal governance in a single framework.

Importantly, the results also shows that the value of governance premium in the security price is not subsumed into the operational profit and executive compensation. Under the optimal dynamic contract, the firm's capital becomes either corporate investment or continuation value, which directly link to the firm's operation profit and executive compensation. However, the security price still reflects the cost of imperfect investor protection and the benefit of firm's governance mechanism as distinct element; country's discount term and firm's governance premium. The distinctive element of governance premium is consistent with the control premium view of the security price. Moreover, because the security price characterization is a result of implementation of optimal contract, the country's discount term and firm's governance premium apply to both private and public corporations. The governance premium is always a part of security price.

An enhancement of internal governance intensifies the incentive alignment of the manager's compensation to the firm's profit. It reshapes the structure of manager's incentive by lessening instantaneous benefit of control while enhancing continuation value due to higher effort cost. As a result, it shifts the weight of manager's compensation from instantaneous benefit of control to future compensation derived from firm's performance. In other words, it reallocates the contribution of instantaneous and intertemporal incentive on executive compensation.

This incentive reallocation due to better governance does not always benefit investor.

[2]This insight is consistent to the market sentiment. The financial market gives a high value on the governance mechanism for a firm with long future. On the contrary, investors in financial market does not concern the corporate governance mechanism when a firm is going to wind up or in reorganization process.

This is beneficial only when the firm is small and in its initial stage, e.g. start-up firm or firm with growth prospect. Under the optimal contract, the higher level of corporate governance motivates the manager to contribute more in order to accelerate the firm's growth and his own benefit. Consequently, in the initial stage, a governance enhancement generates higher profit and it reaches its maximum sooner, at the lower level of manager's continuation value. The better governance intensifies the incentive alignment of the manager in the initial stage. On the contrary, a higher governance level in the mature stage can reduce investor's profit, when the firm has low growth prospect and manager's compensation becomes relatively large due to previous success. This is because the manager requires more compensation for the motivation to create additional growth. Since the additional cost of motivation is excessive in the mature stage, a better governance lowers investor's profit due to large wealth transfer in this stage. Considering the entire dynamics, a governance enhancement increases investor's profit at the initial stage, but decreases it later in the mature stage.

The key insight is following. The corporate governance is costly to investor and it becomes more costly when the firm grows. The investor would benefit from better governance only when it aligns the manager's incentive to generate growth of the firm. In the mature stage, when manager cannot create adequate growth and his compensation becomes large, the higher governance will merely accelerates the transfer from investor's profit to manager's compensation. As a consequence, it lowers investor's profit. In short, the investor benefits from better governance only when the incentive alignment effect exists.

This insight can explain the time-inconsistency in corporate governance, the situation in which the corporate governance is strengthened at the firm's initial stage to attract the capital and relaxed later when the firm finished financing and reached the mature stage. It indicates that it is rational and optimal to improve the governance level at the beginning, when executive compensation is relatively small. This improvement rises investor's profit and accelerate the incoming future profit. After, it is also rational to renegotiate for lower governance level; investor can save cost of motivation and hence retain more profit when the firm has low growth prospect in the mature stage. To the firm's governance mechanism, an enhancement at the beginning and relaxation at the later stage, called time-inconsistency in corporate governance, is a rational decision of both contractual parties.

The insight from security price characterization together with the governance change can explain the recent empirical puzzle and suggestion the potential remedy. The empirical puzzle of corporate governance is the contradictory empirical results about the contribution of corporate governance on equity price. From our results, we claim that there exists a positive contribution of firm's governance on equity price. However, the contribution might not be detected in empirical studies due to a negligence of the effect from the stage of the firm. The unclear contribution of corporate governance in empirical evidences is due to methodological flaw, not the role of governance mechanism. We recommend a consideration of the role of firm's stage into the empirical analysis. For example, a separation of the sampled firms between start-up and mature firm, e.g. growth stocks and value stocks. With the recommended methodological remedy, we can possibly clarify the puzzle in empirical studies of corporate governance and equity price.

This paper relates to two strands of literature. In the recent study on continuous-time dynamic agency literature, this paper follows the line of research of DeMarzo and Sannikov (2006) for corporate finance, Sannikov (2008) for martingale method of dynamic incentive and DeMarzo, Fishman, He and Wang, hereafter DFHW (2010), on dynamic investment model. The theoretical contribution of this paper is to break the direct link between agent's effort decision and the private benefit of control, considered in He (2009) and DFHW (2010). This disengagement gives another choice for modeling the dynamic agency problem. The agency rent needs not to be derived from agent effort only. Furthermore, the model of this paper is the first to incorporate the instantaneous incentive constraint with the dynamic incentive structure. This methodology can be used for the model involving the static incentive constraint and the revelation principle, such as laws, government regulation and taxation.

In corporate finance and governance literature, this paper is among a few that study the corporate governance in the dynamic context. The studies of corporate governance are broad in both theoretical and empirical investigations, but most of them focus on the static framework. It hence cannot address important issues considered here. The reviews of Becht, Bolton and Roell (2007), Denis and McConnell (2003) and Shleifer and Vishny (1997) give excellent summaries of the previous studies. In this paper, using continuous-time dynamic agency model broadens the analysis of firm's governance on the entire relationship between the investor and the manager. Technically, it gives a clear analytical solution and comparative static analysis. In addition, it merges the analysis of dynamic agency with the asset valuation model into a single framework. We can see how governance mechanism affects agency problem in parts of the firm and how firm's value change accordingly. By considering governance mechanism in dynamic context, we can also explore effects of the corporate governance on firm's profit and executive compensation at the different stages of the firm. Further, we can evaluate the dynamic aspect of the governance valuation in security price also. These two issues can not be studied systematically in a static framework. Furthermore, the model provides a method to evaluate the dynamic price of corporate governance from both the country's law and firm's internal governance in a single framework. To the best of my knowledge, this paper is the first to offer theoretical investigation on these issues.

The organization of the paper is following. Section 2 describes the structures of the model. It explains the concept of corporate governance used in the model. It also delineates the investment dynamics, production technology and characteristics of the principal and agent. The formulation of contractual relationship and agency problem is explained here. In Section 3, we derive the optimal dynamic contract using martingale representation of dynamic continuation value and the continuous-time stochastic dynamic programming method, the Hamilton-Jacobi-Bellman (HJB) equation. In Section 4, we derive the security price by an implementation of the optimal contract and exhibits the value of governance mechanism as parts of the price. Section 5 considers the effects of change in governance on the dynamic relationship between investors' profit and executive compensation. We also discuss the results and implication on theoretical insight, time-inconsistency in corporate finance, empirical puzzle of corporate governance on equity price, governmental intervention on firm's governance and substitutability of exit right and internal governance. Section 6 concludes. All proofs are provided in the appendix.

II The Model

In this section, we explain the concept of corporate governance used in the model and formulate the dynamic agency problem with stochastic investment. The corporate governance of a firm composes of the law of the domiciled country and internal governance of the firm. Both law and internal governance contribute to the level of investor protection of a firm. Particularly, firm's internal governance is an agreement between investor and the manager on scope of decision making to run the firm. It limits the scope of damage if the manager steals, but does not guarantee the absence of stealing decision. Based on the agreed governance mechanism, the dynamic contract composes of investment rules, compensation process and terminal time for a given governance level. In the dynamic contracting framework, the capital dynamic is determined by investment and stealing decision. The firm's uncertain cashflow depends on both capital and agent's effort on average productivity. The agent's stealing and effort decisions are unobservable to the principal. The principal hence formulates the optimal dynamic contract to incentivize agent for his own maximal expected profit. Below, we provide precise descriptions of the capital dynamics, productivity and cashflow processes. We next present the utility and decisions of the investor and the manager. We then explain the contracting circumstance in which the optimal dynamic contract will be characterized in the next section.

II.1 Internal Governance Mechanism

The internal governance considered here means the firm's rules and requirements that describe the extent of investor's vote and manager's authority. A good internal governance mechanism promotes opportunities for investors to vote on important corporate decisions. It enhances the investor protection level of the firm, as long as investors are rationale and vote for their benefit, which include preventions from expropriating managerial decisions. The mechanism must be consistent with legal requirement and could put additional rules beyond the law in order to enhance the level of firm's investor protection. The rules and requirements are delineated in the corporate charter and hence legally binding to all contracting parties of the firm, particularly the investor and the manager. In effect, they put more restriction on the managerial decisions in order to prevent managerial expropriations on investors. In this aspect, the internal governance is the agreement between investors and the manager. It provides further protection to investors beyond the law. This definition gives two important implications on our model.

Firstly, the alteration of the internal governance must have mutual consent from both parties. The manager cannot unilaterally change the internal governance without investor's consent, and vice versa. This implication indicates that the mechanism is not the result of optimal decisions of the manager. It is a contract between investors and manager on how the decisions of the firm are made, either by investor's vote or managerial discretion. It will not change along the firm dynamics, until both parties renegotiate and agree on it. If the new internal governance is renegotiated, both parties would be on a new contractual relationship according to the newly agreed parameter of governance. In the model, the internal governance is a parameter that both parties agree upon before initiating the contractual relationship.

Secondly, the internal governance is a mechanism for investor protection beyond legal requirement. The requirement ordinarily provides many aspects of the investor protection mechanisms[3]. However, for a specific firm, investors could find an inadequacy of protection and would increase the protection level by extending the requirements in various aspects[4]. Consequently, the internal governance in this paper reflects the firm's additional protection through extension of mechanisms beyond the law. Based on the measurement methodology provided by LLSV (1998), we summarize the effect of internal governance by the parameter g. The higher level of g represents the better investor protection level from internal governance. If the firm does not provide additional protection from the legal requirement, the internal governance is zero, $g = 0$ and investors are protected solely by the law.

In essence, the internal governance transfers the rights on corporate decision from the manager's discretion to investor's vote. In theoretical terminology, the internal governance mitigates the inefficiency from contract incompleteness by specifying the additional scope of corporate decision to the principal. With higher level of internal governance, the agent has lesser extent of authority on his managerial discretion[5]. This shift of decision rights protects the investors from the risk and possible damage from investor expropriation.

The internal governance creates both cost and benefit. On the cost side, when internal governance level increases, the manager put more effort to keep up the same performance

[3]Becht, Bolton and Roell (2007) summarizes the governance mechanisms into five groups; partial concentration of ownership and control, hostile takeovers, board of directors, executive compensation and fiduciary duties of the manager.

[4]The manager has broad range of discretion on the usage of corporate resources that potentially leads to expropriation without legal restraints, for instance the discretion to determine executive compensation, the power to consider self-dealing transactions, ability to commit mixed-motive actions, the authority on making empire-building investments and the decision for managerial entrenchment. These corporate activities are tainted with agency conflict but cannot be limited by law because the manager is protected by the *Business Judgment Rule*. From Clark (1986), pp. 123, the business judgment rule is *the business judgment of the directors (and manager) that will not be challenged or overturned by courts or shareholders, and the directors (and manager) will not be liable for the consequences of their exercise of the business judgment – even for judgments that appear to have been clear mistakes– unless certain exceptions apply.* The rule puts investors on the various aspects of expropriation unrestrained by the law. Hence the internal governance, by providing additional investors right to determine important managerial decisions and policies or to approval crucial corporate decisions, can provide more protection against the risk of expropriation due to the business judgment rule.

[5]Technically, the internal governance parameter g can be considered as a measure defined on the partially ordered set of managerial decisions. It reflects an agreement between investors and manager to eliminate some actions available in a set of managerial decisions provided by the law. The eliminated choices could be potential expropriations on investors and the firm. Consequently, higher level of internal governance implies a more restricted set on managerial decisions and better investor protection.

due to more stringent requirement on managerial decisions. The investor has to compensate for higher cost in order to motivate the additional effort. On the benefit side, the firm and investors have lower *potential* cost of expropriation, hence have more resources for corporate investment. However, because the internal governance could limit the possible damage from expropriation, it does not guarantee that the manager would not steal the corporate resource for his private benefit. Due to the unobservable decision on stealing, the investor must incentivize the manager away from the stealing and violation of the agreement, which could be an additional cost to the firm. We will explicitly describe the details of cost and benefit of internal governance on the investor and the manager below.

II.2 Investment and Production Technology

We formulate the dynamic investment model. The firm uses capital and agent's effort to generate the stream of cashflow. The capital is considered as a numeraire. The capital dynamics is determined as follow,

$$dK_t = (I_t - \delta K_t - b_t \zeta(g) K_t)\, dt \tag{1}$$

where K_t is capital, I_t is investment, $\delta > 0$ is capital depreciation rate. The $\zeta(g)$ denotes the rate of capital distortion as a function of internal governance level. We assume that the rate of capital distortion is decreasing in internal governance level, $\zeta'(g) < 0$. Note that when the firm only follows the legal requirement on governance mechanism, we have $g = 0$. We also assume that $\zeta(0) > 0$ meaning that the rate of capital distortion is positive when a firm does not put additional governance beyond the law. This implies the imperfect investor protection provided by the law for the considered firm. The imperfection is the consequence of the *business judgment rule*, which opens the room for the investor expropriation. Notice that we currently do not assume the curvature of the $\zeta(\cdot)$ function. With this interpretation of distortion function, we can consider the governance mechanism at both country and firm levels in the same framework. The variable $b_t \in \{0, 1\}$ is a binary decision choice of the agent's decision to respect the overall level of governance. When $b = 0$, it implies agent abides by the governance mechanism and not committing conflicting behavior and $b = 1$ otherwise. This setting underlines the notion that the governance mechanism can limit the damage of expropriation but does not guarantee an absence of stealing by the agent.

The capital dynamics then depends on the investment net of depreciation and the agent's decision to expropriate. The expropriation would divert capital away and retard the process of capital accumulation of the firm.

We assume the linear production technology. The cashflow dynamics derives from the increment of the firm's cashflow from production ($K_t dA_t$) net of cost of capital adjustment ($K_t L(i_t) dt$). The firm's cashflow process is following

$$dY_t = K_t\,(dA_t - L(i_t)dt) \tag{2}$$

where Y_t is the cumulative cashflow at time t, dA_t is instantaneous productivity to be defined below. The function $L(i_t)$ is the adjustment cost of investment per capital, where

$i_t = \frac{I_t}{K_t}$ is the investment per capital. It is defined as $L(i_t) = \frac{L(I_t,K_t)}{K_t} + \frac{I_t}{K_t}$, in which $L(I_t, K_t) = \frac{\theta}{2}\frac{I_t^2}{K_t}$. Then we can write the adjustment cost in the investment per capital unit as $L(i_t) = \frac{\theta}{2}i_t^2 + i_t$ which is the standard convex adjustment cost of investment.

We model the cumulative productivity as A_t, and hence dA_t denotes the instantaneous productivity at time t. The productivity follows the diffusion process in which the drift is determined by the agent's unobservable effort, $a_t \in \mathbb{R}_+$. The productivity volatility is constant and denoted by $\sigma > 0$. The differential form of productivity process reads as follow,

$$dA_t = a_t dt + \sigma dZ_t \tag{3}$$

where Z_t is the standard Brownian motion. Notice that the productivity is subject to uncertainty, due to diffusion term, and the firm possibly incurs loss in spite of positive agent's effort.

The firm is established as contractual relationship between investor and the manager, as the principal and agent respectively. The contract can be ceased when a party unilaterally revoke the relationship and the firm is consequently terminated. We describe the preferences of contractual parties next.

II.3 Modeling Principal and Agent

Due to the separation of ownership and control, the investor, as the principal, employs the manager, or the agent, to run the firm given the technology and agreed governance mechanism. The agent controls the unobservable drift of productivity process in order to generate the incoming cashflow, which is publicly observable and described in equation (2). At any instant, the agent also decides whether to divert the capital for his private benefit. The distortion is limited by the level of governance, as described in equation (1). The principal cannot observe actions of the agent, both the effort provision on the drift of productivity process (a_t), and the decision on capital diversion (b_t). Notice that the governance mechanism could mitigate the agency cost from capital diversion, but does not guarantee agent's decision for not stealing. At any periods, the principal recommends the investment policy and determines the compensation process to the agent. He also decide the terminal period of the contract. These three elements constitute the dynamic contract under the agreed internal governance level.

The agent's preference is following. He derives utility from compensation process, denoted by U_t, and provides the stream of effort ($a_t \in \mathbb{R}_+$) affecting the average productivity level. The cost of effort depends on the effort level and governance mechanism. As previously described, the high governance level put the stringency on available agent's actions in order to keep up the performance. For higher governance level, it hence costs more for a given level of effort. The agent also decides whether to take private benefit from capital diversion. This is a binary decision, $b_t \in \{0, 1\}$. We summarize the preference and decisions of agent as follows.

$$\left(U_t - H(a_t; g)\right) dt + b_t \zeta(g) K_t dt \tag{4}$$

The effort cost function, $H(\cdot;\cdot)$, is increasing and convex in effort (a_t), and governance (g) level. We assume complementarity between two arguments, $\frac{\partial^2 H(a,g)}{\partial a \partial g} > 0$. As mentioned above, at the higher level of internal governance, it takes higher cost for a given level of effort. In notations, for $g_1 > g_0, H(a_t; g_1) > H(a_t; g_0)$. This is the important assumption which drives our key results.

The principal derives the benefit from the cashflow process and pays the agent for his effort given both instantaneous constraints. We describes his benefit in each period as follows,

$$dY_t - U_t^g dt; \qquad \forall t \in [0, \tau]. \tag{5}$$

The principal recovers the termination value when the the firm is ceased at the terminal date. We assume that when the relationship ends, the principal can redeem the value of capital lK_τ, where $l > 0$ is liquidation rate and τ is the terminal date and K_τ is the capital level at terminal time.

II.4 Formulation of Dynamic Contractual Relationship

In our model, both parties maximize their expectation of the discounted benefit over the longevity of the firm, which is the length of contractual relationship. The capital (K_t) and cashflow (Y_t) are observable, while the agent's decisions are not. We assume, without loss of generality, that the discount rate of agent is greater to that of the principal, $\gamma \geqslant r$ respectively[6], and the principal's discount rate (r) is equal to the risk-free rate. We also assume that the principal has the bargaining power and offers the contract[7] to the agent.

The dynamic contractual relationship is formed by the dynamic contract specifying investment (I_t), incentive-compatible compensation (U_t^g) and optimal terminal time (τ), after an agreement on the internal governance mechanism (g). The governance mechanism hold constant along the evolution of the dynamic contract. However, the governance mechanism could change only when both parties agree to renegotiate for the new level in which the dynamic contract will be reformulated accordingly. The dynamic contract is summarized as follows, $\{I_t, U_t^g, \tau; g\}$.

Given the dynamic contract and governance level, the agent determines the effort level and capital diversion decision to maximize,

$$V_0 = \sup_{\{a_t, b_t; t \in [0,\tau]\}} \mathbb{E}^a \left[\int_0^\tau e^{-\gamma t} \left(U_t^g - H(a_t; g) + b_t \zeta(g) K_t \right) dt \right] \tag{6}$$

[6]We can also assume $\gamma > r$, the discount rate of the agent is strictly greater than that of the principal. What is necessary for our analysis is that the discount rate of the principal must not be greater than that of the agent. Otherwise, the principal would postpone the compensation to agent indefinitely. However, the difference between strictly and weakly lower discount rate does not change our insight on the analysis of internal governance

[7]This assumption can be relaxed. It would affect the boundary value of involved parties, but not the main results of optimal contract characterization.

where the expectation is made under the probability measure generated by the effort process $(a_t; t \in [0, \tau])$. The expectation of discounted total benefit composes of two parts, the net benefit from effort and the potential private benefit, or agency rent, from capital distortion.

Suppose that at the time of contracting, the initial capital is $K_0 > 0$ and the agent's initial payoff is denoted by W_0, the objective function of the principal is formulated as follow.

$$F(W_0, K_0) = \sup_{\{I_t, U_t^g; t\in[0,\tau]\}} \mathbb{E}\left[\int_0^\tau e^{-rt} dY_t - \int_0^\tau e^{-rt} U_t^g dt + e^{-r\tau} l K_\tau\right]. \tag{7}$$

which comprises of two elements; the net value of accumulated cashflow and the terminal value.

In the contractual relationship, the principal knows the effort cost function $H(\cdot; g)$, distortion function $\zeta(\cdot)$ and governance level (g), but does not observe the agent's decisions (a_t, b_t). The optimal contract is formulated by the principal's inference on cashflow as a signal of unobservable decisions of the agent. To formulate the optimal dynamic contract, the principal must take into account both instantaneous constraints, equation (8) and (9), and concern about the agent's intertemporal incentive in order to motivate the optimal evolution of the firm. Base on the formulation and information structure described above, we characterize the optimal dynamic contract in the next section.

III Optimal Dynamic Contract with Governance

This section postulates the optimal dynamic contract with governance. The contract composes of two main elements, the agent's incentive structure and principal's profit function. For the incentive structure, it is illustrated by continuation value dynamics. Under the optimal contract, it incorporates the agency rent as the instantaneous benefit and also makes the marginal effort cost to induce intertemporal incentive. The governance mechanism affects both parts of the incentive structure, but in opposite direction. For the profit function, it takes the form of second-order differential equation of the continuation value per capital. The governance level affects the whole shape of profit function; the level, slope and the curvature. The optimal dynamic contract allows agent to gain instantaneous agency rent, but align his dynamic incentive with investor's profit and firm's growth. When we consider the entire dynamic relationship of the firm, the optimal contract creates incentive alignment effect at initial stage of the firm in which the investor's profit grows with agent's continuation value. In the later stage, the investor's profit decreases when continuation value increases, due to wealth transfer effect. These effects are the result of optimal dynamic incentive and they cause the concavity in profit function. We summarize this section with the detailed characteristics of optimal dynamic contract and the figure showing dynamic relationship between continuation value and profit. We use these results for further analysis on security price and governance change in next sections.

To derive optimal dynamic contract, we use the method of stochastic control (HJB equation), the continuous-time dynamic programming, to characterize the optimal contract from principal's perspective. The optimal contract is characterized as a differential equation and boundary conditions. Due to unobservable decisions of the agent, the principal formulate the incentive structure to motivate optimal effort and to ensure an absence of capital diversion in any period. This formulation use the continuation value, proposed by Spear and Srivastava (1987), to keep track of agent's performance and to link it to his future benefit. To characterize the continuation value in continuous-time setting, we employ the martingale representation theorem to recover the continuation value dynamics, as proposed by Sannikov (2008). The dynamic continuation value encapsulates instantaneous and intertemporal incentives of the agent. Based on continuation value dynamics, we characterize the incentive compatible decisions of the agent, characteristics of optimal dynamic contract and investor's profit function as the second-order differential equation. Using continuous-time model and the prescribed methodology, we have a clear and intuitive analytical solution of the optimal dynamic contract and can see how the effects of capital distortion and governance mechanism affects the agent's incentive structure as well as and principal's profit. This advantage yield further benefits in next sections.

This section proceeds as follow. We start with describing the incentive compatible mechanism in both static and dynamic perspective. We employ the direct revelation principle to the stealing decision, which is static in its nature, then consider the evolution of continuation value which capture the dynamic nature of decision on effort. We show that the incentive compatible compensation will be equal to effort cost and control rent in every period, which capture instantaneous incentive of the agent. To capture the dynamic incentive, we construct the continuation value, which is the expectation of discounted net benefit of the agent from current period to the terminal date. The expectation bases on the probability measure generated by effort that determines the drift of productivity process. We then establish the stochastic differential equation of the continuation value based on martingale representation theorem. We next find the necessary conditions for the incentive compatible decisions of the agent and the requirement on continuation value dynamics. With well-behaved continuation value dynamics, we characterize optimal contract by applying HJB equation to the principal's problem. To avoid a complication of many state variables, we normalize the continuation value by capital level and use the continuation value per capital as state variable. This normalization circumvents the problematic interpretation due to firm's size and give an insightful interpretation of optimal contract by focusing on the stage of the firm instead. As a consequence, the optimal contract is formulated as ordinary differential equation of principal's profit function over the continuation value per capital and necessary conditions for the elements of the contract. The elements of optimal contract, investment dynamics and compensation process, will be the function of continuation value. To pinpoint the solution of the ODE of profit function, we describe the boundary conditions and its important property, its curvature. We conclude this section with a collection of conditions for optimal dynamic contract and a figure of profit function.

III.1 Incentive Structure

We now consider agent's incentive structure to work for the principal and to divert corporate resource within any period when both decisions are unobservable. We start with the static incentive by considering the compensation which would induce the no capital diversion decision ($b_t = 0$) for any level of effort provision in any period. We next study the dynamic incentive which motivate the agent to provide optimal flow of effort provision along the firm dynamics given no stealing decision in every period.

Considering static incentive for any given period, we study the allocation profiles, combinations of compensation and capital diversion decision, which would induce no stealing for any level of effort provision ($a_t \in \mathbb{R}_+, b_t = 0$). Our analysis employs the static revelation principle, while will be simplified due to the binary value of stealing decision. It results in two requirements, the instantaneous participation and incentive compatibility constraints.

In our model, the agent will work as long as his net utility from working is not lower than his outside option. Without loss of generality, we normalize the outside option to be zero. This is the *instantaneous participation constraint*. It must be satisfied in any period until the terminal date, τ. The participation constraint hence requires the compensation process to meet cost from the provided effort at any instance. It is described as follows,

$$U_t - H(a_t; g) \geqslant 0; \qquad \forall t \in [0, \tau]. \tag{8}$$

Notice that the compensation for the effort cost does not consider the effect of diversion decision. To incorporate the incentive of capital diversion of the agent, we employ the implementation and truthful revelation mechanism as follow[8].

We define that an absence of capital diversion decision ($b_t = 0$) is *implementable* for any given level of effort provision ($a_t \in \mathbb{R}_+$) if there exists a compensation level in the same period such that the direct mechanism induce the truthful revelation in which the allocation ($U_t^g, b = 0$) yields agent's net benefit not less than ($U_t, b = 1$) for any level of effort provision a_t in any period,

$$U_t^g - H(a_t; g) \geqslant U_t - H(a_t; g) + \zeta(g)K_t; \qquad \forall t \in [0, \tau], \tag{9}$$

In parallel, the revelation mechanism $\{(U_t^g, b_t = 0); (U_t, b_t = 1)\}$ is said to be truthful when the equation (9) holds. This equation is called *instantaneous incentive compatibility constraint*. Notice that U_t^g denotes the compensation process that satisfies the instantaneous incentive constraint under the governance mechanism and U_t represents the compensation process satisfying the participation constraint, equation (8), which consider merely the effort cost at any period.

The intuition of the incentive constraint is following. Because the decision to divert capital is unobservable, the principal would provide the incentive for the agent to ensure

[8] According to the structure of our model, we consider the implementation and truthful revelation mechanism in parallel to Guesnerie and Laffont (1984).

that he will not steal at any instance[9]. We then have the *instantaneous* incentive constraint to guarantee an absence of capital diversion[10].

We take into account for two instantaneous constraints altogether in order to implement the truthful mechanism of no diversion decision. They shape the feasible set compensation level (U_t^g) given the no diversion decision ($b_t = 0$) at any effort level ($a_t \in \mathbb{R}_+$) at any period as follow.

$$U_t^g \geqslant H(a_t; g) + \zeta(g) K_t; \qquad \forall t \in [0, \tau] \tag{10}$$

We show in the proposition 1 that the equation (10) will hold with equality. By static revelation principle, the agent will be indifferent between stealing and no stealing decisions within any period because his net benefit does not differ between the two. The incentive-compatible compensation process (U_t^g) incorporates the agency rent, or the private benefit of control, ($\zeta(g)K_t$) already. It hence satisfies both participation and incentive constraints to guarantee agent's work ($U_t \geqslant H(a_t; g)$) and absence of expropriation ($b_t = 0$) altogether, at any period. This equation restricts the set the incentive-compatible compensation process at any period by ruling out the compensations that could violate the instantaneous participation and incentive constraints.

To consider agent's dynamic incentive, we define the continuation value of the agent with respect to the instantaneous participation and incentive constraints. For given governance mechanism and contract $\{I_t, U_t^g, \tau; g\}$, the agent's continuation value at time $t \in [0, \tau]$ is following,

$$W_t = \mathbb{E}^a \left[\int_t^\tau e^{-\gamma(s-t)} \left(U_s^g - H(a_s; g) \right) ds \right]. \tag{11}$$

The continuation value is the expectation of the discounted net utility of each period under the contract and history of performance from time t to the terminal period τ. At a given period, the principal determines the level of compensation (U_t^g) with respect to instantaneous participation and incentive constrains in order to guarantee an absence of stealing decision. The compensation process must also be aligned with the dynamic incentive under the effort level (a_t) given the current governance mechanism. Notice that the continuation value of the agent does not explicitly take into account the private benefit of control explicitly because the agency rent is incorporated through the instantaneous incentive constraint.

To characterize the evolution of continuation value, we consider the value function of the agent's benefit. The value function of the agent at any time t describes the the net benefit from the initial period up to time t and the expected net benefit in the future through the continuation value. Under the optimal contract, the value function is martingale. Intuitively, this is because the principal would compensate the agent just enough to implement the

[9]This incentive can be incorporated into the contract in the sense that, given the incentive provision the agent would not commit any acts on capital diversion. Otherwise, when the principal can verify the diversion, the agent will be executed with prohibitive punishment.

[10]Because the decision on capital diversion is binary, $b_t \in \{0, 1\}$, we can consider the instantaneous incentive constraint as the static agency problem for a given period.

optimal decisions and to keep the expectation of the compensation level constant over time. He would give a discounted average compensation to the agent just to compensate the costs of actions. Consequently, the value function of the agent would be a martingale, implying its expectation is constant over time.

Because agent's value function is martingale, we claim that there exist a progressively measurable stochastic process λ_t in which its value at any time depends on the information from previous periods only and it makes the evolution of continuation value to be a martingale process. This claim base on the martingale representation theorem. The detail of derivation is provided and proven in the appendix. We summarize the result in the following proposition.

Proposition 1 (**Incentive Structure of Agent**). *Considering agent's static incentive to motivate absence of stealing, there exists an incentive compatible compensation process U_t^g that implements no stealing decision ($b_t = 0$) for any level of effort provision. It make instantaneous constraints binding and hold with equality. Consequently, the incentive compatible compensation process is equal to the summation of the effort cost and agency rent for any given period.*

$$U_t^g = H(a_t; g) + \zeta(g)K_t; \qquad \forall t \in [0, \tau]$$

Considering agent's dynamic incentive to motivate intertemporal effort provision, there exists the progressively measurable process λ_t such that the continuation value of the agent is denoted as follows

$$W_t = W_0 + \int_0^t e^{-\gamma s} \left(\gamma W_s - (U_t^g - H(a_s; g)) \right) ds + \int_0^t e^{-\gamma s} \lambda_s \sigma K_s dZ_s$$

In the differential form,

$$dW_t = \gamma W_t dt - (U_t^g - H(a_t; g))dt + \lambda_t \sigma K_t dZ_t.$$

or equivalently,

$$dW_t = \gamma W_t dt - (U_t^g - H(a_t; g))dt + \lambda_t \left(dY_t - K_t(a_t - L(i_t))dt \right).$$

The proposition provides the incentive compatible compensation (U_t^g) inducing no corporate stealing within every period and characterize the continuation value dynamics which incorporate such compensation to motivate the optimal dynamic incentive for dynamic effort provision. Since the principal must characterize the optimal contract to motivate both instantaneous and intertemporal incentive, the proposition delineates the compensation process that satisfies both types of incentive. Technically, it embeds the instantaneous incentive compatible contract into the dynamic incentive mechanism and use the dynamic revelation principle to characterize the dynamic continuation value in order to design the optimal dynamic contract. The embedding method directly incorporate the effects of instantaneous incentive structure into the dynamic incentive contract. We will use this insight later when we consider the renegotiation on internal governance level as a shift in weight between instantaneous and intertemporal incentive of the agent.

The evolution of continuation value composes of two parts. The first part is the average growth of the continuation value, the drift term of continuation value dynamics. It is the summation of the return from to discount rate and the agency rent from each period. We have agency rent here because the principal must incentivize the agent away from stealing in any period for a given level of governance. The agency rent is incorporated into the continuation value dynamic through instantaneous incentive constraint. We can also express the first part as follow,

$$\gamma W_t dt = \mathbb{E}\left[(U_t^g - H(a_t;g))dt + dW_t\right]. \tag{12}$$

By taking expectation and rearranging terms, the rate of return on continuation value at any period is equal to the agency rent and the expected growth of the continuation value.

The second part is the volatility term. It reflects the sensitivity of the continuation value to the volatility of incoming cashflow. The progressively measurable process λ_t can be considered as a multiplier of the cashflow volatility $(\alpha K_t Z_t)$ of the sensitivity of agent's continuation value. Consequently, the optimal incentive motivates agency's decision by linking his compensation to the firm's performance. The multiplier process induces optimal effort (a_t), by maximizing $(dY_t - K_t(a_t - L(i_t))dt)$, through the multiplication of the net expected cashflow. In essence, optimal contract puts agent to face the risk of performance and to receive return through its multiplication in order to induce the optimal decision under unobservable effort process.

We next consider the incentive compatible decisions of the agent given the dynamic continuation value described previously. We also pinpoint the optimal level of progressively measurable process because there are possibly many processes that satisfy the requirement of proposition 1.

To characterize the incentive compatible decisions $\{a_t, b_t\}$, we use the one-shot deviation principle, a deviation from the optimal decision path at any moment would reduce the total benefit of the agent. The condition inducing no deviation at any moment can be generalized to the whole path of contract. We then use the derived condition to characterize the incentive compatible decisions and the optimal path. The detail of derivation is proven and shown in the appendix. We state the necessary conditions for the optimal decisions here.

Under the optimal dynamic contract, the incentive compatible effort must satisfy the following condition. For $t \in [0,\tau]$, the optimal effort decision must satisfy,

$$(\lambda_t a_t - H(a_t;g)) \geqslant (\lambda_t a_t' - H(a_t';g)), \quad \forall a_t' \neq a_t. \tag{13}$$

For the optimal decision on capital diversion (b_t), we require that

$$b_t(1+\lambda_t)\zeta(g)K_t \leqslant 0, \quad \forall t \in [0,\tau]. \tag{14}$$

Knowing that $\lambda_t \geqslant 0$, $\zeta(g) > 0$ and $K_t > 0$, this condition is satisfied only if $b_t = 0, \forall t \in [0,\tau]$. Hence, taking no capital is incentive compatible according to the continuation value dynamics.

Given both decisions, we now pinpoint the progressive measurable process for the optimal contract. From the requirement in optimal dynamic incentive, it is possible to have many processes satisfying the property. Because the agent's continuation value is cost to the principal, he would choose the lowest possible level of progressively measurable process that satisfies the requirement in proposition 1 and the incentive compatible decisions. We state the condition to pinpoint the progressively measurable process for optimal contract as follow.

$$\lambda_t = min\{\tilde{\lambda_t} \in [0,\infty) : a_t \in argmax_{\{\tilde{a}\in[0,\infty)\}}\{\tilde{\lambda_t}\tilde{a}_t - H(\tilde{a}_t; g)\}\} \tag{15}$$

The necessary conditions for incentive compatible decisions and progressively measurable process in the optimal contract are summarized in the following proposition.

Proposition 2 (Incentive Compatible Decisions). *Given the continuation value dynamics described in proposition 1, agent's decisions on effort provision, capital diversion and multiplier process satisfy the following condition, respectively,*

$$\begin{aligned}
&a_t \in argmax\{\lambda_t\tilde{a}_t - H(\tilde{a}_t; g)\} \quad \forall t \in [0,\tau],,\\
&b_t = 0; \quad \forall t \in [0,\tau],\\
&\lambda_t = min\{\tilde{\lambda_t} \in [0,\infty) : a_t \in argmax_{\{\tilde{a}\in[0,\infty)\}}\{\tilde{\lambda_t}\tilde{a}_t - H(\tilde{a}_t; g)\}\}
\end{aligned}$$

The proposition 2 describes incentive compatible decisions of the agent. Under the optimal incentive, no capital diversion ($b_t = 0$) is optimal because the compensation process already incorporates the agency rent required by the instantaneous incentive constraint. The agent hence does not have motivation to divert more capital from the firm. The optimal effort level (a_t) equilibrates the marginal benefit and marginal effort cost in every period, $\lambda_t = \frac{\partial H(a_t;g)}{\partial a_t}$, in which the marginal benefit is chosen at the lowest possible level that satisfies this condition and proposition 1. This proposition indicates that at any given period t the optimal contract induce efficient decisions of the agent cross-sectionally and dynamically. Based on the dynamic continuation value and optimal decisions, we next consider the profit function and optimal dynamic contract.

III.2 Principal's Profit Function

We now characterize the optimal dynamic contract. We proceed by applying continuous-time dynamic programming method, through the Hamilton-Jacobi-Bellman (HJB) equation, to the principal's profit function $F(W_t, K_t)$, which is a function of two state variables, agent's continuation value (W_t) and capital level (K_t). We transform the optimization into single state variable by scaling down the continuation value with the capital level, denoted by $w_t = \frac{W_t}{K_t}$, in order to simplify the analysis. We then consider the scaled profit function, denoted by $f(w_t) = \frac{F(W_t,K_t)}{K_t}$, in the analysis of optimal contract. We also scale down the related variables in term of unit per capital[11]. The elements of optimal dynamic contract are characterized by optimality and boundary conditions of the HJB equation on scaled profit function and continuation value per capital unit.

From continuation value dynamics and scaled profit function, the HJB equation is following, denoting $u_t^g = \frac{U_t^g}{K_t}$ and $i_t = \frac{I_t}{K_t}$,

$$\begin{aligned} rf(w_t) = \sup_{\{i_t, u_t^g\}} \{ & a_t - u_t^g - L(i_t) + ((\gamma - (i_t - \delta))w_t - (u_t^g - h(a_t; g)))\, f'(w_t) \\ & + \frac{1}{2} f''(w_t)\lambda_t^2 \sigma^2 + f(w_t)(i_t - \delta)\}. \end{aligned} \tag{16}$$

From the HJB equation (16), the optimal investment per capital is determined by the necessary condition

$$f(w_t) - w_t f'(w_t) = L'(i_t). \tag{17}$$

This is the Euler equation for investment per capital. It is consistent with the marginal-q investment decision. It requires that the optimal investment per capital equilibrates the marginal benefit of investment to the value of the firm, or marginal-q, to the marginal cost of capital adjustment. To illustrate the condition, we define the marginal-q as the derivative of total value of the firm with respect to capital, $q_t = \frac{\partial(F(K_t, W_t) + W_t)}{\partial K_t}$ where $F(W_t, K_t) = K_t f(w_t)$. Then $\frac{\partial F(K_t, W_t)}{\partial K_t} = q_t = f(w_t) - w_t f'(w_t)$. The marginal cost of capital adjustment is $L'(i_t)$. The equation (17) states this optimality condition. Substitution the explicit form of marginal cost of capital adjustment, denoted by $L'(i_t) = 1 + \theta i_t$, the optimal policy of investment per capital satisfies the following condition

$$i_t^* = \frac{q_t - 1}{\theta} = \left(\frac{f(w_t) - w_t f'(w_t) - 1}{\theta} \right). \tag{18}$$

We next characterize the optimal compensation process. From the HJB equation (16), the first-order condition with respect to the compensation gives the corner solution, $f'(w_t) = -1$. There is no direct link on optimal dynamic compensation over time. This result is intuitive. We know that the compensation process is costly to the principal and it would be decreased as low as possible at any period. It is hence determined by the instantaneous constraints. Specifically, the compensation process brings the optimal effort provision over time when it meets the effort cost induced by the incentive compatible effort (a_t) and the progressively measurable process (λ_t) required in proposition 2 and 1, respectively. This is because the instantaneous constraints force the compensation process to meet the periodic effort cost at any instance. In turn, it generates the optimal dynamic effort through the λ_t, which generates the optimal dynamic incentive according to continuation value dynamics.

[11] With the transformation, we interpret the dynamic relationship between investor's profit and agent's scaled continuation value as the stage of the firm. We will analyze the relationship later in this section after characterization of optimal dynamic contract.

In short, the compensation process then has indirect link to the optimal dynamic effort through the instantaneous participation and incentive constraints when proposition 1 and 2 hold.

We summarize the differential equation of the profit function and the conditions of optimal investment and compensation $\{i_t, u_t^g\}$ in the following proposition. Note that we substitute the optimal investment policy into the profit function to illustrate the explicit form of differential equation. The detail of derivation of differential equation and necessary conditions is given in the appendix.

Proposition 3 (**Differential Equation of Profit Functions**). *The principal's scaled profit function takes the form of second-order ordinary differential equation as follow,*

$$(r+\delta)f(w_t) = a_t - u_t^g + \frac{(q_t-1)^2}{2\theta} + \left((\gamma+\delta)w_t - (u_t^g - h(a_t;g))\right)f'(w_t) + \frac{1}{2}\lambda^2\sigma f''(w_t).$$

The optimal policy of investment per capital is determined by

$$i_t^* = \frac{f(w_t) - w_t f'(w_t) - 1}{\theta}.$$

The optimal compensation process (u_t^g) is derived from the binding instantaneous participation and incentive constraints under the requirements of the optimal effort provision and agency cost for the given governance level.

To pinpoint the exact solution of profit function, we now need the related boundary conditions. We begin with lower boundary and then consider the upper boundary conditions. We next focus on the important property of the profit function, the curvature.

To consider the condition at the lower boundary, we assume that the outside option of the agent is zero. When the continuation value reaches the zero ($W = 0$), implying agent's expectation for future benefit is zero, and equal to outside option, the agent would not have incentive to work for future benefit. The principal will liquidate the firm and redeem the existing capital[12]. From the setting, the terminal value of the firm is lK_τ. The lower boundary condition is $F(0, K_\tau) = lK_\tau$, or, in the form of scaled profit function,

$$f(0) = l. \tag{19}$$

where l is the liquidation rate of the capital and K_τ is the capital at terminal date when $W_\tau = 0$.

To consider the upper boundary conditions, notice that when the continuation value is excessively high, the principal would find it unprofitable to maintain the contractual relationship with the agent. This is because motivating agency's optimal effort is too costly.

[12]The liquidation of the firm is equivalent to terminate the contract and restructuring the firm, e.g. change the agent. Because the principal employs the agent to work for him, he will terminate the contract when the agent no longer have an incentive to work.

Hence, it is optimal to terminate the existing contract[13]. This claim is consistent with Spear and Wang (2005) and Sannikov (2008). To characterize the continuation value at the upper boundary, we consider the relative value between principal's profit and agent's incentive. Because the principal compensate the agent in the form of sharing transfer from the incoming cashflow, the highest compensation the principal is willing to make is the total amount of incoming cashflow. The principal will not compensate the agent beyond this level, since it will deplete his own wealth to pay the agent. This compensation is in essence the transfer from the principal's profit to the agent's continuation value. Consequently, it costs the principal at most one unit of the profit $f(w_t)$ to increase a unit of agent's wealth, w_t, consistent with DeMarzo and Sannikov (2006). It indicates that the slope of profit function at the upper boundary is a negative unit, implying a unit lesser in principal's profit becomes a unit more in principal's wealth. This is an upper boundary condition of the profit function. We denote the continuation value according to this condition by $\bar{w}_t$. The condition is also called *smooth pasting condition*[14], which requires that, at upper boundary $\bar{w}$,

$$f'(\bar{w}) = -1. \tag{20}$$

Another requirement on upper boundary is *Super Contact Condition.* It requires the rate of change in profit at the upper boundary to be zero. The idea is intuitive. When the principal can transfer the incoming cashflow to the agent in a continuous fashion, at the upper boundary, the slope of profit function should not be affected when the principal can transfer the cashflow in an infinitesimal amount[15]. In short, the rate of profit change by such transfer is zero at the optimal upper boundary. This condition states that

$$f''(\bar{w}) = 0. \tag{21}$$

From the lower and upper boundary conditions, we claim that scaled continuation value under the optimal contract has a range from zero to the upper boundary, $w_t \in [0, \bar{w}]$. The value of profit function is determined accordingly.

Another important property of the profit function over the domain is the curvature. We claim that the profit function is strictly concave over the range, $w_t \in (0, \bar{w})$. We prove the result in the appendix of proposition 4. This non-linear relationship between agent's compensation and principal's profit can be considered as a generalization of the possible linear solution in which the relationship between compensation and profit is fixed along the firm dynamics. The linear solution in the optimal contract is proposed in the early study of intertemporal incentive as in Holmstrom and Milgrom (1987). However, in this paper, we focus our analysis on non-linear relationship as an extension of the linearity result[16].

[13]Equivalently, the principal might cease the existing contract and change the condition of payment to the agent, while keep the contractual relationship between them going.

[14]This idea is similar to instantaneous control problem in which the the principal starts transferring excess cashflow above the upper boundary of the continuation value. The upper boundary is the reflecting boundary in this circumstance.

[15]This condition is that of the determination the boundary of the state variable in instantaneous stochastic control problem, as proven in Dumas (1991) and Dixit (1993).

[16]I gratefully thank Giuseppe Bertola for raising this point and discussion on the linearity result.

We summarize the properties of the non-linear profit function, its curvature and boundary conditions, in the following proposition. The derivation and proof are given in the appendix.

Proposition 4 (Boundary Conditions and Concavity of Profit Function). *The scaled profit function is strictly concave over the interval $w_t \in (0, \bar{w})$ with the following boundary conditions*

1. *Lower Boundary Condition : $f(0) = l$*
2. *Smooth Pasting Condition : $f'(\bar{w}) = -1$*
3. *Super Contact Condition : $f''(\bar{w}) = 0$*

where l is the liquidation rate of capital.

The concavity endogenously arises from the result of second-order differential equation of the profit function. We will show the shape of function and explain the intuition of driving forces for the concavity at the end of this section. We next delineate the complete characteristics of the optimal dynamic contract.

III.3 Optimal Dynamic Contract With Governance

We conclude this section with the proposition 5 describing the complete characteristics of the optimal dynamic contract.It composes of the agent's dynamic continuation value, principal's profit function and elements of contract. We then illustrate the graph of the scaled profit function and continuation value. The explanation on the dynamic relationship between principal and agent is given. This section encapsulates the main results and insight for further analysis for next sections.

Proposition 5 (Characteristics of Optimal Contract with Governance). *There exists an investor' s profit function $F(W_t, K_t)$ which is homogeneous degree one and proportional to capital K_t; $F(W_t, K_t) = f(w_t)K_t$. The function $f(w_t)$ denotes the scaled profit function.*

The scaled profit function is strictly concave and evolves according to the ODE

$$(r+\delta)f(w_t) = a_t - u_t^g + \frac{(q_t-1)^2}{2\theta} + \left((\gamma+\delta)w_t - (u_t^g - h(a_t;g))\right) f'(w_t) + \frac{1}{2}\lambda_t^2\sigma f''(w_t)$$

for $w_t \in (0, \bar{w})$ and $u_t^g = \frac{U_t^g}{K_t}, h(a_t; g) = \frac{H(a_t;g)}{K_t}, q_t = f(w_t) - w_t f'(w_t)$. It satisfies boundary conditions $f(0) = l$, $f'(\bar{w}) = -1$ and $f''(\bar{w}) = 0$.

The continuation value of the agent evolves according to

$$dW_t = \gamma W_t dt - (U_t^g - H(a_t; g))dt + \lambda_t \sigma K_t dZ_t.$$

where $\lambda_t = min\{\tilde{\lambda_t} \in [0,\infty) : a_t \in argmax_{\{\tilde{a}\in[0,\infty)\}}\{\tilde{\lambda}\tilde{a}_t - H(\tilde{a}_t; g)\}\}$*,* K_t *evolves according to* $dK_t = (I_t - \delta K_t)\,dt$ *and* $w_t = \frac{W_t}{K_t}$*.*

Under the optimal contract, agent's effort and stealing decisions depend on the continuation value, $a_t = a(W_t), b_t = b(W_t)$ *and satisfy following conditions, respectively*

$$\begin{aligned}
&a_t \in argmax_{\{\tilde{a}\in[0,\infty)\}}\{\lambda_t\tilde{a}_t - H(\tilde{a}_t; g)\},\\
&b_t = 0; \quad \forall t \in [0,\tau],
\end{aligned}$$

The contract comprises of investment policy, compensation and terminal time, $\{I_t = I(W_t), U^g_t = U^g(W_t), \tau\}$ *satisfying the following conditions,*

$$\frac{I^*_t}{K_t} = i^*_t = \frac{q_t - 1}{\theta} = \left(\frac{f(w_t) - w_t f'(w_t) - 1}{\theta}\right); \quad \forall t \in [0,\tau]$$

$$U^g_t = H(a_t; g) + \zeta(g)K_t; \quad \forall t \in [0,\tau]$$

due to binding instantaneous participation and incentive constraints, as combined in equation (10) The terminal time τ *occurs when the continuation value reaches the lower boundary, with following condition* $W_\tau = w_\tau = 0$*.*

Under the optimal dynamic contract, principal's profit derives from three parts. The first part is the level effect, the contribution from expected cashflow. It is the net effect of productivity and investment in excess of the compensation for the agent. The second part is a contribution from the first-order derivative of the profit, the slope effect. It composes of a benefit from incentive alignment to agent's continuation value and the cost of agency rent. The magnitude of the change in the profit is a consequence of how much the optimal contract align benefit of the agent to the principal's profit, $(\gamma+\delta)w_t$, over the rate of capital distortion, $\zeta(g) = u^g_t - h(a_t; g)$. The third part is the second-order effect of the profit, the curvature effect. It is the contribution from the dynamic incentive effect through the multiplier process, λ^2_t. It affects how much the profit function adjusts to the rate of change from the cashflow fluctuation and its realization.

We conclude this section with graph and explanation of profit function under optimal dynamic contract, shown in figure 1.

Given a liquidation rate $l > 0$, the scaled profit function, $f(w_t)$, is a strictly concave function of the continuation value per capital over the range of lower and upper boundaries, $w_t \in (0, \bar{w})$. The curvature of profit function composes of two parts. The first part has positive slope for $w_t \in (0, w^*)$ and the second part negative slope over $w_t \in (w^*, \bar{w})$. The value w^* pinpoint the change of curvature of the profit function and denote its highest value with the maximal condition $f'(w^*) = 0$. The concavity of the profit function is the result of two effects, the incentive alignment and wealth transfer effects.

The *incentive alignment effect* drives the profit up when the continuation value increases. This effect happens at the initial stage of the firm, e.g. start-up stage, in which the continuation value per capital is relatively low. The profit function has positive slope in this part.

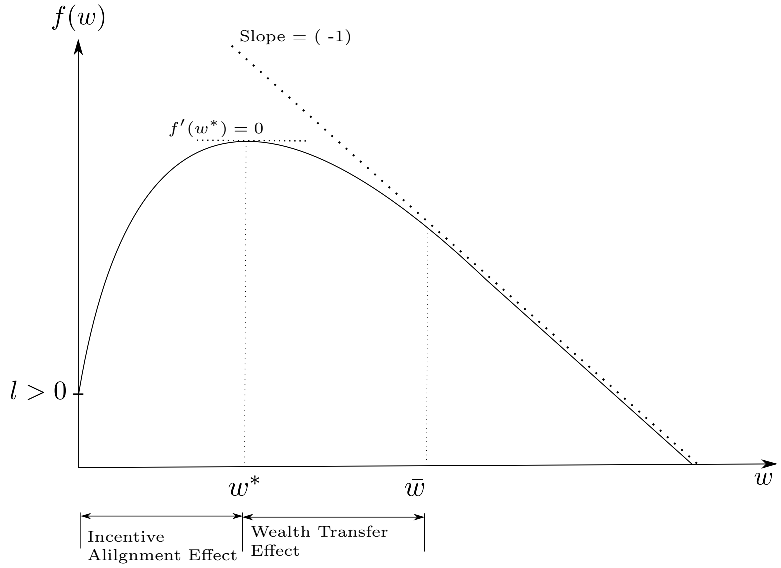

Figure 1: The shape of investor's profit function

This is because, under the optimal dynamic contract, an increase in continuation value per capital (w_t) motivate the agent to create total surplus for sharing between the principal and agent. In the initial stage, the principal's profit share is relatively larger than agent's share from continuation value. As a consequence, an increase in agent's benefit in this range induces higher principal's profit and it accounts for the positive slope of profit function. This effect dominates over the range $w_t \in (0, w^*)$. The incentive alignment effect causes the positive slope of the scaled profit function.

The second is the *wealth transfer effect* which dominates at the later stage of the firm, e.g. mature stage, over the interval $w_t \in (w^*, \bar{w})$. The profit function has negative slope in this part. Because, when agent is motivated to create a surplus, it is largely distributed to agent's continuation value and left with smaller share for principal's profit. Since the continuation value is path dependent and keeps track from previous performance, in the later stage of the firm, when firm is mature from previous track of successes, it is costly to motivate agent's effort to create firm's performance. Consequently, an increase in agent's continuation value reduces the profit of the principal due to large transfer at the mature stage. The wealth transfer effect causes negative slope of the scaled profit function.

Combining two effects, the profit function is strictly concave over the domain of continuation value. With sufficiently low liquidation rate, it rises at the initial stage and declines at the later stage of the firm. We will study the curvature and these effects again when we consider the effect of governance change on optimal contract and dynamic relationship in section V.

IV Security Price and Governance Premium

Since price is a realization of value and investors value the governance mechanism of a firm, this section illustrates the price of corporate governance as a distinct part of firm's security price in both static and dynamic valuation. The theoretical price of corporate governance has many aspects. Firstly, the contribution of governance on security price is distinct from the firm's profit. Specifically, the benefit of firm's governance is not totally subsumed into the operation profit. This gives theoretical ground for the firm's control premium in security price. Secondly, the security price incorporates the effects of governance mechanism from the country's law and firm's governance. This is the the cross-sectional perspective to study the effects corporate governance at the country and firm levels, and also their interrelation, by a single framework. Thirdly, and most importantly, the value of governance mechanism has dynamic context. The value country's law and firm's governance in security price changes over time and depends on the expected longevity of the firm. Intuitively, it depends on how long it can limit agency cost and how much it does. These features of governance mechanism in security price provide theoretical insights of the effects of law and corporate governance on equity price and firm valuation in a systematic view. They can also be useful for an empirical study of the governance effect on security prices and firm's value. It can also explain the recent empirical puzzle of corporate governance when we consider it with the effects of governance change, which will be discussed in next section.

To illustrate the contribution of governance mechanism in the security price under optimal dynamic contract, we proceed in two steps. In the first step, we show that, by defining dynamic capital structure of the firm, an implementation of the optimal contract[17] leads to the security price characterization which composes of principal's profit, agent's continuation value and the effect from the capital distortion of the firm. The first two elements are the standard result, while the third one derives from our modelling on stealing decision and governance mechanism. In the second step, we show the contribution from country's law and firm's internal governance and also their dynamic aspects as parts of a security price, by assigning a specific form of capital distortion function $\zeta(g)$. With explicit characterization of elements in a security price, we call the negative effect from the law in the security price as *country's discount term* and positive effect from internal governance mechanism as *firm's governance premium*. We conclude this section with the proposition of the results and discuss their theoretical implications and potential empirical analysis.

Firstly, we impose firm's capital structure for the security price characterization. The optimal dynamic contract can be implemented by a specific capital structure and financial flexibility which traces the path of continuation value dynamics[18]. We show that under such capital structure the security price, defined as the discounted dividend accumulation

[17] The implementation of optimal contract is not unique, as discussed in DeMarzo and Sannikov (2006) and DFHW (2010).

[18] Because the implementation of the optimal contract through capital structure is not unique, we use the idea of pure-equity firm and financial slack dynamics similar to DFHW (2010) in this section.

over the firm's longevity and the terminal value, includes the effect of country's investor protection from the law and benefit of internal governance of a firm.

We assign that the firm is financed only by equities. The investors require the the dividend to be paid out as a minimum periodic return D_t as follow,

$$dD_t = K_t \left(a_t - L(i_t)\right) dt - (\gamma - r) M_t dt. \tag{22}$$

The first term on the right-hand side is the expected incoming cashflow. The second term is the adjustment term for difference between discounting term of the agent and the principal, which is equal to interest rate[19]. This term takes the form of financial flexibility M_t. The dividend process is an obligation of the agent to pay the principal as his return on investment. If the obligation is not met, the contract and the firm are terminated.

We define financial flexibility dynamics as follow.

$$dM_t = rM_t dt + dY_t - dD_t - dX_t \tag{23}$$

where, dX_t is the cashflow reserved for compensation to the agent, dY_t is the dynamics of incoming cashflow from our setting and is equal to $K_t \left(a_t - L(i_t)\right) dt + \sigma K_t dZ_t$.

The financial flexibility is the extent of the liquidity capacity of the firm in order to operate without financial problem. It can be considered as cash, credit line or other forms of working capital[20]. For our purpose, we define the dynamic financial flexibility equal to the sum of return on risk-free money market ($rM_t dt$) and incoming cashflow (dY_t) in excess of minimum dividend payout (dD_t) and compensation to the agent (dX_t).

To implement optimal dynamic contract, we want the financial flexibility to trace the movement of dynamic continuation value in the sense that the financial flexibility would reach zero whenever the continuation value does. We then assign the financial flexibility to be equal to the continuation value per unit of risk, $M_t = \frac{W_t}{\lambda_t}$. It implies the financial buffer for the short-run downward fluctuation of the productivity and cashflow[21]. The contractual relationship and the firm would continue as long as the financial flexibility does not reach zero. In the proof of proposition 6 provided in the appendix, we show that the dynamic financial flexibility defined in equation (23) and minimum dividend payout process defined in equation (22) would implement the optimal dynamic contract and the incentive compatible decisions of the agent.

We now recover the equity prices from the contract implementation. Using the standard definition of security price as a expectation of discounted future dividends and the terminal value, we write the explicit form of the security price at time t as,

[19]We can assume the equality of two discounting terms without loss of generality. In essence, we require that the dynamic dividend payment captures the expected incoming cashflow of the firm.

[20]There are many forms and interpretations of the financial flexibility, e.g. credit line (DeMarzo and Sannikov (2006)), cash (BMPR (2007))

[21]When $M_t = \frac{W_t}{\lambda_t}$, or equivalently $m_t = \frac{w_t}{\lambda_t}$, from the dynamic continuation value, we have $w_t - 0 = \lambda_t \sigma (Z_t - Z_0)$, then $m_t = \frac{w_t}{\lambda_t} = \sigma Z_t$

$$S_t = \mathbb{E}\left[\int_t^\tau e^{-r(s-t)} dD_s + e^{-r(\tau-t)} lK_\tau\right]. \tag{24}$$

Substitute dD_t and dM_t defined in equation (22) and (23) respectively, we decompose the security price at time t into parts as the contributions from profit, compensation and agency cost, respectively

$$\begin{aligned} S_t =& \mathbb{E}\left[\int_t^\tau e^{-r(s-t)} (dY_s - U_s^g ds) + e^{-r(\tau-t)} lK_\tau\right] \\ &+ \mathbb{E}\left[\int_t^\tau e^{-r(s-t)} (rM_s ds - dM_s)\right] \\ &+ \mathbb{E}\left[\int_t^\tau e^{-r(s-t)} (-\zeta(g) K_s) ds\right]. \end{aligned} \tag{25}$$

In the appendix, we show that the first part is the profit of the principal $(F(W_t, K_t))$, the second part is continuation value per unit of risk $(\frac{W_t}{\lambda_t})$ and the last part is the negative effect from capital distortion. We next extend this part to capture the effect of imperfect legal investor protection and benefit from internal governance of the firm.

We define the capital distortion function as follow.

$$\zeta(g) = \Omega - \psi(g) \tag{26}$$

where Ω is a constant term reflecting the level of capital distortion when the internal governance is zero. This constant captures the possible distortion, or the agency cost, from imperfect investor production due to laws and legal infrastructure of a country where the firm is situated. A country with good legal infrastructure and high level of investor protection from laws has a low level of this constant and vice versa. Notice that if we indicate a country with high legal investor protection with i and another country with low protection with j, we have $\Omega_i < \Omega_j$. For the second element, we define $\psi(g)$ as a measure for the investor protection from the internal governance mechanism (g) of the firm. As previously mentioned, the internal governance gives investor protection against agency conflict in addition to the law. From out setting, it additionally decreases the possible extent of capital distortion committed by the manager. We assume that $\psi(0) = 0$ and $\psi'(g) > 0$. As a consequence, our assumption $\zeta'(g) = -\psi'(g) < 0$ holds with this definition of capital distortion function, $\zeta(\cdot)$. With these two terms, we put corporate governance at country and firm level into the same perspective.

By substitution the definition of capital distortion from equation (26) into the security price, we express the security price at time t from equation (25) in the term of investor's expected profit, agent's continuation value, country-level discount and firm's governance premium. We summarize the characterization of security price in the following proposition. The proofs of results are provided in the appendix. We next discuss its implications and empirical suggestions.

Proposition 6 (Security Price and Governance Premium). *The security price under the optimal dynamic contract is following.*

$$S_t = F(W_t, K_t) + \frac{W_t}{\lambda_t} - \mathbb{E}\left[\int_t^{\tau} e^{-r(s-t)}\Omega K_s ds\right] + \mathbb{E}\left[\int_t^{\tau} e^{-r(s-t)}(\psi(g))K_s ds\right]. \qquad (27)$$

This price derives from an implementation of the optimal dynamic contract in preposition (5). The firm finances solely by equity and pays out dividend according to equation (22) and maintains the strictly positive level of financial flexibility according to equation (23).

The security price at time t composes of four elements. The first is the contribution from investor's expectation of the future profit. The second is the normalized continuation value. The normalization by the multiplier at time (λ_t) is an adjustment for the previous incentive provision when the continuation value (W_t) grows according to the performance history. With the normalization, the contribution from the expected continuation value in the security price is not inflated by the performance history and would reflect the future prospect of the firm. The third element reflects the agency cost due to imperfect investor protection of the law. This is the *country's discount term*. The higher the discount term is, the lower security price would be, ceteris paribus. The fourth term represents the effect of internal governance to limit the agency cost. It reflects the investor's rights and investor protection beyond the legal provision. We call this term *governance premium*. Other things equal, the higher internal governance level is, the larger governance premium and, consequently, the higher the security price would be. Important to note, the values of country's discount term and firm's governance premium vary dynamically and depends on the expected longevity of the firm. The value of law and internal governance, which is static by nature, has a dynamic context. We discuss the details and implications of the security price below.

There are three implications from the security price characterization. Firstly, the value of governance premium in the security price is not subsumed into the operational profit and executive compensation. Under the optimal dynamic contract, the firm's capital becomes either corporate investment or continuation value, which finally contributes to the firm's operation profit and executive compensation, the first two elements of the security price. However, the security price still reflects the cost of imperfect investor protection and the benefit of firm's governance mechanism as distinct elements. This is consistent with the control premium view of the security price[22]. Moreover, because the security price characterization is a result of implementation of optimal contract, the country's discount term and firm's governance premium apply to both private and public corporations.

Secondly, the security price incorporates the effects of governance mechanism at the country and firm levels in a single framework. It gives the cross-sectional perspective on how law and internal governance, and their interrelation, affects the security price. For example, from $\zeta(g) = \Omega - \psi(g)$, we capture the difference of investor protection at the

[22] Essentially, positive value of internal governance reflects the efficiency gain of the investment in corporate capital over the private benefit of control along the expected future of the firm.

country level by varying the constant term Ω. We denote the possible distortion of the capital within the firm as Ω_j for a country j with weak legal investor protection and for a country i with stronger investor protection from the law and legal enforcement Ω_i in which $\Omega_i < \Omega_j$. We then capture the difference of firms' internal governance mechanism through g_i and g_j, respectively. By varying the value of the constant term $\Omega_{i,j}$ and internal governance level $g_{i,j}$, we can consider both effect of governance mechanism for both country and firm levels by a single framework. The security price characterization provides the consistent empirical perspective to consider the effects of law and internal governance level on security price and firm valuation.

Moreover, for an empirical analysis of governance, notice that from $\zeta(g)$ and $\psi(g)$, we have not assumed the shape and curvature of the function. The only requirement is $\zeta'(g) < 0$, or $\psi'(g) > 0$. The shape of the $\psi(g)$ could be a subject to empirical investigation and the governance strategy to enhance the internal governance level. Define $\psi(\cdot) : G \to [0, \Omega]$, with $\psi'(g) > 0$, the function could have the many curvatures; linear, concave, convex or quasi-concave. The characteristics of the curvature brings the relative importance of the elements in governance mechanism, how different elements of governance mechanism affect the security price or firm's value differently. The relative quantitative effect of these elements is an important subject and deserves further empirical investigations.

Thirdly, and most importantly, the value of governance mechanism has dynamic context. The value country's law and firm's governance in security price changes over time and depends on the expected longevity, or *time distance from termination* $(\tau - t)$, of the firm. This result is very intuitive[23]. The value of firm's governance premium depends on how long it limit agency cost, in additional to how much it does. It implies that, on the one hand, the governance premium is large when the firm is in the growing stage or in the mature stage, when $(\tau - t)$ is large. On the other hand, it is small when the firm is close to termination, financial distress or reorganization due to the expected short time to the termination. This insight is consistent with the fact that when a firm is closed to financial distress, its equity price in the financial market takes into account only for the liquidation value and executive compensation, the first and second elements of the security price, and exclude the contribution of governance elements. On the contrary, when a firm is growing or in a mature stage, investors tend to pay attention and value the corporate governance of the firm. The characterization in equation (27) captures this insight of dynamic aspect in the security price.

These features of governance mechanism in security price provide theoretical insights of the effects of law and corporate governance on equity price and firm valuation in a systematic view. They are also useful for further empirical studies. These insights can also explain the recent empirical puzzle of corporate governance when we consider it with the effects of governance change. We discuss it in the next section.

We next consider the effects of a change in governance in the optimal contract, including profit, continuation value, and optimal decisions. Considering the internal governance change gives us an insight of the shifting between instantaneous and intertemporal incentive

[23]However, this result has not yet stated in previous literature, to the best of my knowledge.

structure of the contract. This insight has important implication on how much a government or a regulator can rely on internal governance being determined within a firm.

V Governance Change and Firm Dynamics

This section considers the consequences of governance change on agent's incentive, principal's profit and the firm dynamics. We show that an enhancement of governance mechanism[24], resulting an increase in the governance parameter g in the optimal dynamic contract, reallocate the incentive structure of the agent. The enhanced governance reduces the potential agency cost and heightens marginal cost of effort. It hence reduces the instantaneous incentive constraint, while fortifies the intertemporal incentive through the higher level of multiplication process, λ_t. This change in incentive structure causes a better alignment of agent's compensation to the firm's performance and investor's profit. It intensifies both incentive alignment and wealth transfer effects on the profit function. The profit function has higher slope, in quantitative term, and becomes more concave. Consequently, at the initial stage of the firm, investor receives higher profit for the same level of scaled continuation value. The profit reaches its maximum faster, at the lower level of scaled continuation value. However, after reaching the maximum, in the later stage of the firm, investor profit decreases quickly due to the large wealth transfer effect. The upper boundary will be reached at the lower level of scaled continuation value. In sum, the governance enhancement reallocates agent's incentive structure, accelerates the investor's profit and wealth transfer and consequently change the structure of dynamic relationship of the two. We illustrates this change in figure 2 and discuss its important implications at the end of this section.

The consideration of governance change here gives a more complete analysis than the widely accepted *free cash flow theory*, Jensen (1986). The main thesis of the free cash flow theory implies that a governance enhancement through several ways, resulting in an improvement in investor protection and lower agency cost, will increase investor's profit. This key result bases on the static perspective of corporate governance. However, in this paper, we propose the dynamic perspective of the governance mechanism, in the sense that, even though the corporate governance is static and does not change frequently along the firm's longevity, its effects on contractual parties do change along the dynamic of the firm. This dynamic changing is due to the role of dynamic contract. As a consequence, our dynamic analysis of governance mechanism subsumes the main insight and analysis of free cash flow theory as one of its cases. Specifically, we can obtain the analysis of free cash flow theory when firm is in its initial stage of our model. Consequently, the dynamic analysis of governance change gives a broader perspective on the analysis of corporate governance on a firm. It also provides better understanding on corporate governance conundrums, including

[24] We consider the case of enhancement in internal governance, higher level of g. The consideration in the case of lower governance level is similar. Its analysis is parallel to the case of governance enhancement, but in the opposite direction. Hence, we skip the analysis of lowering governance mechanism.

time inconsistency in corporate finance and recent empirical puzzle in corporate governance. We study these phenomena at the end of this section.

To study the effects of governance change, we use the comparative static analysis of dynamic contract proposed by DeMarzo and Sannikov (2006). We proceed in two steps. The first step is to consider the effect of change in governance[25] to the profit function defined on the whole path of continuation value. We apply the Keynman-Fac formula to obtain the solution of differential equation in the expectation form. We differentiate ODE of the profit function with respect to a given parameter, ϕ, holding the scaled continuation value (w) constant. We then evaluate the change on the whole path of profit function at the boundaries using upper and lower boundary conditions, $f'(\bar{w}) = -1$ and $f''(\bar{w}) = 0$. Note that we assume the effect of governance change on the level effect of the profit function to be zero in order to focus on qualitative effects on the slope and curvature. The second step is to consider the total derivative on the interested value. We consider the boundary and the relevant conditions. We then consider the conditions related to three important point of the agent's continuation value; the initial continuation value[26], the turning point and the upper boundary, $\{w_0, w^*, \bar{w}\}$ respectively. We use total derivative on the related conditions and the previous results to derive the conclusion. The results of the comparative static analysis are concluded in the proposition 7. The detail of derivations and the proof is given in the appendix.

We now consider an increase in the internal governance and its effects of agent's incentive, principal's profit and hence the firm dynamics as an interplay of the two. The summary of the governance change on firm dynamics can be illustrated in figure 2.

An enhancement of governance level reallocates agent's incentive structure. To consider the reallocation, we consider the continuation value dynamics, show in equation (28). The governance enhancement reduces the instantaneous incentive by decreasing the periodic agency rent, a reduction in rate of capital diversion $\zeta(g)$. It simultaneously intensifies the intertemporal incentive by increasing the multiplication process which is equal to the marginal effort cost, $\lambda_t = \frac{\partial H(a_t;g)}{\partial a_t}$,

$$dW_t = \gamma W_t dt - \zeta(g) K_t dt + \lambda_t \sigma K_t dZ_t \tag{28}$$

equivalently, in the scaled-down form,

$$dw_t = \left((\gamma + \delta - i_t) w_t - \zeta(g)\right) dt + \lambda_t \sigma dZ_t \tag{29}$$

From the continuation value dynamics, under the optimal contract, an increase in governance level reduces the potential manager's benefit from agency rent ($\zeta(g)$), or the private benefit of control, which derives from the revelation principal and instantaneous constraints, as shown in proposition 1. This is a reduction in instantaneous incentive. On the dynamic incentive, the volatility part of the continuation value dynamic captures the intertemporal

[25] The method can be applied to the comparative static analysis of other parameters.

[26] This is the continuation value when agent enters into the contractual relationship with the principal under agreed governance and other parameters.

motivation that aligns the agent's incentive to principal's benefits because the volatility term is equal to expected net incoming cashflow to the firm, which is the principal's benefit, $\sigma K_t dZ_t = dY_t - K_t(a_t - L(i_t))dt$. An increase in internal governance level is in essence a shift the weight in agent's incentive structure from instantaneous to intertemporal incentive in order to reduce the private benefit of control and to make agent's benefit more depending on performances. Hence, agent's compensation is better aligned with investor's profit and the performance of the firm, the growth of the firm.

From the principal's perspective, an increase in internal governance has consequences on the level, slope and the curvature of the profit function. However, the change in governance does not affect the boundary conditions. Both lower and upper boundary conditions hold the same under the circumstance of increased internal governance. We hence consider the differential equation of the profit function under optimal dynamic contract as shown in equation (30),

$$(r+\delta)f(w_t) = a_t - h(a_t;g) - \zeta(g) + \frac{(q_t-1)^2}{2\theta} + \left((\gamma+\delta)w_t - \zeta(g)\right) f'(w_t) + \frac{1}{2}\lambda_t^2 \sigma f''(w_t). \quad (30)$$

To focus on the qualitative aspect of the governance change on the relationship between principal's profit and agent's incentive, we neutralize the effect of of governance change on the level effect of the profit function[27]. In other words, we consider how a governance change influences the relationship between principal and agent along the firm dynamics, while hold the their expected benefit from the change unaltered. In notation, we know from our assumptions, $\frac{\partial h(a_t;g)}{\partial g} > 0$ and $\frac{d\zeta(g)}{dg} < 0$. Considering level-effect neutralization, we assume the equality of the magnitude of governance change on effort cost and on diversion function; $|\frac{\partial h(a_t;g)}{\partial g}| = |\frac{d\zeta(g)}{dg}|$. In effect, when we change the governance level, the direct consequence on expected cost and benefit on principal's profit would be equal and canceled out to keep the expected profit constant. In our comparative static analysis of governance, the neutrality on the level of profit function holds.

There are two important reasons to consider the case of neutrality on the level effect of profit function. Firstly, the governance level which induces the neutrality on level of profit from its change indicates the efficient level. If the neutrality does not hold, the principal would raise the governance level to gain benefit from lower diversion rate and pay the higher effort cost to the agent in order to convince him for better governance in the optimal contract. With the more enhanced governance level, profit function would shift up and become more concave. This process would keep occurring until the governance attains the efficient level and its change induces the equality of the cost and benefit. At this level, the governance change would not affect the average, or the expected level of, benefits of principal as denoted by equation (7). In the analysis of governance change, we consider this situation in which the neutrality holds and the governance mechanism reaches the efficient level.

[27]If we do not neutralize the level effect on profit function, the net effect will merely shift the entire level of profit function, without changing the qualitative property. The neutralization would make the qualitative analysis of governance change clearer and more intuitive.

Secondly, the neutrality guarantees the possibility of a renegotiation on governance change. The neutrality on the profit level requires the equality of net cost and benefit from the principal's perspective, while he must compensate the agent to lure him for the agreement. As long as the neutrality holds, no party is worse-off from the governance change. Consequently, when the profit-level neutralization holds, no party would veto on the renegotiation on the governance alteration, either increase or decrease. The neutralization is required for a renegotiation on governance change between the principal and agent.

With the neutrality on the level effect, we then consider the slope and curvature, equivalently the first-order and second-order effects, of governance change on the profit function. From equation (30), both slope and curvature effects are more pronounced due to lower rate of capital distortion and higher marginal effort cost. The coefficient of the first-order derivative of profit function increases due to the lower capital distortion. Also, the coefficient of the second-order derivative increases from larger multiplication process. The graph of scaled continuation value and the profit function, $(w, f(w))$ will become steeper and more concave when the governance level increases. The increase in slope and curvature of the profit function is the result of more intensified incentive alignment of the agent. With more intensified incentive, agent is more motivated to generate the firm's cashflow and the growth in order to rise his own compensation. As a consequence, in the initial stage of the firm, incentive alignment effect increases and principal would have higher profit for the same level agent's continuation value. The profit reaches the maximum sooner, at the lower level of continuation value. In the later stage, an governance enhancement also fortifies the wealth transfer effect and increases the speed of wealth sharing from investor's profit to agent's compensation. Considering the entire firm dynamics, an enhancement of governance level increases the size of slope of profit function and it becomes more concave. We illustrate the changing in profit function and the firm dynamics in figure 2.

From the figure 2, we compare the graphs of profit function between two governance level; $g_1 < g_2$. At the low value of continuation value $w_t \in (0, w^*(g_2))$, the strengthened governance mechanism helps the principal to achieve the higher profit and reach its maximum sooner and less costly in term of the agent's continuation value, $w^*(g_2) < w^*(g_1)$. However, after reaching the highest profit, an increase in internal governance also intensifies the wealth transfer effect due to the the higher intertemporal incentive. In the later stage of the firm when the scaled continuation value is relatively large from previous success, $w_t \in (w^*(g_1), \bar{w}(g_2))$, investor's profit is lessened when the governance level is raised.

We now consider the stage of the firm, or the timing, when both parties renegotiate for the new internal governance level. Under the optimal dynamic contract, the enhanced internal governance intensifies agent's incentive to firm's performance and reallocate the weight of agent' s motivation from instantaneous to intertemporal incentive. The principal can gain benefit from this reallocation only when the incentive alignment effect persists, where an governance enhancement would reinforces the agent's incentive to generate more profit at higher rate. This benefit of better governance occurs in the initial stage of the firm. Otherwise, when incentive alignment effect disappears, better governance would rather lower investor's profit due to more accelerated wealth transfer effect to agent's compensation.

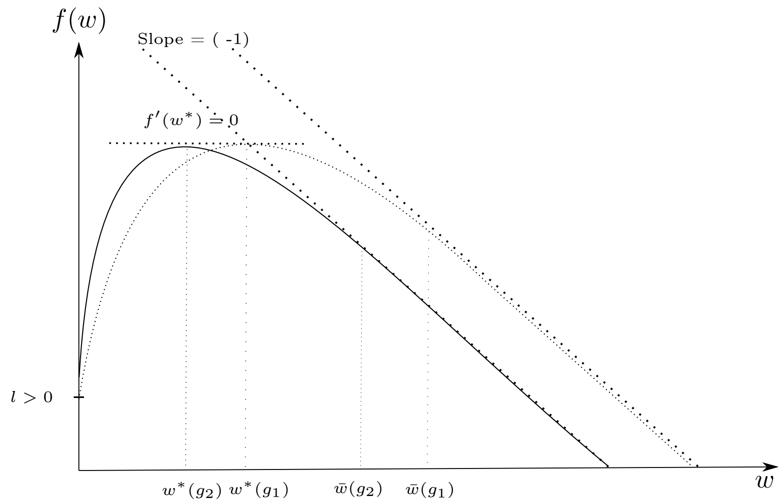

Figure 2: The shape of profit function with increased level of internal governance keeping the effect on level constant; $g_2 > g_1$. We neutralize effect of governance enhancement on the level of profit function.

To be precise, we illustrate the consequences of governance change on the expected investor's profit in figure 3 that summarizes the intuition. When the governance mechanism is strengthened from g_1 to g_2, the scaled profit function becomes more concave, holding the lower boundary and upper boundary condition unchanged. The investor's profit changes track from $f(w_t; g_1)$ to $f(w_t; g_2)$ As a consequence, the turning point of the higher governance level is lower than before, $w^*(g_2) < w^*(g_1)$. Then, there exists a level of scaled continuation value at which the level of scaled profit does not change, denoted by $\hat{w}$. It is between the two turning points of profit curves of the old and new level of governance, $\hat{w} \in [w^*(g_2), w^*(g_1)]$. It hence divide the interval of scaled continuation value into two parts, $(0, \hat{w})$ and $(\hat{w}, \bar{w})$. When we increase governance level in the initial stage, in which $w_t \in (0, \hat{w})$, the investor's profit is expected to be higher on average. On the contrary, when we increase governance level at mature stage, $w_t \in (\hat{w}, \bar{w})$, the investor's profit is expected to be lower on average. The figure 3 illustrates the value of $\hat{w}$ and the intuition on governance enhancement and investor's profit.

We hence conclude the results of governance enhancement and its consequence on investor's profit as following. There exists $\hat{w} \in [w^*(g_2), w^*(g_1)]$ for $g_2 > g_1$, such that $\frac{\partial f(\hat{w})}{\partial g} = 0$ and the following results hold.

$$\frac{\partial f(w)}{\partial g} \begin{cases} > 0; & \text{for } w \in (0, \hat{w}) \\ < 0; & \text{for } w \in (\hat{w}, \bar{w}) \end{cases} \tag{31}$$

Our dynamic model provides a more complete analysis of the effects of governance change on the firm than the static framework, which bases on the *free cash flow theory*, proposed by Jensen (1986). The main thesis of the static view of governance implies higher profit

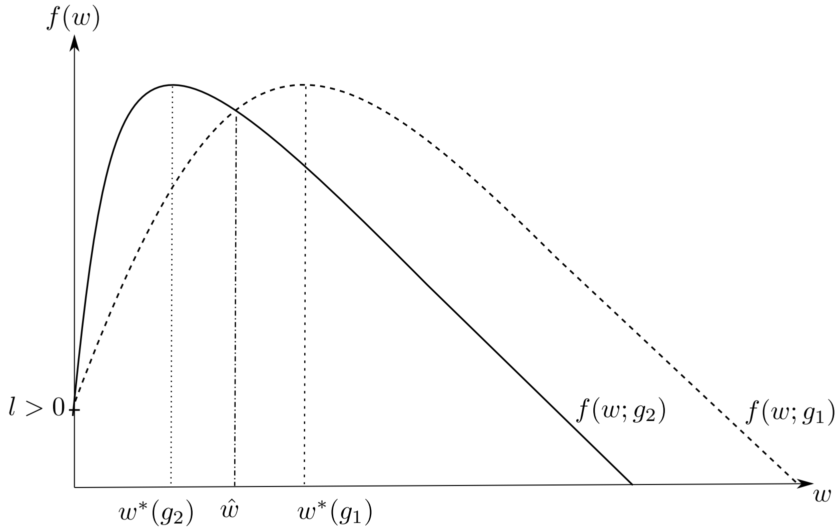

Figure 3: At $\hat{w}$, the scaled profit does not change with governance enhancement in which $\frac{df(\hat{w})}{dg} = 0$, for $g_2 > g_1$.

when governance level increases. This implication is consistent with the case of governance enhancement at the initial stage of the firm, when $w_t \in (0, \hat{w})$, in the our dynamic framework. Consequently, the dynamic analysis of governance subsumes the main thesis of the static analysis as one of its case. It extends the static analysis by considering the case of governance enhancement at mature stage of the firm, when $w_t \in (\hat{w}, \bar{w})$, as explained previously. The broader scope of analysis is a result of the recent advance in dynamic contract theory providing the analytical framework on firm and investment dynamics with agency problem. This extension of the analysis gives significant improvement on the understanding of various phenomena on corporate governance, such as the time inconsistency problem in corporate finance, empirical puzzle of corporate governance and equity price and government intervention on firm's internal governance. We will use the additional insight of dynamic analysis of corporate governance to explain these phenomena after stating the proposition below.

We conclude the results on the effects of changes in internal governance on the firm dynamics in the following proposition. We discuss the theoretical implications and the effects on executive compensation, investor's profit after the proposition. The detailed proofs are provided in the appendix.

Proposition 7 (**Governance Change and Firm Dynamics**)**.** *Suppose that governance change does not influence the level effect of investor's profit on average. Under optimal dynamic contract, an increase in governance level reallocates agent's incentive structure by shifting the weight from instantaneous to intertemporal incentive in the continuation value. The initial continuation value of the agent after governance enhancement will be lessened with* *higher governance level,* $\frac{\partial w_0}{\partial g} < 0$*. The governance enhancement intensifies the alignment of agent's compensation to principal's profit. It fortifies the incentive alignment and wealth transfer effects on the profit function. Hence, the profit function becomes more concave.*

As a consequence, the profit function becomes more concave. The principal has higher expected profit at the initial stage of the firm, when $w_t \in (0, \hat{w})$*, reaches the maximal profit at the lower level of scaled continuation value,* $\frac{\partial w^*}{\partial g} < 0$*, and has lower expected profit at the later stage of the firm, when* $w_t \in (\hat{w}, \bar{w})$*.*

We now discuss the implications from the analysis and insight. We begin with the theoretical implication from the contract theory perspective. We next consider the time inconsistency problem in corporate governance and provides the rationale based on our dynamic analysis of governance. We then explain the recent empirical puzzle in corporate governance and equity price. We finish this section with a discussion on the effects of governmental intervention on firm's governance.

A. Theoretical Implication

In the theoretical aspect, an increase in internal governance alleviates the contract incompleteness by reducing manager's scope of authority on the important managerial issues and requiring investors' vote and approval on them. Consequently, an enhancement of governance mechanism under optimal dynamic contract makes the contractual agreement on the management between manager and investor more complete and turn the contractual incompleteness into hidden-action problem, which can be solved with optimal dynamic contract.

Since a governance enhancement reduces the potential agency rent and raises the marginal effort cost of the manager, we can consider it as an economic exchange between investor and manager. When two parties agree on higher level of internal governance, the manager would have lower authority in managerial decisions because he needs to ask for investor's approval before the implementation. He has lower ability to gain potential private benefit of control, the agency cost, due to divested authority. It is reflected by lower level of rate of capital diversion, lower $\zeta(g)$. In return, the investor would pay for the additional voting power, on the marginal extent of corporate decision, by compensating the manager through higher marginal effort cost, a higher $\frac{\partial h(a_t;g)}{\partial g}$. From the investor's perspective, an agreement on the governance improvement is in fact a purchase of managerial power by paying the cost in the form of higher marginal effort cost from the governance, $\frac{\partial^2 h(a_t;g)}{\partial g^2} > 0$, and receiving the benefit as smaller amount of agency rent, $\zeta'(g) < 0$, over the longevity of the firm.

B. Implication on Time Inconsistency in Corporate Governance

The time-inconsistency in corporate governance, or topsy-curvy incentive, is a situation in which the governance level is strengthened at the initial stage to attract the capital and relaxed later when the firm finished capital raising. This situation is well-concerned and studied as a problem of commitment to the predetermined governance level, as in Tirole (2006), or a problem of coordination of firms, see Acharya and Volpin (2010). However, our analysis provides a rationale for the situation. In other words, our dynamic framework of governance shows that a decrease in governance level in the later stage, when the firm is mature and expect low growth prospect, is rational for both investor and manager, both parties are better off and agree to lower the governance level in the later of the firm.

A decrease in governance level in the firm's later stage is rational because at this stage the cost to induce agent's effort for the growth is very costly due to previous successes. The investor can save this cost by lowering governance level in which the marginal effort cost will be lessened, according to $\frac{\partial^2 h(a_t;g)}{\partial g^2} > 0$. From the proposition 7 and equation (31), a reduction in governance level in the later stage of the firm increases investor's profit. The manager will not be worse off and would agree on the governance change because the level of continuation value will not increase. Notice that the manager could refuse the governance decrease if it lowers his compensation because a change in governance requires an agreement between both parties, not a unilateral decision. In effect, a lower governance level merely reallocates the weight of manager's incentive, putting more weight on instantaneous incentive and less on intertermporal incentive. This reallocation also benefits the investor because he can save the high compensation for growth inducement while tolerates a smaller amount for agency rent[28]. In sum, lowering governance level in the later stage of the firm increases investor's profit and does not reduce manager's compensation and hence it is rational and being Parato improvement.

Considering the entire firm dynamics from the initial to the later stage of the firm, moving the scaled continuation value w_t along the horizontal axis from small to large value in figure 2, our dynamic perspective of governance mechanism provides a rationale for the time inconsistency problem. From the proposition 7 and its implication from equation (31), an increase in governance level at the initial stage, or start-up stage, of the firm and a decrease in later stage, or mature stage, are driven by self-interest motive of contractual parties and being beneficial to the firm, both investor and manager.

The intuition is simple. It is rational and optimal to improve the governance level at the beginning, when executive compensation is relatively small. This improvement rises investor's profit and accelerate the incoming future profit. After, when executive compensation is very high, it is also rational to renegotiate for lower governance level; investor can save cost of motivation and hence retain more profit when the firm has low growth prospect in the mature stage. On the firm's governance mechanism, an enhancement at the beginning and relaxation at the later stage is a rational decision of both contractual parties.

C. Explaining Empirical Puzzle of Corporate Governance on Equity Price

The dynamic analysis of governance mechanism can explain the empirical puzzle of corporate governance on equity price and hence provides a possible remedy on empirical methodology. The recent empirical studies of corporate governance do not provide consistent evidences about the contribution of governance mechanism on equity price and the rate of return. Some of them confirm a positive contribution of internal governance on equity price supporting the

[28]Simply said, for a mature firm, it is better to allow perks than to pay for lucks.

main thesis of free cash flow theory, a better governance mechanism increases equity price, see Gompers, Ishii and Metrick (2003), Bebchuk, Cohen and Ferrell (2008) and Lewellen and Metrick (2010). However, many studies with the same governance database reconsider and come up with contradictory conclusion, no significant positive effect of governance on equity price, see Cremers and Nair (2005), Core, Guay and Rusticus (2006), Johnson, Moorman and Sorescu (2009) and Price, Roman and Rountree (2011). These contradictory results raise the empirical puzzle of corporate governance on equity price.

From previous results on security price characterization and governance change, proposition 6 and 7 respectively, an enhancement of governance level increases the governance premium, from equation (27), but its effect on investor's operation profit, which is a part of security price, depends on the firm's stage when the enhancement occurs, equation (31). We can explain the empirical puzzle by considering the effect of governance enhancement on security price through the governance premium and the profit elements, holding other things equal. Supposedly, we consider the case of governance enhancement from pure legal requirement ($g = 0$) to high level of internal governance ($g \gg 0$). Consider two firm groups which are different only in their operating stage, one is in its initial stage and another in mature stage. The enhancement would increase the governance premium element of both groups, but rise the profit only for firms in their initial stage while lower the profit for the others in mature stage. If a study does not control the effect of firm's stage or not separate the sample group according to the stage, the consequence of governance enhancement on security price could disappear or become insignificant due to the mix of samples and averaging-out consequence on sampled prices. In sum, the empirical puzzle can be a result of neglected procedure in methodology in order to capture the distinctive contribution of corporate governance on equity price.

We recommend a possible remedy on the empirical methodology here. To clearly capture the effect of governance mechanism on equity price, a study should separate the samples according to their operating stage or to control the effect from firm's stage. The results of our model suggest that governance enhancement has a positive contribution to security price of the firms in their initial stage, e.g. start-up firms or the growth stocks. Conversely, the enhancement has a negative effect on equity price of firms in their mature stage; e.g. value-stock firms. With the remedy, our model suggests future empirical studies on the effect of corporate governance on equity price which could possibly clarify the empirical puzzle.

D. Implication on Governmental Intervention and Exit Rights

To consider the governmental intervention on the governance level of a firm and investor's exit right, we must consider the private motivation to improve the internal governance whether the firm's relevant parties have an incentive to improve the governance level beyond the law. From previous results, the investor would benefit from enhancement in internal governance only when the incentive alignment effect persists, while manager would be indifferent to governance change as long as the optimal dynamic contract is implemented. Hence, if both parties have private interest to adjust the governance level, the government or regulator shall not intervene on firm's governance because they would adjust to the op-

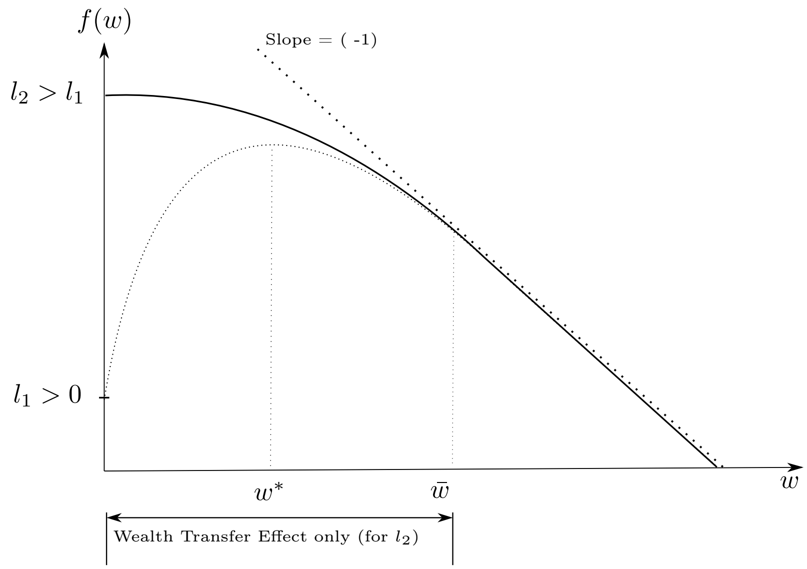

Figure 4: The shape of investor's profit functions with high and low liquidation rate ($l_2 > l_1$)

timal level of internal governance of the firm, consistent with empirical study by Larcker, Ormazabal and Taylor (2011). However, in some firms or industries, the legal governance level is inadequate for investor protection and there is no mutually agreeable possibility for an improvement in firm's governance. It hence calls for government intervention on firm's governance. We now discuss the nature of business or industries that have this problem.

Under optimal dynamic contract, a firm with high liquidation rate will have small incentive alignment effect and this effect disappears when the liquidation rate is adequately large. The figure 4 shows the profit functions of two different firms with low (l_1) and high (l_2) liquidation rates, consistent with DeMarzo and Sannikov (2006), Sannikov (2008) and DFHW (2010). The firm with high liquidation does not have the incentive alignment effect. The principal, or investors of this firm, will not have benefit from an increase in internal governance. The higher internal governance would deteriorate principal's profit along the firm dynamics, as shown in the figure 5. This is the business that would call for governmental intervention in order to strengthen investor protection of the firm. The high liquidation rate of the firm provides a rationale of industry regulations on firm's governance mechanism by the government.

The high liquidation rate implies that the market value of capital is high[29] and investor have better opportunity to exit from the contractual relationship. For example, the value of money in the financial market is as same as the value of money capital for the money management business, such as mutual fund. The money management industry would have a high liquidation rate and call for additional governmental intervention if the existing governance mechanism is inadequate to protect investors. On the contrary, high technological

[29]The high liquidation rate also implies the low contribution of the agent in the production and profit.

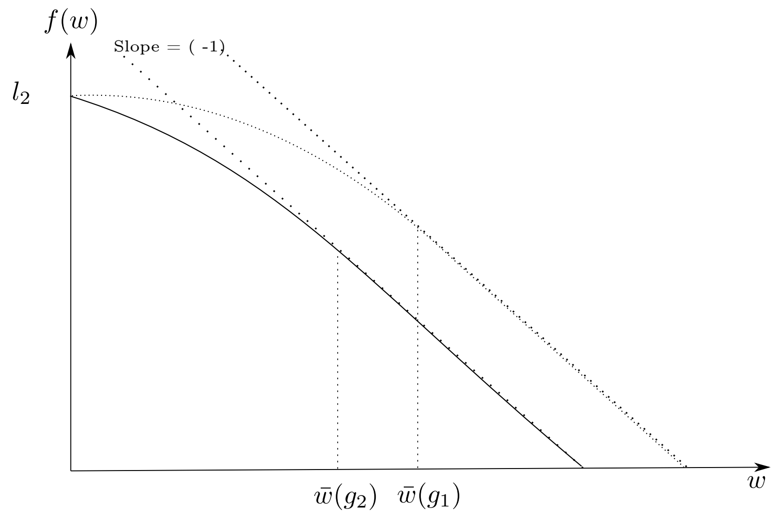

Figure 5: The shape of investor's profit function without incentive alignment effect and higher governance $g_2 > g_1$

firms have low liquidation rate. The market value of capital used by hi-tech firm is much lower than its value under firm's operation. The contractual parties of this firm have considerable possibility for mutual agreement to improve the internal governance mechanism of the firm. The examples indicate two issues on the importance of liquidation rate on internal governance. Firstly, a high liquidation rate indicates high potential to exit from the contractual relationship when investor protection is low and a mutual agreement to enhance the internal governance level is limited. This issue underlines the substitutability of exit right and governance mechanism of the firm. Secondly, as long as the optimal dynamic contract is implemented, distinct industries would require different degree of governmental intervention on firm's governance mechanism.

VI Conclusions

This paper provides the dynamic analysis of corporate governance on investor, manager and the evolution of the firm, using continuous-time dynamic agency model. It gives a more complete analysis which bases on the static framework, *free cash flow theory* and subsume the main thesis of the static framework into its analysis. We develop the dynamic framework in three steps; derivation of optimal dynamic contract of governance, characterization of security price under the optimal contract implementation and consideration of the governance change and its implications. From the results provided in all steps, we give a clarification on time-inconsistency situation in corporate finance, explain the empirical puzzle of corporate governance on equity price and suggest the methodological remedy. We also provides the insight on the legitimacy of government intervention on firm's governance and substitutability of exit right and internal governance. The dynamic framework essentially gives systematic explanation of important phenomena and important analysis for the understanding corporate governance of a firm.

Under the optimal dynamic contract, manager does not divert capital, but enjoy the same amount as a part of compensation. The effort provision is determined by the equality of the proportional profit sharing and marginal effort cost. The optimal investment follows the marginal-q. The contract terminates when the manager expects future benefit to be equal to his outside option, which is normalized to zero, and when the investor cannot gain benefit from the manager's contribution because his compensation is unprofitably high. The analysis of governance mechanism in dynamic incentive framework indicates that there is no free cashflow problem under optimal contract. The optimal dynamic contract incorporates the private benefit of control into the his compensation, according to instantaneous incentive constraint and revelation principle. However, the benefit of control portion in the compensation is limited by the internal governance, which is negotiable between the investor and the manager. If both parties agree to lessen the benefit of control by raising the internal governance level, the manager will be compensated by higher pay from future growth of the firm. Consequently, the optimal contract motivates the manager to invest all the available capital for future incoming cashflow of the firm in order to gain higher future compensation.

An implementation of optimal contract illustrates the dynamic value of internal governance as a part of the security price. We show that the security price composes of four parts; investor's profit,manager's normalized compensation, country's discount term and firm's governance premium. The country's discount term captures the inefficiency of the laws on investor protection of a country. The firm's governance premium reflects the value of internal governance of a specific firm. There are three important features in our security price characterization. Firstly, the value of governance premium in the security price is not subsumed into the operational profit and executive compensation. The security price still reflects the cost of imperfect investor protection and the benefit of firm's governance mechanism as distinct element; country's discount term and firm's governance premium. The distinctive element of governance premium is consistent with the control premium view of the security price. Secondly, the security price incorporates the effects of governance mechanism at the country and firm levels in a single framework. It gives the cross-sectional perspective on how law and internal governance, and their interrelation, affects the security price. Thirdly, and most importantly, the dynamic property the corporate governance mechanism, for both country's discount term and firm's governance premium. Their value are different overtime, even though the law and internal governance mechanism are constant. Intuitively, the dynamic valuation of the corporate governance elements in security price depends on the longevity of the firm, which is the distance between the current time and the expected termination period. This result has important consequence on dynamic valuation of corporate governance in the security price and the firm.

We then consider the effect of governance change on optimal dynamic contract and firm dynamics. An enhancement of internal governance intensifies the incentive alignment of the manager's compensation to the firm's profit. It reshapes the structure of manager's incentive by lessening instantaneous benefit of control while enhancing continuation value due to higher effort cost. As a result, it shifts the weight of manager's compensation from instantaneous benefit of control to future compensation derived from firm's performance. However, the more intensified incentive alignment is not always beneficial to the investor. Its benefit depends on the firm dynamics; the firm's stage that governance enhancement occurs. It makes investor better off only when the firm is small and in its initial stage, e.g. start-up firm or firm with growth prospect. The governance enhancement generates higher profit and it reaches its maximum sooner, at the lower level of manager's continuation value. However, governance improvement in the mature stage can reduce the profit when the firm has low growth prospect and manager's compensation is relatively high due to his track of successes. To recapitulate, governance enhancement increases investor's profit at the initial stage, but decreases it in the later stage of the firm.

The previous results give more insight and better understanding on recent important phenomena. It can explain the time-inconsistency situation in corporate finance. To the firm's governance mechanism, an enhancement at the beginning and relaxation at the later stage, called time-inconsistency in corporate finance, is a rational decision of both contractual parties. The insight from security price characterization and the governance change altogether can explain the recent empirical puzzle and suggestion the potential remedy. The empirical puzzle of corporate governance on equity price is caused by methodological flaw in methodology. We recommend a consideration of the role of firm's stage into the empirical analysis. With the recommended methodological remedy, we can possibly clarify the puzzle in empirical studies of corporate governance and equity price.

The analysis corporate governance in dynamic framework extends the static framework in many strands. It also provides better understandings of the effect of corporate governance to the firm and useful explanation of recent important phenomena of corporate governance. It also opens room for empirical studies, including an empirical identification of the relative importance of governance mechanism and a methodological improvement for the study of corporate governance on security price. They are important agenda for future research on corporate governance.

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Appendix

A Proofs of Propositions

Proof of Proposition 1. The logic of the proposition is to require the incentive compatible compensation (U_t^g) to satisfy instantaneous incentive constrain first, and then design it to motivate optimal effort provision via the continuation value dynamics. Technically, the idea is to *embed* the static incentive into the dynamic incentive and the optimal contract would characterize the optimal compensation process U_t^g that induces no stealing and optimal effort provision.

The mechanism of the proof is to initially require a specification on instantaneous incentive constraint (U_t^g) for the absence of corporate stealing, and then use such incentive compatible compensation to create the continuation value dynamics (dW_t). Firstly, to formally characterize the instantaneous incentive compatible compensation, I apply the static revelation principle and the implementation of the contract as used in Guesnerie and Laffont (1984) and Laffont and Martimort (2002), which is the standard method for static incentive problem. When I embed the instantaneous incentive compatible compensation into the dynamic incentive problem, I use the same method of dynamic revelation principle as in Sannikov (2008) to characterize the continuation value that induces the optimal intertemporal incentive.

Note that this method of considering both types of incentives at the same time is different to Sannikov (2008). In Sannikov's paper, the model does not directly address the instantaneous incentive issue, because there is no characterization on optimal static incentive compatible decision, but infer to the relative importance of dynamic and static incentive via the outside option[30]. In this paper, we directly address both types of incentives because we have two key decisions with distinct incentive structure for the governance problem. The result of the method in this paper is the ability to illustrate a shift or a reallocation in the weight between instantaneous and intertemporal incentive through the renegotiation on the governance. We will use this insight later when we consider the renegotiation on internal governance level.

Starting with the instantaneous incentive within any period, we follow the logical step of Guesnerie and Laffont (1984) due to the similar structure of preferences and allocation mechanism. However, the proof is for the binary choice of agent's decision on capital diversion. Initially, we define the compensation scheme $\tilde{U}_t : \{0,1\} \to \tilde{U}(b_t)$. The compensation scheme associates a net compensation at any period t with a choice of stealing decision $b_t \in \{0,1\}$.

Given the defined compensation scheme, the agent with hidden decision on capital diversion (b_t), for any given level of effort provision (a_t), solves the following program.

[30] This is because the main objective of the paper is to provide the characterization of optimal dynamic incentive decision.

$$\sup_{\{U_t, b_t\}} \{U_t - H(a_t; g) + b_t \zeta(g); \quad \text{for} \quad U_t \leqslant \tilde{U}_t\} \tag{A.1}$$

From our definition of implementability and truthful revelation mechanism as shown in equation (9), the principal can implement an absence of stealing decision and also match the compensation scheme associated with each decision profile; $\{(\tilde{U}_t^g, b_t = 0); (\tilde{U}_t, b_t = 1)\}$ due to the monotonicity of principal's preference on compensation level. Because the compensation is costly to the principal, he would choose the lowest possible compensation level to implement agent's no stealing decision. We then denote such implementing compensation level by $U_t^g = \tilde{U}_t^g$ in which the agent's solve the program in equation (A.1) and both instantaneous constraints, as summarized in equation (10), are binding with equality. As a consequence, the incentive compatible compensation process at any given effort level is equal to the summation of the effort cost and agency rent for any period as shown in proposition 1,

$$U_t^g = H(a_t; g) + \zeta(g) K_t; \quad \forall t \in [0, \tau].$$

By static revelation principle, the agent will be indifferent between stealing ($b_t = 1$) and no stealing ($b_t = 0$) decisions at any time because his net benefit does not differ between the two. Intuitively, the revelation principle unveils the hidden action of stealing decision and makes the extent of capital diversion ($\zeta(g)$) to be a part of compensation process (U_t^g). As a consequence, both contractual parties can negotiation the amount of private benefit of control, hence compensation, through the corporate governance mechanism of the firm, which is an agreement between the two.

For agent's dynamic incentive, in order to recover the dynamic continuation value, we write the value of total expected benefit given information at time t,

$$V_t = \mathbb{E}^a \left[\int_0^t e^{-\gamma s} (U_s^g - H(a_s; g)) ds + e^{-\gamma t} W_t \right]$$

Differentiation with respect to time gives us,

$$dV_t = e^{-\gamma t} (U_t^g - H(a_t; g)) dt + d(e^{-\gamma t} W_t)$$

or, equivalently

$$dV_t = e^{-\gamma t} \left[(U_t^g - H(a_t; g)) dt + dW_t - \gamma W_t dt \right] \tag{A.2}$$

where $d(e^{-\gamma t} W_t) = e^{-\gamma t} dW_t - \gamma W_t e^{-\gamma t} dt$.

Similarly, we can write the value of total expected benefit in another form. By martingale representation theorem, similar to Sannikov (2008), there exists λ_t that makes the total expected benefit a martingale.

$$V_t = V_0 + \int_0^t e^{-\gamma s} \lambda_s \left(dY_s - K_s (a_s - L(i_s)) ds \right)$$

in which $\sigma K_t dZ_t = dY_t - K_t(a_t - L(i_t))dt$ [31]

By taking differentiation, we write another form of the dynamics of agent's total value.

$$dV_t = e^{-\gamma t}\lambda_t \sigma K_t dZ_t \tag{A.3}$$

From the equality of (A.2) and (A.3),

$$e^{-\gamma t}\left[(U_t^g - H(a_t; g))dt + dW_t - \gamma W_t dt\right] = e^{-\gamma t}\lambda_t \sigma K_t dZ_t,$$

we write the dynamics of continuation value as follow,

$$dW_t = \gamma W_t dt - (U_t^g - H(a_t; g))dt + \lambda_t \sigma K_t dZ_t. \tag{A.4}$$

This is the differential form of continuation value dynamics. We can show the integral form this stochastic differential equation by integrating the differential form with respect to time (t) given its initial value (W_0). The result is summarized in the proposition 1. □

Proof of Proposition 2. Given the dynamic continuation value, we need to ensure that the decisions are incentive compatible in the sense that $\{b_t = 0; \forall t \in [0, \tau]\}$ and a_t maximizes the profit. We consider the deviation from the optimal decision during the time 0 and t. Suppose that the agent deviates from optimal decisions and chooses $\{a'_t \neq a_t, b_t = 1\}$ from time $0 \to t$ and chooses the optimal decisions $\{a_t, b_t = 0\}$ from time $t \to \tau$. The total benefit of the agent is following.

$$V_t = \int_0^t e^{-\gamma s}(U_s^g - H(a'_s; g) + \zeta(g)K_s)ds + e^{-\gamma t}W_t$$

We consider the value in the deviation period. We show that the deviation from the optimal decisions would never be positive. The agent does not have incentive to deviate from optimal decisions under the dynamic continuation value described in proposition 1. Taking derivative on V_t gives us,

$$dV_t = e^{-\gamma t}(U_t^g - H(a'_t; g) + \zeta(g)K_t)dt - e^{-\gamma t}(U_t^g - H(a_t; g))dt + e^{-\gamma t}\lambda_t \sigma K_t dZ_t$$

where $\left(-e^{-\gamma t}(U_t^g - H(a_t; g))dt + e^{-\gamma t}\lambda_t \sigma K_t dZ_t\right) = d(e^{-\gamma t}W_t)$, using result from proposition 1.

From $\sigma K_t Z_t(a) = \sigma K_t Z_t(a'_t) + \int_0^t ((a'_s - a_s)dt + b_s \zeta(g)K_s dt)$, we write the diffusion term as $\sigma K_t dZ_t(a) = \sigma K_t dZ_t(a') + (a'_t - a_t)dt + b_t \zeta(g)K_t dt$. We reformulate the dV_t as follow

$$\begin{aligned} dV_t = \quad & e^{-\gamma t}\left[(U_t^g - H(a'_t; g) + \zeta(g)K_t)dt - (U_t^g - H(a_t; g))dt\right] \\ & + e^{-\gamma t}\left[\lambda_t \sigma K_t dZ_t(a') + \lambda_t(a'_t - a_t)dt + \lambda_t b_t \zeta(g)K_t dt\right] \end{aligned}$$

[31] We have this relationship from the cashflow and the productivity processes. From $dY_t = K_t(dA_t - L(i_t)dt)$ and $dA_t = a_t dt + \sigma dZ_t$, we write $dY_t = K_t(a_t - L(i_t))dt + \sigma K_t dZ_t$.

and canceling terms gives

$$\begin{aligned} dV_t = \quad & e^{-\gamma t}\left[H(a_t;g) - H(a'_t;g) + \lambda_t(a'_t - a_t) + (1+\lambda_t)b_t\zeta(g)K_t\right]dt \\ & + e^{-\gamma t}\lambda_t\sigma K_t dZ_t(a'). \end{aligned} \tag{A.5}$$

To consider the incentive compatible decisions $\{a_t, b_t\}$, we consider the case that expectation of the deviation would yield non-positive return, $\mathbb{E}(dV_t) \leqslant 0$. We then consider the drift term of equation (A.5).

For the effort choice (a_t), we require that

$$\big((\lambda_t a'_t - H(a'_t;g)) - (\lambda_t a_t - H(a_t;g))\big) \leqslant 0$$

or, equivalently,

$$(\lambda_t a_t - H(a_t;g)) \geqslant (\lambda_t a'_t - H(a'_t;g)), \quad \forall a'_t \neq a_t. \tag{A.6}$$

For the decision on capital diversion (b_t), we require that

$$b_t(1+\lambda_t)\zeta(g)K_t \leqslant 0. \tag{A.7}$$

The requirement holds only when $b_t = 0, \forall t \in [0,\tau]$.

Note that from the equation (A.6), λ_t is not unique and can take different processes. Because the λ_t determines the agent's incentive through the volatility term of the continuation value dynamics, it is costly to the principal for high value of λ_t. In addition to the incentive compatible decisions, we also require that

$$\lambda_t = min\{\tilde{\lambda_t} \in [0,\infty) : a_t \in argmax_{\{\tilde{a}\in[0,\infty)\}}\{\tilde{\lambda_t}\tilde{a}_t - H(\tilde{a}_t;g)\}\} \tag{A.8}$$

□

Proof of Proposition 3. Defining the scaled continuation value denoted by w_t, the scaled profit function satisfies the following condition[32]

$$f(w_t) = F(1, w_t) = \frac{1}{K_t}F(K_t, W_t).$$

Similarly, we scale down the dynamics of continuation value by the application of the Ito's lemma to dW_t, from proposition 1, and capital dynamics (dK_t). The derivation of the dynamics of scaled continuation value is following.

$$\begin{aligned} dw_t &= d(\frac{W_t}{K_t}) = d(W_t \cdot K_t^{-1}) \\ &= W_t dK_t^{-1} + K_t^{-1}dW_t + dW_t \cdot dK_t^{-1} \end{aligned}$$

[32]From homogeneity degree one in profit function, $F(\beta K_t, \beta W_t) = \beta F(K_t, W_t)$ in which $\beta = \frac{1}{K_t}$

Under incentive compatible decisions, the capital dynamics is $dK_t = (I_t - \delta K_t)dt$, then we have $dK_t^{-1} = -K_t^{-1}(i_t - \delta)dt$ where $i_t = \frac{I_t}{K_t}$. Let $\frac{U_t^g}{K_t} = u_t^g$ and $\frac{H(a_t;g)}{K_t} = h(a_t;g)$. We write the dynamic scaled continuation value as follow.

$$\begin{aligned} dw_t &= -\frac{W_t}{K_t}(i_t - \delta)dt + \frac{1}{K_t}\left(\gamma W_t - (U_t^g - H(a_t;g))\right)dt + \frac{1}{K_t}\lambda_t \sigma K_t dZ_t \\ &= (\delta - i_t)w_t dt + (\gamma w_t - (u_t^g - h(a_t;g)))dt + \lambda_t \sigma dZ_t \\ &= \left((\gamma + \delta - i_t)w_t - (u_t^g - h(a_t;g))\right)dt + \lambda_t \sigma dZ_t \end{aligned} \tag{A.9}$$

We now consider the optimal decisions of the principal using HJB equation of the scaled profit function. We use Ito lemma to transform profit function into the scaled profit function; $F(K_t, W_t) = K_t f(w_t)$.

$$dF(K_t, W_t) = d(K_t f(w_t)) = K_t d(f(w_t) + f(w_t)dK_t$$

where $dK_t = (I_t - \delta K_t)dt$ in which $b_t = 0$. From $df(w_t) = f'(w_t)dw_t + \frac{1}{2}f''(w_t)(dw_t)^2$, we have

$$\begin{aligned} df(w_t) &= \left((\gamma + \delta - i_t)w_t - (u_t^g - h(a_t;g))\right)f'(w_t)dt + f'(w_t)\lambda_t \sigma dZ_t \\ &\quad + \frac{1}{2}f''(w_t)\lambda^2\sigma^2 dt \\ &= \left((\gamma + \delta - i_t)w_t - (u_t^g - h(a_t;g))f'(w_t) + \frac{1}{2}f''(w_t)\lambda_t^2\sigma^2\right)dt + f'(w_t)\lambda_t \sigma dZ_t \end{aligned}$$

We formulate the HJB equation of the profit function, $F(K_t, W_t)$. The instantaneous return of the principal is $K_t(a_t - L(i_t) - u_t^g)dt$ [33], where $u_t^g = \frac{U_t^g}{K_t}$,

$$\begin{aligned} rF(K_t, W_t) &= \sup_{\{i_t, u_t^g\}} K_t(a_t - L(i_t) - u_t^g) + \mathbb{E}[K_t df(w_t) + f(w_t)dK_t] \\ &= \sup_{\{i_t, u_t^g\}} K_t(a_t - L(i_t) - u_t^g) + K_t\mathbb{E}[df(w_t) + f(w_t)(i_t - \delta)dt] \end{aligned}$$

Dividing through by K_t and substitute $f(w_t)$ from above, we have HJB equation in the form of scaled profit function as follow.

$$\begin{aligned} rf(w_t) = \sup_{\{i_t, u_t^g; t \in [0,\tau]\}} &\{a_t - u_t^g - L(i_t) + \left((\gamma - (i_t - \delta))w_t - (u_t^g - h(a_t;g))\right)f'(w_t) \\ &+ \frac{1}{2}f''(w_t)\lambda_t^2\sigma^2 + f(w_t)(i_t - \delta)\} \end{aligned} \tag{A.10}$$

[33] This term is from the drift of $dY_t - U_t^g dt = K(a_t - L(i_t) - u_t^g)dt + K_t \sigma dZ_t$

We now investigate the optimal decisions of the principal. From the HJB equation (A.10), the necessary condition for optimal investment reads,

$$f(w_t) - w_t f'(w_t) = L'(i_t). \tag{A.11}$$

This is the Euler equation for investment per capital. The necessary condition coincides with the classical marginal-q theory saying that the optimal investment equalizes the marginal-q and the marginal cost of capital adjustment. We define the marginal-q as the derivative of total value of the firm with respect to capital, $q_t = \frac{\partial(F(K_t,W_t)+W_t)}{\partial K_t}$ where $F(K_t, W_t) = K_t f(w_t)$. Then $\frac{\partial F(K_t,W_t)}{\partial K_t} = q_t = -w_t f'(w_t) + f(w_t)$. Hence $q_t = f(w_t) - w_t f'(w_t)$. Substitute the marginal cost of capital adjustment, $L'(i_t) = 1 + \theta i_t$, the necessary condition requires that

$$f(w_t) - w_t f'(w_t) = 1 + \theta i_t,$$

or, equivalently, in term of optimal investment

$$i_t^* = \frac{q_t - 1}{\theta} = \left(\frac{f(w_t) - w_t f'(w_t) - 1}{\theta}\right). \tag{A.12}$$

The optimal investment at any period is determined by usual necessary condition, the equality of marginal benefit and cost. The agency conflict deteriorate the level of scaled profit $f(w_t)$ through $u_t^g > 0$ and $u_t^g - h(a_t, g) > 0$ as shown in equation (A.10) and hence the marginal-q, the marginal benefit of investment. The effect of agency conflict slow down the investment dynamics. With corporate governance to limit the extent of agency conflict, it consequently increases the optimal level of investment by increasing profit at any instance.

For the optimal compensation process, we know that the first-order condition from the HJB equation (16) gives us the corner solution, $f'(w_t) = -1$. There is no optimal dynamic link of the compensation process. This result is intuitive. We know that the compensation process is costly to the principal and would be decreased as low as possible at any time. It is hence determined by the instantaneous constraints, rather than the dynamic optimality condition. In other words, the optimal compensation process derives from the instantaneous participation and incentive compatibility constraint at all periods[34].The optimal condition for the compensation process consequently is derived from the optimal effort, individual rationality and incentive compatibility constraints. From equation (5), we see that compensation is costly to the the principal and hence the optimal compensation process is the result from the equalities of the instantaneous constraints given the optimal effort level. The optimal compensation is derived from the equalities of equation (9) and (8).

We substitute the optimal investment into HJB equation (A.10). With a rearrangement, we have

[34]This result is consistent with the previous works of DFHW (2010), which does not consider the compensation process explicitly. However their result states that the principal does not pay any compensation to the agent until the state variable reach the upper boundary, which is also the boundary condition as shown in this paper.

$$
\begin{aligned}
(r+\delta)f(w_t) =& \{a_t - u_t^g + ((\gamma+\delta))w_t - (u_t^g - h(a_t;g)))\, f'(w_t) + \frac{1}{2}\lambda_t^2\sigma^2 f''(w_t) \\
& - L(i_t^*) - i_t^* w_t f'(w_t) + i_t^* f(w_t)\}
\end{aligned}
$$

With explicit functional form of cost of capital adjustment, we rewrite the terms related to optimal investment. Suppressing asterisk, we write $-L(i_t) - i_t w_t f'(w_t) + i_t f(w_t) = -i_t - \frac{\theta}{2} i_t^2 + i_t(f(w_t) - w_t f(w_t))$, and from $f(w_t) - w_t f(w_t) = q_t = (1+\theta i_t)$, we have

$$
\begin{aligned}
-L(i_t) + i_t q_t &= -i_t - \frac{\theta}{2} i_t^2 + i_t(1+\theta i_t) \\
&= \frac{\theta}{2} i_t^2 \quad = \frac{\theta}{2}\frac{(q_t-1)^2}{\theta^2} \\
&= \frac{(q_t-1)^2}{2\theta}
\end{aligned}
$$

□

Proof of Concavity of Profit Function in Proposition 4. With previous boundary conditions, we investigate its property through the second-order derivative over the interval $w_t \in (0, \bar{w})$. We differentiate the ODE of profit function, described in proposition 3, with respect to the scaled continuation value.

$$
\begin{aligned}
(r+\delta)f'(w_t) =& \frac{1}{2\theta}\frac{d\left(f(w_t) - w_t f'(w_t) - 1\right)^2}{dw_t} + (\gamma+\delta)\frac{d(w_t f'(w_t))}{dw_t} \\
& - (u_t^g - h(a_t;g))\frac{d(f'(w_t))}{dw_t} + \frac{\lambda^2\sigma^2}{2}\frac{(df''(w_t))}{dw_t}
\end{aligned}
$$

Rearranging the terms gives us

$$
\begin{aligned}
(r+\delta)f'(w_t) =& \frac{1}{\theta}(f(w_t) - w_t f'(w_t) - 1)(f'(w_t) - w_t f''(w_t) - f'(w_t)) \\
& + (\gamma+\delta)(w_t f''(w_t) + f'(w_t)) - (u_t^g - h(a_t;g))f''(w_t) + \frac{\lambda^2\sigma^2}{2} f'''(w_t) \\
=& \frac{1}{\theta}(f(w_t) - w_t f'(w_t) - 1)(-w_t f''(w_t)) \\
& + (\gamma+\delta)(w_t f''(w_t) + f'(w_t)) - (u_t^g - h(a_t;g))f''(w_t) + \frac{\lambda^2\sigma^2}{2} f'''(w_t).
\end{aligned}
$$

We then evaluate the ODE at the upper boundary $\bar{w}$ and use boundary conditions, equation (20) and (21). We have the following result.

$$-(r+\delta) = -(\gamma+\delta) + \frac{\lambda^2\sigma^2}{2} f'''(\bar{w})$$
$$(\gamma - r) = \frac{\lambda^2\sigma^2}{2} f'''(\bar{w})$$

From the assumption $\gamma \geqslant r$, we consider the case of strict inequality here $\gamma > r$, without loss of generality. we conclude that $f'''(\bar{w})$ is positive. We consider the behavior of $f''(\bar{w})$ at the upper boundary. We know that $f''(\bar{w})$ is locally increasing around $\bar{w}$ due to $f'''(\bar{w}) > 0$ and the super contact condition guarantee that $f''(\bar{w}) = 0$, then we conclude that $f''(\bar{w}-\epsilon) < 0$ for $\epsilon > 0$. We can extend the value of ϵ over the interval $(0, \bar{w})$. We then claim that the profit function is concave on the range of interest[35].

□

Proof of Security Price and Governance Premium in Proposition 6. The financial slack is proportional to λ_t which is equal to marginal effort cost under optimal contract. So we can see that $\lambda_t = \frac{\partial h(a;g)}{\partial a}$. The financial flexibility depends on the past performance (w_t) and inversely varies to marginal effort cost. The financial flexibility can take many forms such as cash reserve or credit line. Basically it reflects the firm's asset to absorb the short-run fluctuation without termination or change of managerial control.

To implement the optimal contract through design of financial securities, we want to find the combination of financial assets that induce the optimal decisions in the same way as the optimal contract does. Hereafter, we interpret the financial flexibility as cash reserve of the firm that gains the return at risk-free rate (r). We then assign the dynamic of financial flexibility as follow.

$$dM_t = rM_t dt + dY_t - dD_t - dX_t \tag{A.13}$$

where, dX_t is the cashflow reserved for compensation to the agent, dY_t is the dynamics of incoming cashflow,

$$dY_t = K_t \left(a_t - L(i_t)\right) dt + \sigma K_t dZ_t,$$

dD_t is the dividend process dynamics that investor requires as periodic return to his investment,

$$dD_t = K_t \left(a_t - L(i_t)\right) dt - (\gamma - r) M_t dt. \tag{A.14}$$

The first term on RHS is the expected cashflow and the second term is the adjustment term for difference between discounting term and interest rate.

[35]We can also check the strict concavity of the profit function over $(0, \bar{w})$ by considering the non-existence of $\tilde{w} < \bar{w}$ in which $f''(\tilde{w}) = 0$ and $f'''(\tilde{w}) > 0$. However, the key argument depends the one we already use, namely $f''(\bar{w}-\epsilon) < 0$ for $\epsilon > 0$. We then skip the formal prove of strict concavity of profit function. The interested reader is referred to the complete proof in DFHW (2010)

We write the explicit form of financial flexibility dynamics by substitute the composing dynamics as follow.

$$\begin{aligned} dM_t &= rM_t dt + K_t(a_t - L(i_t))dt + \sigma K_t dZ_t - K_t(a_t - L(i_t))dt + (\gamma - r)M_t dt - dX_t \\ &= rM_t dt + (\gamma - r)M_t dt - dX_t + \sigma K_t dZ_t \end{aligned}$$

Notice that $(\gamma - r)M_t dt$ is an adjustment term for the different between agent's discount term and risk-free rate, which is equal to principal's discount term. At this step, we can assume without loss of generality that $\gamma = r$ in order to skip the term in the implementation[36].

Assuming the equality of the agent's discount and risk-free rate, the dynamic financial flexibility reads

$$dM_t = rM_t dt - dX_t + \sigma K_t dZ_t. \tag{A.15}$$

We write the scaled financial flexibility dynamics $m_t = \frac{M_t}{K_t}$ and $x_t = \frac{X_t}{K_t}$,

$$dm_t = (\gamma - (i_t - \delta))\, m_t dt - dx_t + \sigma dZ_t. \tag{A.16}$$

We verify that dM_t will induce optimal decisions in the sense that dM_t lead to dW_t in the optimal contract as derived above.

We define the continuation value of agent as a value function, $W_t = V(W_t, K_t)$. Then, from $M_t = \frac{W_t}{\lambda_t}$, we have $\lambda_t M_t = V(M_t, K_t)$. We verify dM_t by constructing HJB equation and check the decision induced by the such HJB equation based on dM_t. From $dK_t = (I_t - \delta K_t - b_t \zeta(g) K_t)dt$, equation (1), and $dM_t = rM_t dt - dX_t + \sigma K_t dZ_t$, equation (A.15), at any point in time, we must have $\lambda M = V(M, K)$. We have the results from partial derivatives such that $V_M = \lambda, V_{MM} = 0, V_K = 0, V_{KK} = 0, V_{MK} = 0$.

By Ito lemma, we have

$$\begin{aligned} dV(M,K) &= V_M dM_t + V_K dK_t + \frac{1}{2} V_{MM} (dM_t)^2 + \frac{1}{2} V_{KK} (dK_t)^2 + V_{MK}(dM_t \cdot dK_t) \\ &= \lambda\left(\delta M_t dt - dX_t + \sigma K_t dZ_t\right) \end{aligned}$$

We assign that the instantaneous return is equal to λdX_t. We write the HJB induced by M_t with agent's discount term γ as

$$\begin{aligned} \gamma \lambda M_t &= \sup_{\{a_t \in [0,\infty), b_t \in \{0,1\}\}} \{\lambda dX_t + \lambda(\gamma M_t - dX - t)\} \\ &= \sup_{\{a_t \in [0,\infty), b_t \in \{0,1\}\}} \{\lambda \gamma M_t\} \end{aligned}$$

[36] We previously assume that $\gamma > r$ in the derivation of optimal contract. We can maintain the assumption for the analysis of implementation. However, the main result and insight from the implementation do not change when we assume the equality of the two.

We hence conclude that the financial flexibility, dM_t defined in equation (A.15), implements the decisions induced by the optimal contract.

We now recover the equity prices from the implementation induced by the financial flexibility. From the definition of security price as a expectation of discounted dividend and the terminal value for the investor, we write the explicit form of the security price at time t as,

$$S_t = \mathbb{E}\left[\int_t^\tau e^{-r(s-t)} dD_s + e^{-r(\tau-t)} lK_\tau\right]. \tag{A.17}$$

From dM_t and dD_t defined in equation (A.13) and (A.14) respectively, we have security price at time t in the form of

$$\begin{aligned} S_t =& \mathbb{E}\left[\int_t^\tau e^{-r(s-t)}(dY_s - U_s^g ds) + e^{-r(\tau-t)} lK_\tau\right] \\ &+ \mathbb{E}\left[\int_t^\tau e^{-r(s-t)}(rM_s ds - dM_s)\right] \\ &+ \mathbb{E}\left[\int_t^\tau e^{-r(s-t)}(-\zeta(g)K_s)ds\right]. \end{aligned} \tag{A.18}$$

The security price comprises of three parts. Firstly, we know from the objective of the principal that

$$F(W_t, K_t) = \mathbb{E}\left[\int_t^\tau e^{-r(s-t)}(dY_s - U_s^g ds) + e^{-r(\tau-t)} lK_\tau\right].$$

Secondly, we apply the integration by part to the second term of the right hand side of equation (A.18),

$$\begin{aligned} \int_t^\tau e^{-r(s-t)} dM_s =& e^{-r(s-t)} dM_s|_t^\tau + \int_t^\tau re^{-r(s-t)} M_s ds \\ =& \left(e^{-r(\tau-t)} M_\tau - M_t\right) + \int_t^\tau re^{-r(s-t)} M_s ds \\ =& - M_t + \int_t^\tau re^{-r(s-t)} M_s ds, \end{aligned}$$

because $M_\tau = 0$. We then have,

$$\begin{aligned} \mathbb{E}\left[\int_t^\tau e^{-r(s-t)} rM_s ds - dM_s\right] =& \mathbb{E}\left[\int_t^\tau e^{-r(s-t)}(rM_s ds) + M_t - \int_t^\tau e^{-r(s-t)} rM_s ds\right] \\ =& E[M_t] \\ =& M_t \end{aligned}$$

where $M_t = \frac{W_t}{\lambda_t}$ by definition.

Finally, we see that the third term on RHS of equation (A.18) is the discounted agency rent from the time t to the terminal time. From our definition of corporate governance as a mechanism to reduce the agency conflict and cost, we can transform this term to highlight the role of corporate governance. We define the distortion function as follow.

$$\zeta(g) = \Omega - \psi(g) \tag{A.19}$$

where Ω is a constant term reflecting the level of capital distortion when the internal governance is zero. This constant captures the possible distortion from inefficient governance mechanism based on the country's legal environment and investor protection. Then, a country with good legal infrastructure for investor protection, the level of Ω is low. We define $\psi(g)$ as a measure for the investor protection from the internal governance mechanism of the firm. We assume that $\psi'(g) > 0$. Hence, our assumption $\zeta'(g) = -\psi'(g) < 0$ holds with the definition of $\zeta(\cdot)$ function.

From our definition of distortion function $\zeta(g)$, we write the third term on RHS of equation (A.18), as follow,

$$\begin{aligned}
\mathbb{E}\left[\int_t^\tau e^{-r(s-t)}(-\zeta(g)K_s)ds\right] =& \mathbb{E}\left[\int_t^\tau e^{-r(s-t)}(\psi(g) - \Omega)K_s ds\right] \\
=& -\mathbb{E}\left[\int_t^\tau e^{-r(s-t)}\Omega)K_s ds\right] \\
&+ \mathbb{E}\left[\int_t^\tau e^{-r(s-t)}(\psi(g))K_s ds\right]
\end{aligned} \tag{A.20}$$

The first term reflects the agency cost at country level due to the inefficient law and legal enforcement for investor protection. This is the country's discount term. The second term capture the effect of internal governance for each firm. It represents the amount of capital saved from agency rent. We call the second term the *governance premium* of the security price. This term captures the effect of internal governance of the firm that reduces the agency rent for a given country's legal infrastructure. The value of internal governance is equal to the discounted capital being recovered from possible distortion under optimal contract. With two terms put corporate governance at country and firm level into the same perspective.

We now write the security price at time t in the term of investor's expected profit, agent's continuation value, country-level discount and firm's governance premium, as follow,

$$S_t = F(W_t, K_t) + \frac{W_t}{\lambda_t} - \mathbb{E}\left[\int_t^\tau e^{-r(s-t)}\Omega K_s ds\right] + \mathbb{E}\left[\int_t^\tau e^{-r(s-t)}(\psi(g))K_s ds\right]. \tag{A.21}$$

Other things equal, the security price in the country with weak legal investor protection, Ω is large, is lower than the country with the stronger one. Within a country, having identical legal infrastructure for investor protection, a firm with higher internal governance level, g and $\zeta(g)$ are large, has higher security price than the firm with lower level, *ceteris paribus.*

Equivalently, we can also express the security price in term of the scaled profit function and financial flexibility as

$$S_t = \left(f(\lambda_t M_t) + m_t - \mathbb{E}\left[\int_t^\tau e^{-r(s-t)}\Omega ds\right] + \mathbb{E}\left[\int_t^\tau e^{-r(s-t)}(\psi(g))ds\right]\right) K_t.$$

□

Proof of Comparative Static Analysis in Proposition 7. We conduct the comparative analysis to highlight the consequences of change in internal governance on the principal's profit function. We follow the methodology used in DeMarzo and Sannikov (2006). We proceed in two steps. The first step is to consider the effect of change in governance[37] on the profit for the whole path of continuation value using Keynman-Fac formula. The second step is to consider the total derivative on boundary value or interested value of agent's profit to consider the change in governance on

For the first step, we denote the parameter of interest, including governance level, as ϕ[38]. We differentiate ODE of the profit function with respect to ϕ holding continuation value (w) constant and then evaluate at the upper boundary, $\bar{w}$. Then from ODE of profit function, differentiation with respect to the parameter and using upper boundary conditions, $f'(\bar{w}) = -1$ and $f''(\bar{w}) = 0$, gives

$$\begin{aligned}
(r+\delta)\frac{\partial f(w)}{\partial \phi} + f(w)\frac{\partial (r+\delta)}{\partial \phi} =& \frac{\partial a}{\partial \phi} - \frac{\partial u^g}{\partial \phi} + \frac{f(w) - wf'(w) - 1}{\theta}\left(\frac{\partial f(w)}{\partial \phi} - w\frac{\partial f'(w)}{\partial \phi}\right) \\
&+ (\gamma+\delta)w\frac{\partial f'(w)}{\partial \phi} + wf'(w)\frac{\partial(\gamma+\delta)}{\partial \phi} \\
&- \zeta(g)\frac{\partial f'(w)}{\partial \phi} - f'(w)\frac{\partial \zeta(g)}{\partial \phi} \\
&+ \frac{\lambda^2\sigma^2}{2}\frac{\partial f''(w)}{\partial \phi} + \frac{f''(w)}{2}\frac{\partial(\lambda^2\sigma^2)}{\partial \phi}.
\end{aligned}$$

From the upper boundary conditions, we have $\frac{\partial f'(w)}{\partial \phi}|_{w=\bar{w}} = 0$ and $\frac{\partial f''(w)}{\partial \phi}|_{w=\bar{w}} = 0$. We now rearrange the terms in order to apply the Feynman-Kac formula as the solution to PDE,

$$\begin{aligned}
(r+\delta)\frac{\partial f(w)}{\partial \phi} =& - f(w)\frac{\partial (r+\delta)}{\partial \phi} + \frac{\partial a}{\partial \phi} - \frac{\partial u^g}{\partial \phi} + \left(\frac{f(w) - wf'(w) - 1}{\theta}\right)\frac{\partial f(w)}{\partial \phi} \\
&+ wf'(w)\frac{\partial(\gamma+\delta)}{\partial \phi} - f'(w)\frac{\partial \zeta(g)}{\partial \phi} + \frac{f''(w)}{2}\frac{\partial(\lambda^2\sigma^2)}{\partial \phi}.
\end{aligned}$$

[37]The method can be applied to the comparative static analysis of other parameters.

[38]We denote ϕ as a representative of general parameters of the model, including discount rate of the agent (γ), variance of the cashflow process (σ^2), liquidation rate (l) and internal governance level (g).

Applying Feynman-Kac formula to express the solution of $\frac{\partial f(w_t)}{\partial \phi}$, with terminal period τ in which investor derives the terminal value at the rate of liquidation (l),

$$
\begin{aligned}
\frac{\partial f(w_t)}{\partial \phi} = \mathbb{E}^{w_0=w}\Bigg[\int_0^{\tau} e^{-(r+\delta)t}\Bigg(&-f(w_t)\frac{\partial(r+\delta)}{\partial \phi} + \frac{\partial a_t}{\partial \phi} - \frac{\partial u_t^g}{\partial \phi} \\
&+ \left(\frac{f(w_t) - w_t f'(w_t) - 1}{\theta}\right)\frac{\partial f(w_t)}{\partial \phi} + wf'(w)\frac{\partial(\gamma+\delta)}{\partial \phi} \\
&- f'(w)\frac{\partial \zeta(g)}{\partial \phi} + \frac{f''(w)}{2}\frac{\partial(\lambda^2\sigma^2)}{\partial \phi}\Bigg)dt \\
&+ e^{-(r+\delta)\tau}\frac{\partial l}{\partial \phi}\Bigg]
\end{aligned}
\tag{A.22}
$$

For $\phi = g$, the partial derivative of constant terms and functions which are not directly affected by g are zero. From the binding instantaneous constraints, we have $u_t^g = h(a_t; g) + \zeta(g)$ and $\frac{\partial u_t^g}{\partial g} = \frac{\partial h(a_t;g)}{\partial g} + \frac{d\zeta(g)}{dg}$. From the previous assumptions, we know that $\frac{\partial h(a_t;g)}{\partial g} > 0$ and $\frac{d\zeta(g)}{dg} < 0$. To focus on the effect of governance change on incentive structure of the manager on the profit function along firm dynamics, we consider the case which neutralizes the consequences of governance change on the level of profit function. In other words, we consider how a governance change influences the relationship between principal and agent at different stages of the firms, while hold the their expected benefit from the change unaltered. Equivalently, we assume the equality of the magnitude of governance change on effort cost and on diversion function; $|\frac{\partial h(a_t;g)}{\partial g}| = |\frac{d\zeta(g)}{dg}|$. In effect, when we change the governance level, the direct consequence on expected cost and benefit on both principal and agent would be equal and canceled out to keep the expected profit constant. In our comparative static analysis of governance, the neutrality on the level of profit function holds.

Moreover, the change in governance under optimal contract would not change the agent's expected value from the contractual relationship (V_t) as well. This is the reason why agent would agree to renegotiate on governance alteration. If V_t decreases, the agent would not agree on governance change and would veto the governance alteration proposed by the principal. Likewise, if V_t increase, the change would cause principal to have lower expected benefit and hence he would not propose or would veto the governance change. So, under optimal contract, the governance change should not alter the V_t. The rationale is following.

Under the optimal contract, the principal would design the contract to make agent to have expected value as low as possible, given that he still participates. When we assume a constant outside option, which is reasonable due to the irrelevance of outside payoff to the contractual relationship, the agent's expected value would be constant overtime, V_t is hence constant. The agent's intertemporal incentive is induced by the multiplier process (λ_t) in the continuation value (W_t) which only relates to the diffusion term with zero expectation. The change in governance affects the agent's value of contract through the drifts of V_t and W_t, yet on the opposite direction.

Technically, when we raise the governance level, we decrease the drift of V_t and increase the drift of W_t and two effects cancel out. Intuitively, when we enhance the governance level, it will lessen the periodic control rent $u^g - h(a; g) = \zeta(g)$, hence the drift of V_t will be lower. This amount of lesser control rent would become additional corporate investment and induce higher expected cashflow into the firm. As a consequence, the expected compensation to the agent due to higher corporate investment and cashflow, which is shown by the drift of continuation value ($u^g - h(a; g) = \zeta(g)$ in W_t), would increase. Overall, when we enhance the governance level under optimal contract, we rearrange the agent's incentive structure away from instantaneous, through the periodic control rent, to intertemporal one, through the higher continuation value from both drift and diffusion terms being realized only when higher performance occurs.

There are two important reasons to consider the case of neutrality on the level effect of profit function. Firstly, the governance level, which induces unchanged profit when the governance changes, indicate the efficient level of the governance itself. Consider the contradiction in the case in which the governance changes causes a change on the expected level of profit function. If the neutrality does not hold, the principal would raise the governance level to gain benefit from lower diversion rate and pay the higher effort cost to the agent in order to convince him for better governance in the optimal contract. With more enhanced governance level, profit function would shift up and become more concave, as shown in figure 6. This process would keep occurring until the governance attains the efficient level and its change induces the equality of the cost and benefit. At the efficient level , the change of internal governance would not affect the average, or the expected level of, benefits of principal and agent under optimal contract, as denoted by equations (7) and (6) respectively. In the analysis of governance change, we consider this situation in which the neutrality holds and the governance mechanism reaches the efficient level.

Secondly, the neutrality guarantees the possibility of a renegotiation on governance change. The neutrality on the profit level requires the equality of net cost and benefit of both parties. No party is worse-off from the governance change. As a consequence, when the neutrality on the profit level holds, no party would veto on the renegotiation on the governance alteration, either increase or decrease. As a consequence, neutrality is required for a renegotiation on governance change between the principal and agent.

Technically, the neutralization implies that a change in distortion from governance change is equal to the change in effort cost of the manager,$|\frac{\partial h(a_t;g)}{\partial g}| = |\frac{d\zeta(g)}{dg}|$. As a consequence, the first step of comparative statics with respect to governance (g) gives the following result.

$$\frac{\partial f(w)}{\partial g} = \mathbb{E}^{w_0=w}\left[\int_0^\tau e^{-(r+\delta)t}(-f'(w_t))\zeta'(g)dt\right] \tag{A.23}$$

The sign of derivative depends on the initial value when both parties agree to change the governance level $w_0 = w$, because the value of derivative is evaluated for the change in profit function for the entire path of w_t. We know that $\zeta'(g) < 0$. The sign of $\frac{\partial f(w)}{\partial g}$ depends on the sign of $f'(w_t)$. Due to the strict concavity, the value of $f'(w) > 0$ for $w \in (0, w^*)$ and $f'(w) < 0$ for $w \in (w^*, \bar{w})$.

To be precise, we consider the details of governance enhancement as following. When the governance mechanism is strengthened from g_1 to g_2 in which $g_2 > g_1$, the scaled profit

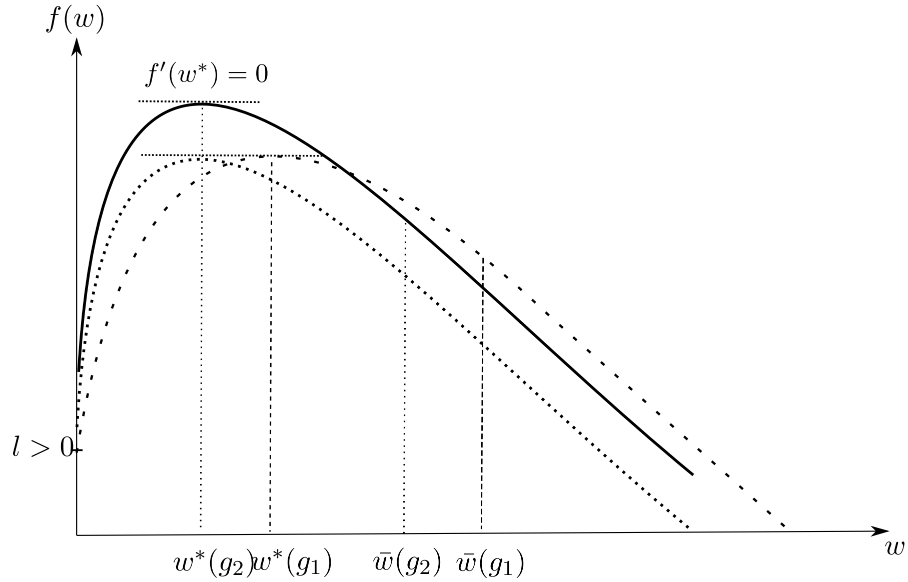

Figure 6: This figure shows the case when the neutrality does not hold. The black line illustrates the shape of investor's profit function with better governance ($g_2 > g_1$) with positive net level effect. The investor attains net positive benefit from the enhancement of internal governance.

function becomes more concave, holding the lower boundary and upper boundary condition unchanged. As a consequence, the turning point of the higher governance level is lower than before, $w^*(g_2) < w^*(g_1)$. Then, there exists a level of scaled continuation value at which the level of scaled profit does not change, denoted by $\hat{w}$. It is between the two turning points of profit curves of the old and new level of governance, $\hat{w} \in [w^*(g_2), w^*(g_1)]$. It hence divide the interval of scaled continuation value into two parts, $(0, \hat{w})$ and $(\hat{w}, \bar{w})$, which reflects the initial and mature stage of the firm accordingly. When we increase governance level in the initial stage, in which $w_t \in (0, \hat{w})$, the investor's profit is expected to be higher on average. On the contrary, when we increase governance level at mature stage, $w_t \in (\hat{w}, \bar{w})$, the investor's profit is expected to be lower on average.

We hence summarize the results of governance enhancement and its consequence on investor's profit as following. There exists $\hat{w} \in [w^*(g_2), w^*(g_1)]$ for $g_2 > g_1$, such that $\frac{\partial f(\hat{w})}{\partial g} = 0$ and the following results hold.

$$\frac{\partial f(w)}{\partial g} \begin{cases} > 0; & \text{for } w \in (0, \hat{w}) \\ < 0; & \text{for } w \in (\hat{w}, \bar{w}) \end{cases} \tag{A.24}$$

The intuition of the equation (A.24) is following. When the governance level is raised, the entire profit under optimal dynamic contract would change the track along the firm dynamics and become more concave. Consider a small neighborhood of scaled continuation value when we raise the governance level, $(w_0 - \epsilon, w_0 + \epsilon)$ for small positive number (ϵ), the value of $f'(w_t)$ locally maintains its sign according to the slope of concave function mentioned above. When we consider along the firm dynamics, because w_t is driven by Brownian Motion, it has an equal chance to be greater and lower than its initial value. When we in crease the governance level at $w_1 \in (0, w^*)$, it is more likely to terminate at the lower boundary than the upper boundary and hence has high propensity to generate higher profit than the lower profit before the terminal period. On average, its the expected value of the equation (A.23) would be greater than zero. On the contrary, when the governance is enhanced at $w_2 \in (w^*, \bar{w})$, it is more likely to terminate at the upper boundary than the lower boundary and hence has high propensity to generate lower profit than the higher profit before the terminal period at the upper boundary. As a consequence, the expected profit change from the governance enhancement in this case is negative.

In the second step of comparative static analysis, we consider the boundary and the relevant conditions. Our interest is the effect of change in governance on the investor's profit and the firm dynamics. We then consider the conditions related to three important point of the agent's continuation value; the initial continuation value[39], the turning point and the upper boundary, $\{w_0, w^*, \bar{w}\}$ respectively. We use total derivative on the related conditions and the previous results to derive the conclusion.

For w_0, we use the condition $f(w_0) = \alpha$ saying that the outside option of the principal, or investor, does not depend on the continuation value of the agent and can be represented by a constant term, α. The total derivative is then $\frac{\partial f(w_0)}{\partial \phi} + f'(w_0)\frac{\partial w_0}{\partial \phi} = 0$. We then have the effect of change in governance on the initial continuation value as,

$$\frac{\partial w_0}{\partial \phi} = -\frac{\left(\frac{\partial f(w_0)}{\partial \phi}\right)}{f'(w_0)} \tag{A.25}$$

where $f'(w_0) = \frac{df(w)}{dw}\Big|_{w=w_0}$.

For w^*, we use the condition $f'(w^*) = 0$, characterizing the point that the slope of profit function changes from positive to negative. By total derivative, we have $\frac{\partial f'(w^*)}{\partial \phi} + f''(w^*)\frac{\partial w^*}{\partial \phi} = 0$, and

$$\frac{\partial w^*}{\partial \phi} = -\frac{\left(\frac{\partial f'(w^*)}{\partial \phi}\right)}{f''(w^*)} \tag{A.26}$$

where $\frac{\partial f'(w^*)}{\partial \phi} = \frac{\partial^2 f(w^*)}{\partial \phi \partial w}\Big|_{w=w^*}$.

For $\bar{w}$, we use the ODE evaluated at the upper boundary value as the condition. Applying the total derivative to ODE of the profit function at upper boundary, $(r+\delta)f(\bar{w}) = a_t - u_t^g + \frac{(f(\bar{w})+\bar{w})^2}{2\theta} - (\gamma+\delta)\bar{w}$, we have

[39]This is the continuation value when agent enters into the contractual relationship with the principal under agreed governance and other parameters.

$$
\begin{aligned}
(r+\delta)\left[\frac{\partial f(\bar{w})}{\partial \phi}+f'(\bar{w})\frac{\partial \bar{w}}{\partial \phi}\right] =& \frac{da(w)}{dw}\frac{\partial w}{\partial \phi}\Big|_{w=\bar{w}}-\frac{du^{g}(w)}{dw}\frac{\partial w}{\partial \phi}\Big|_{w=\bar{w}} \\
&+\frac{1}{\theta}(f(\bar{w})+\bar{w})\frac{d(f(\bar{w})+\bar{w})}{d\phi}-(\gamma+\delta)\frac{\partial \bar{w}}{\partial \phi} \\
(r+\delta)\left[\frac{\partial f(\bar{w})}{\partial \phi}-\frac{\partial \bar{w}}{\partial \phi}\right] =& \frac{1}{\theta}(f(\bar{w})+\bar{w})\left[\frac{\partial f(\bar{w})}{\partial \phi}+f'(\bar{w})\frac{\partial \bar{w}}{\partial \phi}+\frac{\partial \bar{w}}{\partial \phi}\right]-(\gamma+\delta)\frac{\partial \bar{w}}{\partial \phi} \\
(r-\gamma)\frac{\partial \bar{w}}{\partial \phi} =& \left[(r+\delta)-\frac{(f(\bar{w})+\bar{w})}{\theta}\right]\frac{\partial f(\bar{w})}{\partial \phi}.
\end{aligned}
$$

We then have

$$
\frac{\partial \bar{w}}{\partial \phi}=\left[\frac{r+\delta}{r-\gamma}-\frac{(f(\bar{w})+\bar{w})}{\theta(r-\gamma)}\right]\frac{\partial f(\bar{w})}{\partial \phi}.
$$

We cannot determine the sign of the coefficient term of $\frac{\partial f(\bar{w})}{\partial \phi}$ because it depends on the quantitative value of the parameters, profit function and continuation value.

We now consider the specific case for $\phi = g$ for $\{w_0, w^*\}$. We then need another auxiliary result.

$$
\frac{\partial f'(w^*)}{\partial g}=\frac{\partial\left(\frac{\partial f(w)}{\partial g}\right)}{\partial w}\Bigg|_{w=w^*}=\mathbb{E}^{w_0=w}\left[\int_0^{\tau}e^{-(r+\delta)t}(-f''(w_t))\zeta'(g)dt\right]<0 \tag{A.27}
$$

because $f''(w_t)<0$ from concavity and $\zeta'(g)<0$ by the assumption.

The effect of change in governance on the turning point (w^*) is following

$$
\frac{\partial w^*}{\partial g}=\frac{-\left(\frac{\partial f'(w^*)}{\partial g}\right)}{f''(w^*)}<0 \tag{A.28}
$$

due to equation (A.27) and concavity of profit function.

The effect of change in governance on the initial continuation value of the agent (w_0) depends on the sign of $f'(w_0)=f'(w)\big|_{w=w_0}$ and $\frac{\partial f(w_0)}{\partial g}=\frac{\partial f(w)}{\partial g}\big|_{w=w_0}$. From equation (A.24), we consider the possible value of relevant elements. For $w_0\in(0,\hat{w})$, $f'(w_0)>0$ and $\frac{\partial f(w_0)}{\partial g}>0$, we then have $\frac{\partial w_0}{\partial g}<0$. For $w_0\in(\hat{w},\bar{w})$, $f'(w_0)<0$ and $\frac{\partial f(w_0)}{\partial g}<0$, the result is $\frac{\partial w_0}{\partial g}<0$. We conclude that under the optimal contract, the effect of change in governance on initial continuation value is negative,

$$
\frac{\partial w_0}{\partial g}=\frac{-\left(\frac{\partial f(w_0)}{\partial g}\right)}{f'(w_0)}<0 \tag{A.29}
$$

□