Search frictions in good markets and CPI inflation *

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Abstract

This study explores the impact of frictional goods markets on consumer price inflation dynamics using a New Keynesian dynamic stochastic general equilibrium model that incorporates both frictional goods markets and endogenous product entry. We introduce search friction between producers and retailers, utilizing the shift in consumption from brick-and-mortar towards online retailers during the COVID-19 pandemic as a natural experiment, given that online retailers have higher search efficiency than brick-and-mortar retailers. Our findings, derived from the New Keynesian Phillips Curve, suggest that as consumers shift towards online retailers, they gain access to retailers who charge a lower wedge between consumer and producer prices, consequently reducing CPI inflation. The model highlights the importance of shocks to online retail sales share in explaining the dynamics of prices in the United Kingdom during the COVID-19 pandemic, emphasising the role of frictional goods markets in macroeconomic and policy analysis.

JEL codes: **Keywords:**

*Any views expressed are solely those of the authors and so cannot be taken to represent those of the Bank of England and Siam Commercial Bank or members of their committees.

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1 Introduction

Retailers play a crucial role in the economy, occupying the final stage in the journey of goods from production to consumption. Their economic value lies in their ability to search for and match household demand with the variety of goods produced in the economy. As intermediaries, retailers charge households a retail margin on top of the wholesale prices paid to producers, compensating for the costs incurred through the search and matching process. This retail margin introduces a wedge between producer and consumer prices, the magnitude of which is determined by the retailers' search efficiency. Higher search efficiency translates into lower search costs and a narrower gap between consumer and producer prices, highlighting the significant impact of retailers' effectiveness in aligning product offerings with consumer preferences on the prices households ultimately pay. Empirical evidence underscores the crucial role retailers play in determining the final prices paid by consumers. Nakamura (2008) conducted an extensive study using detailed price and quantity data from US grocery stores, revealing that the majority of observed price fluctuations originated at the retail level rather than from manufacturers. This study laid the foundation for subsequent research examining retailers' influence on the wedge between consumer and producer prices. Hottman et al. (2016), for example, utilized comprehensive barcode data to measure retail markups across US stores, finding substantial markup dispersions that varied with store characteristics such as size and product variety. These variations in retail markups significantly contribute to aggregate price dispersion and the gap between the prices producers receive and the prices consumers ultimately pay, emphasizing the critical role retailers play in shaping the final costs borne by households.

One of the most significant trends in retail markets over the past 15 years has been the rapid rise of online retail platforms, such as Alibaba, Amazon Marketplace, and eBay. These platforms have become essential to the e-commerce ecosystem, enabling digital search and matching between buyers and sellers through various interfaces. The increasing prominence of online shopping has led to a notable shift away from traditional physical stores, with the share of online retail sales in the UK surging from less than 10% in 2010 to nearly 30% by 2023, highlighting the transformative impact of e-commerce on the retail landscape. The COVID-19 pandemic further accelerated this trend, as online retail became relatively more desirable compared to in-person shopping, initially catalyzing a considerable shift towards online retail sales. However, upon the reopening of brick-and-mortar stores, this shift partially reverted, providing evidence of possible short-run temporary shifts in consumer preferences towards online retail markets (Figure ??).

Although a universal definition for these platforms is not yet established, Fradkin (2017) suggests that they share the common characteristic of leveraging technology to improve search efficiency in frictional goods markets. By analyzing user behavior and using advanced algorithms, online platforms optimize the search and matching process, potentially surpassing the performance of traditional brick-and-mortar retailers in this aspect. If search friction in retail markets is relevant, the difference in search efficiency suggests that online retailers should have lower retail margins. Empirical evidence supports this argument, as Cavallo (2017) conducted a study comparing brick-and-mortar and online retail prices collected simultaneously from physical stores and online platforms of four large multi-channel retailers in the UK between March and May 2015, finding that, on average, online retail prices were one percent lower than brick-and-mortar retail prices. This finding, combined with the observed short-run temporary shifts in consumer preferences towards online retail markets during the pandemic, implies that these shifts could have implications for short-run fluctuations in CPI inflation. The COVID-19 pandemic serves as a natural experiment, providing an opportunity to explore the influence of short-run fluctuations in consumer preferences towards online retail markets on inflation dynamics and examine the role of retailers in setting final prices paid by consumers and the potential impact of such shocks on the broader economy.

We develop empirical evidence. First, we assess the response of the increase in the share of

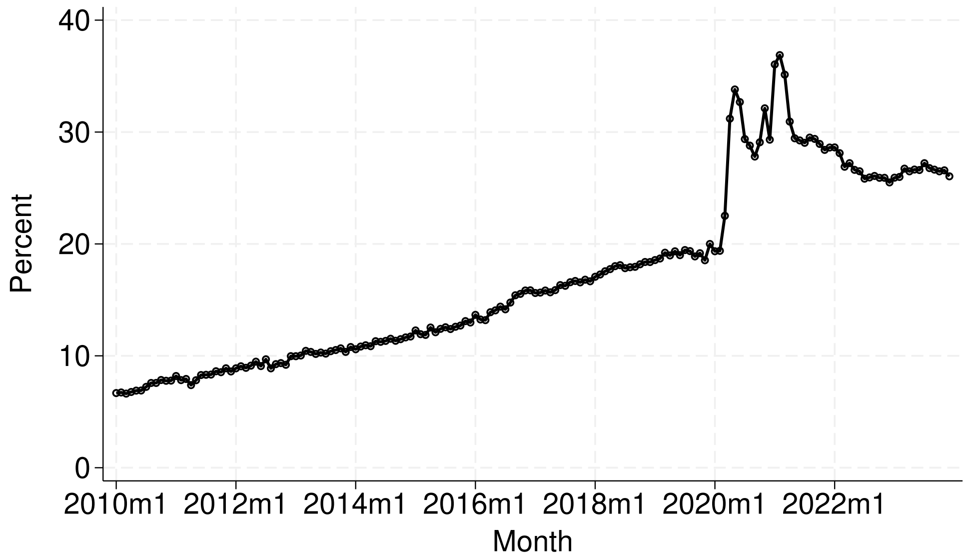


Figure 1: Share of online retail sales to total retail sales in the UK

online retail sales share on a CPI inflation trajectory. To establish the impulse responses, We adopted the local projection method (LP) introduced by (Jordà, 2005), [1] We are interested in characterising the path of the cumulative response of CPI inflation (CPI) on a percent change in the share of the online retail sales to total retail sales ($ONLINE$),

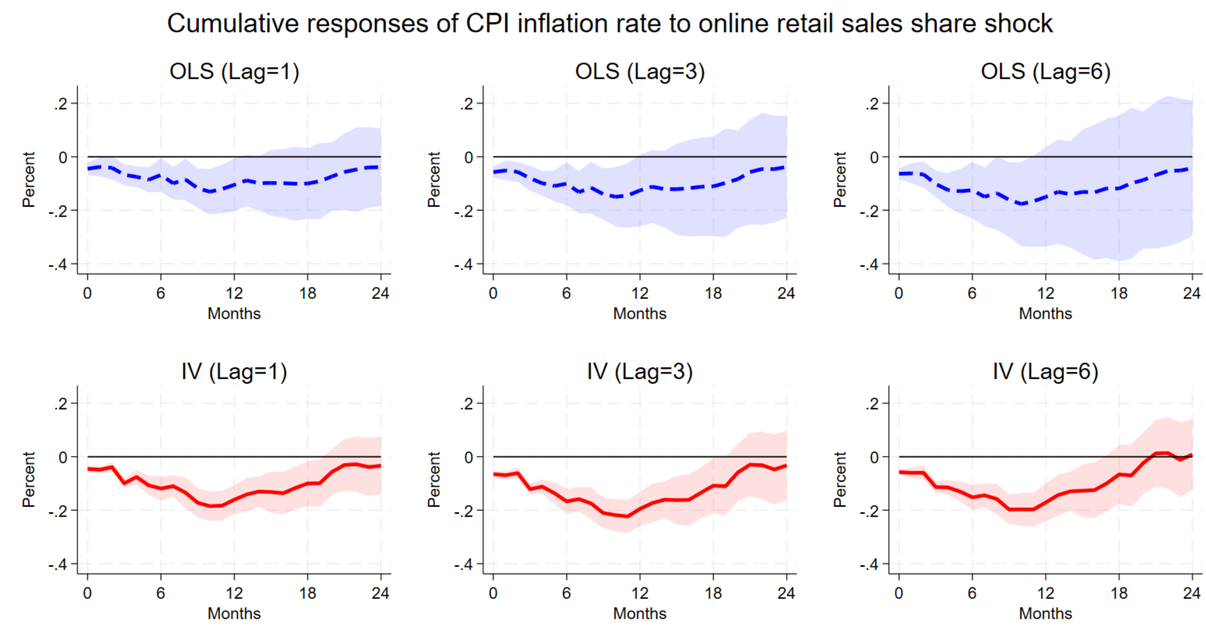
$$CR(\Delta_h CPI_{t+h}, \delta) = E_t\left(\Delta_h CPI_{t+h} | ONLINE_t = \overline{ONLINE} + \delta; X_t, X_{t-1}, ...\right) \quad (1)$$

$$- E_t\left(\Delta_h CPI_{t+h} | ONLINE_t = \overline{ONLINE}; X_t, X_{t-1}, ...\right) \quad (2)$$

where $CR(\Delta_h CPI_{t+h}, \delta)$ denotes the average cumulative response of CPI inflation, given a size δ change in the percentage change in the share of the online retail sales to total retail sales. The path of CPI inflation is conditional on $X_t, X_{t-1}, ...$, which contains the history of CPI inflation, PPI inflation, and a percent change in the share of online retail sales. This path accounts for the persistent effects of the change in the PPI inflation or a percentage change

[1] The LP method directly estimates how a targeted variable responds to shocks over time. This approach uses simple regressions for each time period, making it easier to estimate and understand than VAR models, which require a lot of parameter estimation and matrix manipulation. Also, the LP method does not need a fully specified model of the whole dynamic system, so it's less likely to have errors from incorrect model specifications. This makes the LP method's estimates of impulse responses more reliable when studying complex relationships between variables.

in the share of online retail sales. We employ a dataset comprising monthly observations, spanning a total of 156 data points from the first quarter of 2010 to the fourth quarter of 2022. [2]



Notes: 90 percent confidence bands

Figure 2: Response of CPI inflation to an increase in the share of online retail sales

The top panel of Figure 2 reports results from OLS estimation, controlling for CPI inflation, PPI inflation, and the share of online retail sales in the previous lags (1, 3, 6). The results suggest that the cumulative response of CPI inflation rate to a one percent increase in the share of online retail sales is consistently negative for 12 consecutive months, attaining statistical significance within the 90% confidence interval. Throughout a 12-month period, the cumulative response in the UK peaks after 9 months at around -0.15%. The response is robust to the number of lags that capture the history of CPI inflation, PPI inflation, and the share of online retail sales, as our results remain significant at the 10% level in the first 12 months for all lag specifications.

It is crucial to note that share of online retail sales is influenced by a myriad of factors that

[2] Please refer to Appendix A for the description of the dataset.

could simultaneously affect CPI inflation, presenting identification problems. To identify the shock to consumer preferences towards online retail sales, we conduct an instrumental variable local projection (LP-IV) estimation, using the number of deaths due to the COVID-19 pandemic as an instrument for the percentage change in the share of online retail sales. The number of COVID-19 deaths is considered exogenous, as it influences preferences towards online shopping while not being directly subject to CPI inflation. The LP-IV results, reported in the bottom panel of Figure 2, confirm the causal response of CPI inflation to an increase in the share of online retail sales. We find similar point estimates of the impulse responses for every specification, and the results are more significant, with the response being statistically significant at the 90% level for up to 18 months. These findings confirm that shocks to consumer preferences for online retail markets have negative and significant effects on CPI inflation, supporting the relevance of search frictions in retail markets in determining price dynamics.

Empirical evidence from local projection exercises motivate us to construct an economic framework to study how short-run preference shocks towards online versus offline retail can impact pricing dynamics and inflation measures. This paper constructs and estimates a New Keynesian dynamic stochastic general equilibrium (DSGE) model that incorporates frictional goods markets with search and matching between retailers offering differentiated products and monopolistic producers. Our framework distinguishes between online and brick-and-mortar retailers, accounting for potential differences in search efficiency. Leveraging the demand shifts during the COVID-19 pandemic, we analyze how shocks to the share of online retail sales impact pricing dynamics and the New Keynesian Phillips Curve relationship between inflation and economic activity. Our findings indicate that shifts in consumer preferences towards online retailers contribute to a reduction in CPI inflation. As consumers shift towards online retailers, they gain access to sellers who charge a lower wedge between consumer and producer prices, consequently lowering consumer prices. The model highlights the importance of shocks to online retail sales share in explaining the dynamics of prices in the United Kingdom during the COVID-19 pandemic, emphasizing the role of frictional goods markets in macroeconomic and policy analysis.

Our research also contributes to the macroeconomic literature that focuses on customer bases and trade partners. A significant influence on our work is Gourio and Rudanko (2014), who underscores the importance of trade partner acquisition costs and the value of long-term trading relationships. This paper argues that trade partner acquisition costs, assumed to be convex, engender the critical role of long-term customer relationships as essential assets shaping a firm's rational choices. On the production front of the model, our paper is a natural extension of the works by Bilbiie et al. (2008) and Bilbiie et al. (2014). Bilbiie et al. (2008) introduces a New Keynesian Phillips curve that links inflation dynamics to marginal cost, which is adjusted based on the number of firms in the economy. The study reveals an additional pathway for monetary policy transmission via the number of firms. Moreover, with the number of firms acting as a state variable, the model produces persistent inflation dynamics that align with empirical observations. Bilbiie et al. (2014) builds upon Bilbiie et al. (2008), emphasizing the derivation of optimal monetary policy.

Our research is closely related to Dong et al. (2021), who developed a New Keynesian DSGE model to explore the relationship between product life cycles and staggered pricing in the retail industry. The paper incorporates endogenous product entry by integrating the entry of new retailers into the frictional goods markets. It demonstrates how demand shocks affect the entry of new retailers, the total number of retailers, and the tightness of the frictional goods markets. The level of market tightness then influences the proportion of products undergoing price adjustments. In essence, the model endogenises Calvo's parameter, directly linking product dynamics to price inflation. While our paper shares the same mechanism for adjusting to demand shocks, it diverges by focusing on the shift in consumer preferences towards online retailers, who are more efficient in matching supply with demand than traditional brick-and-mortar retailers. Our primary contribution is to explore how this shift impacts the pass-through from producer prices to consumer price inflation and the monetary transmission.

The paper is structured as follows: Section 2 introduces our NK-DSGE model with endogenous product entry and frictional goods markets. Section 3 examines modifications to the New Keynesian Phillips Curve due to these market frictions. Section 4 detail our calibration and estimation strategies, respectively, alongside simulation results. The paper concludes in Section 5, summarising our key findings and implications.

2 Model

We move on to the main investigation where we introduce dynamic stochastic general equilibrium model (DSGE) that features search and matching functions in good markets. We build our model upon Bilbiie, Ghironi, and Melitz (2008), the pioneer work that introduce the endogenous linkages between the entry of heterogeneous firms and monopolistic competition in a DSGE model with sticky prices. We develop good market search and matching friction from Michaillat and Saez (2015).

2.1 Demand for retail goods

The novel feature is our model relative to Bilbiie, Ghironi, and Melitz (2008) is the introduction of retailing and good market search and matching frictions. The model economy consists of a continuum of atomistic households, each identical. We denominate all contracts and prices in nominal terms. We construct the similar consumer problem to simplified model. At time t, household consumes final goods offered by two types of retailers: online retailers (O) and brick-and-mortar (indexed by B). The basket of goods is thus defined as

$$C_t = \left(\frac{C_{O,t}}{\alpha_t}\right)^{\alpha_t} \left(\frac{C_{B,t}}{1-\alpha_t}\right)^{1-\alpha_t} \quad ,$$

where α_t is the expenditure share of retail goods from brick-and-mortar retailers, which we assume to be exogenous and follow an AR(1) process in percent deviation from its steady-state level with an *i.i.d.* normal error term. Let $P_{j,t}$ denotes the price of the retail goods offered by a retailer of type $j \in \{O, B\}$ at time t. The consumption-based price index of the final goods is then

$$P_t = P_{B,t}^{\alpha_t} P_{O,t}^{1-\alpha_t}, \tag{3}$$

and the household's demand for retail goods from each retailer is

$$C_{O,t} = \alpha_t \frac{P_t C_t}{P_{O,t}} \text{ and } C_{B,t} = (1 - \alpha_t) \frac{P_t C_t}{P_{B,t}}.$$

We can express the consumption-based price index of the final goods as and household's demand for retail goods in real term relative to the consumer price index as

$$1 = \rho_{O,t}^{\alpha_t} \rho_{B,t}^{1-\alpha_t}, \tag{4}$$

where $\rho_{O,t} = P_{O,t}/P_t$ and $\rho_{B,t} = P_{B,t}/P_t$, respectively. Furthermore, we derive the representative household's optimal demand for retail goods as a function of real retail price as

$$C_{O,t} = \frac{\alpha_t C_t}{\rho_{O,t}} \text{ and } C_{B,t} = \frac{(1 - \alpha_t) C_t}{\rho_{B,t}}, \tag{5}$$

respectively

2.2 Retailers and good market search and matching

A retailer of type $j \in \{O, B\}$ purchase varieties indexed ω, $y_t(\omega)$, from a continuum of varieties, Ω, available in each period. They aggregate varieties into retail goods $Y_{j,t}$ using a

CES aggregator that takes the form

$$Y_{j,t} = V_{j,t} \left(\int_{\omega_i} y_{j,t} (\omega)^{\frac{\sigma_t - 1}{\sigma_t}} d\omega \right)^{\frac{\sigma_t}{\sigma t - 1}}, \tag{6}$$

where $y_{j,t}$ is the demand of retailer of type j for variety ω and $V_{j,t} \equiv N_{j,t}^{\psi - \frac{1}{\sigma - 1}}$ in which $N_{j,t}$ stands for the number of varieties to which the retailer of type j has access. ψ stands for the marginal utility resulting from a unit increase in the number of varieties as discussed in Benassy (1996).[3] $\sigma_t > 1$ is the stochastic elasticity of substitution between varieties. Importantly, σ_t determines the stochastic markup in the goods market. Following Smets and Wouters (2003), we interpret shock to this parameter as a cost-push shock to the inflation equation. Cost-push shock is exogenous and follows an AR(1) process in percent deviation from its steady-state level with an *i.i.d.* normal error term. We assume that brick-and-mortar and online retailers have access to the same set of varieties and buy all varieties. It implies that $N_{B,t} = N_{O,t} = N_t$ and $V_{B,t} = V_{O,t} = V_t$, respectively. The number of varieties that the retailer purchases is determined by a matching function, which takes the form

$$Y_{j,t} = \left(\left(\zeta_j Y_{j,t}^{Search} \right)^{-\lambda} + N_t^{-\lambda} \right)^{-1/\lambda} \tag{7}$$

where $\zeta_j Y_{j,t}^{Search}$ is defined as efficiency-adjusted search efforts. $Y_{j,t}^{Search}$ is the retail goods that a retailer of type j pays for matching efforts, where

$$Y_{j,t}^{Search} = Y_{j,t} - Y_{j,t}^{Sales}. \tag{8}$$

$Y_{j,t}$ and $Y_{j,t}^{Sales}$ denote the total output purchased from producers and the output sold to consumers and the new entrants. In each period, there are H_t entrant firms who purchase baskets of retail goods to pay for the sunk entry cost f_E. Assume that this basket has exactly the same composition of retail goods as consumption, demand for retail goods O and B from

[3] If we set $\psi = \frac{1}{\sigma - 1}$, the consumption basket dissolved to the one discussed in Dixit and Stiglitz (1977).

prospective entrants are

$$\frac{\alpha_t H_t f_{E,t}}{\rho_{O,t}} \text{ and } \frac{(1-\alpha_t) H_t f_{E,t}}{\rho_{B,t}}$$

respectively, ζ_j is product-market search efficiency. Higher ζ_j implies that a unit increase in $Y_{j,t}^{Search}$ contributes more as an input to the matching process. translating to a higher chance of being matched with producers. The parameter $\lambda > 0$ governs the elasticity of substitution of efficiency-adjusted matching efforts and the number of producers.

We define product market tightness $\mathcal{T}_{j,t}$ as the ratio of the efficiency-adjusted matching efforts to the number of producers, that is, $\mathcal{T}_{j,t} = \zeta_j Y_{j,t}^S / N_t$. Tightness determines the probability that a product is sold to retailer j, $\mathcal{P}_{j,t} = Y_{j,t}^W / N_t$, and a unit of efficiency-adjusted matching effort that is successful, $\mathcal{Q}_{j,t} = Y_{j,t}^W / \left(\zeta_j Y_{j,t}^S\right)$, which can be interpreted as the probability that the retailer of type j being able to acquire varieties. We can show that when the product market tightness increases, producers can sell variety more easily, but it will be harder for retailers to find varieties. Note that we can write a total purchase of final goods,

$$Y_{j,t} = \mathcal{Q}_{j,t} \zeta_j Y_{j,t}^S. \tag{9}$$

We assume that retailers are identical within each type and operate in perfectly competitive markets. The retailer of type j maximises the following profit measured in real terms relative to the consumer price index,

$$d_{j,t} = \rho_{j,t} Y_{j,t}^{Sales} - \int_{\omega} \rho_t(\omega)\, y_{j,t}(\omega)\, d\omega$$

subject to (6), (8), and (9). The first order condition with respect to total retail goods sold by type-j retailers suggests that real retail prices set by the retailer of type j, are given by

$$\rho_{j,t} = \underbrace{\left(1 - \frac{1}{\mathcal{Q}_{j,t}\zeta_j}\right)^{-1}}_{\equiv \mathcal{M}_{j,t}} \rho_{P,t} \tag{10}$$

where $\rho_{P,t}$ is the real aggregate producer price and $\mathcal{M}_{j,t}$ is interpreted as the markup that retailers j set to cover the cost of search activity, $Y_{j,t}^{Search}$, paid in the unit of retail goods. Hereafter, we will refer to this markup as the search cost. search cost is positively related to the probability of the retailer being able to acquire variety. brick-and-mortar and online retailers are different in the search efficiency parameter. We assume that $\zeta_O > \zeta_B$ online retailers are more efficient in searching for varieties than brick-and-mortar retailers . Lastly, the first order condition states that the retailer j's demand for each variety ω is

$$y_{j,t}(\omega) = V_t^{\sigma-1}\left(\frac{\rho_t(\omega)}{\rho_{P,t}}\right)^{-\sigma} Y_{j,t}\ . \tag{11}$$

2.3 Firm entry and exit

In each period, there exists an unbounded number of potential entrants, who are forward-looking and able to accurately forecast their future expected profits, $d_t(\omega)$, for every period t. We assume a one-period time-to-build lag, that is, the entrants at time B commence production at time $t+1$. Additionally, they are aware of the probability, δ, of encountering a death shock that necessitates firm exit at the end of the period, after production and entry. Entrants in period B calculate their anticipated post-entry value, represented by the present discounted value of their expected profit stream, $v_t(\omega)$[4].

As mentioned above, prior to entry, firms face the sunk entry cost of $f_{E,t}$ units of consumption goods.[5] Entry continues until the anticipated post-entry value matches the cost of entry,

[4] $v_t(\omega)$ also represents the average value of incumbent firms subsequent to production, as both new entrants and incumbents face an identical survival probability of $1-\delta$.

[5] The change in $f_{E,t}$ can be interpreted as the changes in product market regulation that facilitate or hinder firm entry.

constituting the free entry condition,

$$v_t(\omega) = f_{E,t}.$$

Measuring sunk entry cost in units of consumption goods instead of units of effective labour implies a positive response of firm entry to monetary expansion, assuming that a sunk entry cost is constant.[6] This feature aligns the model with the empirical findings outlined in Bergin and Corsetti (2005) and Lewis (2006). A monetary policy shock that reduces the *ex ante* real interest rate between B and $t+1$ brings about the expansion in consumption demand. Furthermore, since producers do not pay sunk entry cost in units of effective labour, we rule out the sectoral reallocation of labour between firm entry and the production of existing producers. As a result, the expansionary monetary policy shock induces firm entry.[7] Finally, the one-period time-to-build lag dictates that the number of producers during period B follows $N_t = (1-\delta)(N_{t-1} + H_{t-1})$.

2.4 Producers and producing decision

The setup of the producers is similar to that of the firms in Bilbiie, Ghironi, and Melitz (2008). The difference lies in that firms in Bilbiie, Ghironi, and Melitz (2008) sell varieties directly to representative households, whereas our model assumes that producers sell varieties to brick-and-mortar and online retailers . There is a continuum of monopolistically competitive producers. Each producer manufactures a variety, $\omega \in \Omega$. producer ω produces requires labour, $l_t(\omega)$, to produce the output $y_t(\omega) = Z_t l_t(\omega)$ where Z_t denotes the aggregate labour productivity which represents the effectiveness of a unit of labour. Aggregate labour productivity is exogenous and follows an AR(1) process in percent deviation from its

[6] This assumption does not imply that the post-entry value of the firms in a data-consistent unit or that in nominal term is constant.

[7] If we assume that $f_{E,t} = 1$, investment in our model behaves closely to the standard RBC model without capital adjustment cost. To avoid the problem such that the no-arbitrage condition between bonds and shares features only forward variables, exposing the model to indeterminacy, we must restrict our interest rate rules to current inflation rather than expected inflation.

steady-state level with an *i.i.d.* normal error term. The unit cost of production is w_t/Z_t, measured in units of consumption goods, where $w_t = W_t/P_t$ is the wage in real terms relative to the consumer price index. We assume no fixed production costs, thus all firms produce.

A producer ω also faces nominal rigidity in the form of a quadratic price adjustment cost, $pac_t\left(\omega\right)$. The price adjustment cost could be interpreted as the quantity of marketing materials a firm must purchase when it changes its prices. Mathematically, we specify the price adjustment cost as the real cost that is incurred when individual price inflation deviates from a steady-state level, which is equal to 0, and assume the cost is proportional to real revenue from production relative to the consumer price index:

$$pac_t\left(\omega\right) = \frac{\kappa}{2}\left(\frac{p_t\left(\omega\right)}{p_{t-1}\left(\omega\right)} - 1\right)^2 \rho_t\left(\omega\right) y_t\left(\omega\right), \tag{12}$$

where $p_t\left(\omega\right)$ is the individual nominal price of the producer ω. For simplicity, we assume that the price adjustment cost is in units of the composite basket that has the same composition as the basket of retail goods. Also, we define the aggregate price adjustment cost, PAC_t, and assume that the price adjustment cost is symmetric across producers. Therefore, we can derive $PAC_t \equiv N_t pac_t$. Total demand for outputs of producer ω thus comes from brick-and-mortar and online retailers , from producers themselves as price adjustment cost, and from the firm entry cost:

$$y_t\left(\omega\right) = \left(\frac{\rho_t\left(\omega\right)}{\rho_{P,t}}\right)^{-\sigma} Y_t = \left(\frac{\rho_t\left(\omega\right)}{\rho_{P,t}}\right)^{-\sigma} \left(Y_{B,t} + Y_{O,t} + PAC_t\right)$$

Producer firms choose l_t and p_t to maximise current profit plus their real value at time t. which is the expected present discounted value of the future stream of profits. A one-period time-to-build lag implies that we must start accumulating profits from $t+1$ on. We apply the household's stochastic discount factor on the future profits as the household owns producers:

$$v_t(\omega) = \mathbb{E}_t \sum_{s=t+1}^{\infty} \Lambda_{t,s} d_s(\omega) \tag{13}$$

whereas

$$d_t(\omega) = \rho_t(\omega) y_t(\omega) - w_t l_t(\omega) - \frac{\kappa}{2}\left(\frac{p_t(\omega)}{p_{t-1}(\omega)} - 1\right)^2 \rho_t(\omega) y_t(\omega).$$

and $\Lambda_{t,s} \equiv [\beta(1-\delta)]^{s-t} U_C(C_s, L_s)/U_C(C_t, L_t)$, subject to total demand. The first-order condition with respect to the individual firm producing price gives the individual producing price equation:

$$\rho_t(\omega) = \mu_t(\omega) \frac{w_t}{Z_t} \tag{14}$$

where

$$\mu_t(\omega) = \frac{\sigma_t y_t(\omega)}{(\sigma_t - 1) y_t(\omega)\left(1 - \frac{\kappa}{2}\left(\frac{p_t(\omega)}{p_{t-1}(\omega)} - 1\right)^2\right) + \kappa \Upsilon_t} \tag{15}$$

and

$$\Upsilon_t \equiv y_t(\omega) \frac{p_t(\omega)}{p_{t-1}(\omega)}\left(\frac{p_t(\omega)}{p_{t-1}(\omega)} - 1\right) - \mathbb{E}_t\left[\Lambda_{t,t+1} y_{t+1}(\omega) \frac{P_t}{P_{t+1}}\left(\frac{p_t(\omega)}{p_{t-1}(\omega)} - 1\right)\left(\frac{p_t(\omega)}{p_{t-1}(\omega)}\right)^2\right] \tag{16}$$

Equation (14) states that individual producer price of production is a markup over marginal costs. In the absence of nominal rigidity, $\kappa = 0$, the markup reduces to $\sigma_t/(\sigma_t - 1)$. producers earn

$$d_t(\omega) = \left(1 - \frac{1}{\mu_t(\omega)} - \frac{\kappa}{2}\left(\frac{p_t(\omega)}{p_{t-1}(\omega)} - 1\right)^2\right) \frac{\rho_{P,t} Y_t}{N_t}. \tag{17}$$

2.5 Symmetric firm equilibrium

In equilibrium, we assume that all producers make identical decisions. Therefore, $p_t(\omega) = p_t$, $\mu_t(\omega) = \mu_t$, $\rho_t(\omega) = \rho_t$, $y_t(\omega) = y_t$, $pac_t(\omega) = pac_t$, $d_t(\omega) = d_t$, and $v_t(\omega) = v_t$, Under symmetric equilibrium across producers,

$$\rho_t = \mu_t \frac{w_t}{Z_t} \tag{18}$$

where

$$\mu_t = \frac{\sigma_t y_t}{(\sigma_t - 1)\, y_t \left(1 - \frac{\kappa}{2}\pi_t^2\right) + \kappa \Upsilon_t}$$

and

$$\Upsilon_t = \pi_t\,(1+\pi_t) - \mathbb{E}_t \Lambda_{t,t+1} \frac{y_{t+1}}{y_t} \frac{1+\pi_{t+1}}{1+\pi_{t+1}^C} \pi_{t+1}\,(1+\pi_{t+1})\,.$$

Since in symmetric equilibrium, $\rho_{P,t} Y_t = N_t \rho_t y_t$ where Y_t is the aggregate production, we substitute y_t with $\rho_{P,t} Y_t / (N_t \rho_t)$ such that

$$\Upsilon_t = \pi_t\,(1+\pi_t) - \mathbb{E}_t \Lambda_{t,t+1} \frac{\rho_{t+1}}{\rho_t} \frac{Y_{t+1}}{Y_t} \frac{N_t}{N_{t+1}} \frac{\rho_t}{\rho_{P,t+1}} \frac{1+\pi_{t+1}}{1+\pi_{t+1}^C} \pi_{t+1}\,(1+\pi_{t+1})\,.$$

Put in the definition, $\frac{\rho_t}{\rho_{P,t+1}} \frac{1+\pi_{t+1}}{1+\pi_{t+1}^C} = 1$, and expand the stochastic discount factor $\Lambda_{t,t+1}$ to obtain

$$\Upsilon_t = \pi_t\,(1+\pi_t) - \beta\,(1-\delta)\,\mathbb{E}_t \left(\frac{C_t}{C_{t+1}}\right) \frac{\rho_{P,t+1}}{\rho_{P,t}} \frac{Y_{t+1}}{Y_t} \frac{N_t}{N_{t+1}} \pi_{t+1}\,(1+\pi_{t+1})\,.$$

Thus,

$$\rho_t = \frac{\sigma_t}{(\sigma_t - 1)\left(1 - \frac{\kappa}{2}(\pi_t)^2\right) + \kappa\left(\pi_t\,(1+\pi_t) - \beta\,(1-\delta)\,\mathbb{E}_t \frac{C_t}{C_{t+1}} \frac{N_t}{N_{t+1}} \frac{\rho_{P,t+1}}{\rho_{P,t}} \frac{Y_{t+1}}{Y_t} \pi_{t+1}\,(1+\pi_{t+1})\right)} \frac{w_t}{Z_t}. \tag{19}$$

The real individual producer price constitutes the aggregate producer price. Following Melitz (2003), in an equilibrium characterised by a mass of firms, N_t, the aggregate producer price in real terms is given by

$$\rho_{P,t} = N_t^{-\psi} \rho_t, \tag{20}$$

which is the variety effect equation. The term $N_t^{-\psi}$ captures variety effects on aggregate producer price.

The setup of our model allows for the decomposition of consumer price into several key elements. We start off by writing Equation (10) in nominal terms and put it in Equation (3):

$$P_t = \mathcal{M}_{O,t}^{\alpha_t} \mathcal{M}_{B,t}^{1-\alpha_t} P_{P,t}$$

where $P_{P,t}$ denotes the aggregate producer price in nominal terms. Decomposing $P_{P,t}$ into the individual producer price and variety effects following Equation (20) yields

$$P_t = \mathcal{M}_{O,t}^{\alpha_t} \mathcal{M}_{B,t}^{1-\alpha_t} N_t^{-\psi} p_t.$$

The individual producer price p_t can be substituted by the pricing rule from Equation (18), written in nominal terms:

$$P_t = \underbrace{\mathcal{M}_t}_{\text{Aggregate search cost}} \underbrace{N_t^{-\psi}}_{\text{Variety effect}} \underbrace{\mu_t}_{\text{Producer markup}} \underbrace{\frac{W_t}{Z_t}}_{\text{Marginal cost}}, \tag{21}$$

where we define the aggregate search cost, $\mathcal{M}_t = \mathcal{M}_{O,t}^{\alpha_t} \mathcal{M}_{B,t}^{1-\alpha_t}$, and

$$\mathcal{M}_{O,t} = \left(1 - \frac{1}{\mathcal{Q}_{O,t}\zeta_O}\right)^{-1} \text{ and } \mathcal{M}_{B,t} = \left(1 - \frac{1}{\mathcal{Q}_{B,t}\zeta_B}\right)^{-1}. \tag{22}$$

Equation (21) decomposes consumer price into marginal cost, producer's monopolistic markup and variety effects as in Bilbiie, Ghironi, and Melitz (2008). Importantly, we show that search cost, which is the Cobb-Douglas aggregation of brick-and-mortar retailers and online retailers ' search cost, also contributes to consumer price dynamics. The decomposition of consumer price also suggests that the change in consumer price is attributable to the relative expenditure share of the brick-and-mortar retail sales to online retail sales, α_t. The role of good market search friction in determining CPI inflation will be discussed in detail in the following section where we illustrate how good market search friction alters the New Keynesian Philip Curve equation.

2.6 Household

The model economy consists of a continuum of atomistic households, each identical. We denominate all contracts and prices in nominal terms. Each household maximizes an intertemporal utility function given by $E_0 = \sum_{t=0}^{\infty} \beta^t U_t$, where $\beta \in (0,1)$ is the subjective discount factor. The period utility function is separable in consumption, C_t, and labour, L_t, taking the form

$$U_t = \left(\ln C_t - \varepsilon_{L,t} \chi \frac{L_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right) . \tag{23}$$

In equation (23), χ captures the disutility of labour supply and $\varphi \geq 0$ is the Frisch elasticity of labour supply to wages. The period utility function contains the labour supply shock, $\varepsilon_{L,t}$, which is exogenous and follows an AR(1) process in percent deviation from its steady-state level with an *i.i.d.* normal error term.[8]

Household budget constraint written in real terms follows

$$\frac{\mathcal{B}_{t+1}}{P_t} + C_t + x_{t+1} \left(N_t + H_t \right) v_t = \left(1 + r_t \right) \frac{\mathcal{B}_t}{P_t} + L_t w_t + x_t N_t \left(v_t + d_t \right) + d_{B,t} + d_{O,t}. \tag{24}$$

On the use of budget, the household consumes C_t, buying x_{t+1} shares of the mutual funds constructed from the share of existing firms, N_t, and new firms, H_t, at the share price,

[8] We choose separable preferences and logarithmic utility from consumption following Bilbiie, Ghironi, and Melitz (2008). Using separable preferences, the logarithmic utility of consumption ensures that the income and substitution effects of the real wage on labour supply neutralise each other when the real wage varies from its steady state value.

v_t, as well as purchasing nominal government bonds $\mathcal{B}_{t+1}$. On the source of budget, the household supplies labour, L_t, to earn income at real wage, w_t. Households hold producers' shares through mutual funds, thus will receive dividend income, d_t, every period they hold the shares and could sell the share at the real value v_t. Households also receive profits from online retailers, $d_{O,t}$, and brick-and-mortar retailers, $d_{B,t}$, and on a lump-sum basis. Lastly, the representative household receives the principal and return of bond holdings where $1+r_t \equiv (1+i_{t-1}) / \left(1+\pi_t^C\right)$ designates the gross real interest rate on bond holdings between $t-1$ and t.

In each period t, the representative household chooses consumption, C_t, producer's shareholding for each active retailer, x_{t+1}, and the labour supply, L_t, to maximize the expected utility function subject to the budget constraint. The first-order condition with respect to consumption yields the labour supply equation

$$\varepsilon_{L,t}\chi L_t^{\frac{1}{\varphi}} = \varepsilon_{B,t}\left(1+\tau_L\right)\frac{w_t}{C_t}, \tag{25}$$

which suggests that the representative household will allocate labour efforts until the marginal disutility of labour is equal to the marginal utility from consuming the real wage translated from an additional unit of labour. The Euler equation for bond holding is

$$v_t = \beta\left(1-\delta\right)\mathbb{E}_t\frac{C_t}{C_{t+1}}\left(v_{t+1}+d_{t+1}\right). \tag{26}$$

Lastly, the Euler equation for shareholdings is

$$1 = \beta\mathbb{E}_t\left[\frac{1+i_t}{1+\pi_{t+1}^C}\frac{C_t}{C_{t+1}}\right], \tag{27}$$

where with $1+\pi_t^C \equiv P_t/P_{t-1}$.

2.7 Model equilibrium

We impose good market clearing to derive the aggregate equilibrium. Aggregate accounting identity suggests that

$$C_t + H_t v_t = w_t L_t + N_t d_t + d_{B,t} + d_{O,t}. \tag{28}$$

The model consists of 31 endogenous variables and 31 equilibrium conditions, including the equation that governs the nominal interest rate setting by the monetary authority and the setting of the labour subsidy to eliminate the inefficiency generated by monopolistic competition among producers, in order to achieve the efficient equilibrium. Endogenous variables are ρ_t, μ_t, Υ_t, $\rho_{P,t}$, $\rho_{B,t}$, $\rho_{O,t}$, w_t, π_t, π_t^C, Y_t, $Y_{B,t}$, $Y_{O,t}$, $Y_{B,t}^{Search}$, $Y_{O,t}^{Search}$, $\mathcal{P}_{B,t}$, $\mathcal{P}_{O,t}$, $\mathcal{Q}_{B,t}$, $\mathcal{Q}_{O,t}$, $\mathcal{T}_{B,t}$, $\mathcal{T}_{O,t}$, $\mathcal{M}_t$, $\mathcal{M}_{O,t}$, $\mathcal{M}_{B,t}$, C_t, N_t, H_t, d_t, v_t, L_t, $\tau_{L,t}$, and i_t. N_t constitutes a state variable in the system. Table 1 and 2 summarise model equations.

3 The New Keynesian Phillips Curve

We log-linearise the model around the efficient steady state with zero inflation and derive the New Keynesian Philips Curve. We denote log-linearised variables in San Serif fonts or in Serif fonts capped with Tildes.

We start with log-linearizing Equation 2 around the steady state of μ_t which is equal to $\sigma/(\sigma-1)$.

$$\pi_t = \beta(1-\delta)\mathbb{E}_t\pi_{t+1} - \frac{\sigma-1}{\kappa}\widetilde{\mu}_t \tag{29}$$

where individual producer price inflation rate π_t is also percent deviations of gross inflation from zero steady-state while $\widetilde{\mu}_t$ denotes the percent deviations of marginal cost from the

Description	Equation
Producer pricing	$\rho_t = \mu_t \frac{w_t}{Z_t}$
Producer markup	$\mu_t = \frac{\sigma_t}{(\sigma_t - 1)\left(1 - \frac{\kappa}{2}(\pi_t)^2\right) + \kappa \Upsilon_t}$
Definition	$\Upsilon_t = \pi_t (1 + \pi_t)$
	$-\beta (1-\delta) \mathbb{E}_t \frac{C_t}{C_{t+1}} \frac{N_t}{N_{t+1}} \frac{\rho_{P,t+1}}{\rho_{P,t}} \frac{Y_{t+1}}{Y_t} \pi_{t+1} (1 + \pi_{t+1})$.
Variety effects	$\rho_t = N_t^{\psi} \rho_{P,t}$
Search cost (Online)	$\mathcal{M}_{O,t} = \left(\frac{1}{1 - \zeta_{O,t} \mathcal{Q}_{O,t}}\right)^{-1}$
Search cost (Brick-and-mortar)	$\mathcal{M}_{B,t} = \left(\frac{1}{1 - \zeta_{B,t} \mathcal{Q}_{B,t}}\right)^{-1}$
Aggregate search cost	$\mathcal{M}_t = \mathcal{M}_{O,t}^{\alpha_t} \mathcal{M}_{B,t}^{1-\alpha_t}$
Retail price (Online)	$\rho_{O,t} = \mathcal{M}_{O,t} \rho_{P,t}$
Retail price (Brick-and-mortar)	$\rho_{B,t} = \mathcal{M}_{B,t} \rho_{P,t}$
Real CPI	$1 = \rho_{O,t}^{\alpha_t} \rho_{B,t}^{1-\alpha_t}$
Producer profits	$d_t = \left(1 - \frac{1}{\mu_t} - \frac{\kappa}{2} (\pi_t)^2\right) \frac{\rho_{P,t} Y_t}{N_t}$
Free entry condition for producers	$v_t = f_{E,t}$
Motion of producers	$N_{t+1} = (1-\delta)(N_t + H_t)$
Euler equation for producers	$v_t = \beta (1-\delta) \mathbb{E}_t \frac{C_t}{C_{t+1}} (v_{t+1} + d_{t+1})$
Optimal labour supply	$\varepsilon_{L,t} \chi L_t^{\frac{1}{\varphi}} = (1 + \tau_L) \frac{w_t}{C_t}$
Matching function (Online)	$Y_{O,t} = \left(\left(\zeta_O Y_{O,t}^{Search}\right)^{-\lambda} + N_t^{-\lambda}\right)^{-1/\lambda}$
Matching function (Brick-and-mortar)	$Y_{B,t} = \left(\left(\zeta_B Y_{B,t}^{Search}\right)^{-\lambda} + N_t^{-\lambda}\right)^{-1/\lambda}$
Tightness (Online)	$\mathcal{T}_{O,t} = \frac{\zeta_O Y_{O,t}^{Search}}{N_t}$
Tightness (Brick-and-mortar)	$\mathcal{T}_{B,t} = \frac{\zeta_B Y_{B,t}^{Search}}{N_t}$
Prob. of producer matching (Online)	$\mathcal{P}_{O,t} = \frac{Y_{O,t}}{N_t}$
Prob. of producer matching (Brick-and-mortar)	$\mathcal{P}_{B,t} = \frac{Y_{B,t}}{N_t}$

Table 1: Summary of equations

Description	Equation
Prob. of retailer matching (Online)	$\mathcal{Q}_{O,t} = \frac{Y_{O,t}}{\zeta_O Y_{O,t}^{Search}}$
Prob. of retailer matching (Brick-and-mortar)	$\mathcal{Q}_{B,t} = \frac{Y_{B,t}}{\zeta_B Y_{B,t}^{Search}}$
Use of retail goods (Online)	$Y_{O,t} = C_{O,t} + \alpha_t \frac{H_t f_{E,t}}{\rho_{O,t}} + Y_{O,t}^{Search}$
Use of retail goods (Brick-and-mortar)	$Y_{B,t} = C_{B,t} + (1-\alpha_t) \frac{H_t f_{E,t}}{\rho_{B,t}} + Y_{B,t}^{Search}$
Total production	$\rho_{P,t} Y_t = \rho_{P,t} \left(Y_{O,t} + Y_{B,t}\right) + \frac{\kappa}{2} \left(\pi_t\right)^2 \rho_{P,t} Y_t$
Good market clearing	$C_t + H_t v_t = w_t L_t + N_t d_t$
CPI inflation	$\frac{1+\pi_t}{1+\pi_t^C} = \frac{\rho_t}{\rho_{t-1}}$
Euler equation for bonds	$1 = \beta \mathbb{E}_t \frac{C_t}{C_{t+1}} \left(\frac{1+i_t}{1+\pi_{t+1}^C}\right)$

Table 2: Summary of equations (Continued)

steady state. From real CPI equation, and retail prices,

$$1 = \rho_{O,t}^{\alpha_t} \rho_{B,t}^{1-\alpha_t}$$
$$1 = \mathcal{M}_{O,t}^{\alpha_t} \mathcal{M}_{B,t}^{1-\alpha_t} \rho_{P,t}.$$

Along with pricing rule in Equation X and $\rho_t = N_t^{\psi} \rho_{P,t}$ and we get

$$\mathcal{M}_{O,t}^{\alpha_t} \mathcal{M}_{B,t}^{1-\alpha_t} \mu_t \frac{w_t}{Z_t} = N_t^{\psi} \tag{30}$$

Log linearise Equation (30)[9], we get

$$\hat{\mu}_t = \psi \mathsf{N}_t - \alpha \left(\ln \mathcal{M}_{O,t} - \ln \mathcal{M}_{B,t}\right) \tilde{\alpha}_t + \alpha \tilde{\mathcal{M}}_{O,t} + (1-\alpha) \tilde{\mathcal{M}}_{B,t} - \left(\mathsf{w}_t - \mathsf{Z}_t\right).$$

Put this expression in Equation (29),

$$\begin{aligned} \pi_t =& \beta (1-\delta) \mathbb{E}_t \pi_{t+1} + \frac{\sigma-1}{\kappa} \left(\mathsf{w}_t - \mathsf{Z}_t\right) - \frac{\sigma-1}{\kappa} \psi \mathsf{N}_t \\ &+ \frac{\sigma-1}{\kappa} \left(\alpha \left(\ln \mathcal{M}_{O,t} - \ln \mathcal{M}_{B,t}\right) \tilde{\alpha}_t + \alpha \tilde{\mathcal{M}}_{O,t} + (1-\alpha) \tilde{\mathcal{M}}_{B,t}\right). \end{aligned} \tag{31}$$

[9]Please refer to B for detailed calculations

Equation 31 represents the New Keynesian Phillips Curve. Variables that are written in Serif fonts are measured in terms of the deviation from the steady state. The term $E_t(\pi_{t+1})$ stands for expected inflation in the next period, suggesting that current inflation depends on what household and firms think inflation will be in the future. This highlights the forward-looking nature of economic decisions. The expression $w_t - Z_t$ represents marginal costs of hiring additional unit of labour. When real wages increase, it raises the cost of labour, passing on to higher inflation. Conversely, when productivity rises, companies can produce the same output with fewer workers, reducing labour demand and inflationary pressures. As in Bilbiie, Ghironi, and Melitz (2008), our version of the New Keynesian Phillips Curve engenders the marginal cost that captures the number of producers.

We propose novel implications of goods market search frictions on inflation, specifically the effects of temporary changes in the share of online retail sales to total expenditure on consumer price inflation. These effects are captured in the terms

$$\frac{\sigma - 1}{\kappa}\left(\alpha\left(\ln \mathcal{M}_{O,t} - \ln \mathcal{M}_{B,t}\right)\tilde{\alpha}_t + \alpha\tilde{\mathcal{M}}_{O,t} + (1-\alpha)\,\tilde{\mathcal{M}}_{B,t}\right)$$

which can be decomposed into two channels.

The first channel, which we call the composition channel, arises from the term $\frac{\sigma-1}{\kappa}\alpha\left(\ln \mathcal{M}_{O,t} - \ln \mathcal{M}_{B,t}\right)$. It suggests that an increase in the share of online retail sales (a positive deviation of α_t from the steady state) alters inflation. As online retailers are more efficient at matching than brick-and-mortar retailers, $\ln \mathcal{M}_{O,t} - \ln \mathcal{M}_{B,t}$ is negative. Given the restrictions on $\alpha - 1$ and κ, this term implies that price inflation decreases in response to a rise in the share of online retail sales. Intuitively, as consumers shift their preferences towards online retail, they purchase from online retailers who charge a lower wedge on top of producer prices, resulting in lower consumer prices and a decline in aggregate consumer prices (or a negative inflation rate).

The second channel, which we refer to as the arbitrage channel, is captured by $\frac{\sigma-1}{\kappa}\left(\alpha\tilde{\mathcal{M}}_{O,t}+(1-\alpha)\,\tilde{\mathcal{M}}_{B,t}\right)$. As customers shift their preferences towards online retail markets, the marginal benefits of search for online retailers increase, leading them to exert more effort in searching. This increased search effort tightens the online frictional market, decreasing the probability of retailers finding firms and raising search costs. Consequently, online retailers must increase the wedge between consumer and producer prices to cover these costs. Conversely, as customer demand shifts away from brick-and-mortar stores, the marginal benefits of search for these retailers decrease, causing them to exert less effort in searching. This reduced search effort loosens the traditional frictional market, increasing the probability of retailers finding firms and lowering search costs. As a result, brick-and-mortar stores will charge a lower wedge. In the next section, we carry the model to the data to estimate net effects of these two channels.

4 Estimation and Interpretation of the results

4.1 Estimation

We estimate the model using quarterly UK data. Our baseline spans over the period between 2010:I and 2022:IV. We are restricted to this period because we suffer from a lack of data. The data set consists of five variables: CPI nominal inflation, PPI nominal inflation, the share of online retail sales, the growth of the nominal value of the online retail sales, producers' aggregate productivity, and producers' average real profit growth. All the variables are measured in terms of year-over-year growth rates.

4.1.1 Priors

We follow Smets and Wouters (2007) in forming priors. For the monetary policy rule parameters, we use a normal distribution to describe the long-term responses to inflation, ϕ_π, and the output gap, ϕ_c, with means of 1.5 and 0.125, and standard deviations of 0.125 and 0.05, respectively. The rule's persistence is captured by the coefficient on the lagged interest rate, assumed to be normally distributed with a mean of 0.75 and a standard deviation of 0.1. We set the prior mean of the producer price adjustment cost, denoted by κ, at 300, closely following the estimates for producer price adjustment costs found in the study by Harrison and Oomen (2010) using UK data. This parameter significantly influences the response of real profits and the subsequent adjustment in the number of producers to a monetary policy shock. To attenuate the positive reaction of consumer prices to an expansionary monetary policy shock, allowing for an increase in real profits and encouraging firm entry, κ must be sufficiently large. Consequently, an expansionary monetary policy shock results in an increase in the number of firms. Lastly, we assume the persistence of stochastic processes to be a beta distribution, having an average of 0.5 and a standard deviation of 0.2. We assume the standard deviation of the processes to follow an inverse-gamma distribution, with an average value of 0.10.

4.1.2 Posterior estimation

Our estimates reveal that shocks to the elasticity of substitution between different varieties, σ, and productivity shocks, Z, are the most enduring over time. This indicates that, over the long term, these shocks explain most of the variations in economic variables in the model. On the other hand, monetary policy shocks, $MONET$, are less persistent and less volatile, suggesting that adjustments to the interest rates set by the central bank closely follow the expected pattern according to the Taylor rule.

Importantly, we found the AR(1) coefficient and the standard deviation for the shock to online retail sales share, α to be 0.656 and 0.099, respectively. This shows that online retail sales shocks last longer than monetary policy shocks, $MONET$, but not as long as technology shocks, Z, implying that online retail sales shocks affect the business cycle in the medium term, unlike monetary policy shocks, $MONET$, that have a short-term impact and technology shocks that influence the long term, Z.

Parameters	Prior distribution			Posterior distribution				
	Distr.	Mean	St.Dev.	Mode	Mean	St.Dev.	5%	95%
λ	Invgamma	5.000	0.500	9.769	9.286	1.886	7.823	10.891
ϕ_π	Normal	1.500	0.250	1.674	1.739	0.205	1.414	2.068
ϕ_c	Normal	0.120	0.050	0.292	0.307	0.034	0.253	0.358
κ	Normal	300.000	30.000	300.018	291.343	27.777	246.091	336.802
ρ_r	Beta	0.500	0.200	0.041	0.061	0.032	0.007	0.113
ρ_α	Beta	0.500	0.200	0.667	0.656	0.099	0.493	0.825
ρ_{MONET}	Beta	0.500	0.200	0.106	0.148	0.072	0.024	0.263
ρ_z	Beta	0.500	0.200	0.929	0.836	0.059	0.658	0.995
ρ_σ	Beta	0.500	0.200	0.923	0.917	0.057	0.864	0.973
ρ_l	Beta	0.500	0.200	0.500	0.498	0.277	0.173	0.834
σ_α	Invgamma	0.100	2.000	0.076	0.079	0.008	0.064	0.091
σ_{MONET}	Invgamma	0.100	2.000	0.013	0.014	0.001	0.012	0.016
σ_z	Invgamma	0.100	2.000	0.038	0.038	0.004	0.032	0.045
σ_σ	Invgamma	0.100	2.000	0.361	0.399	0.151	0.244	0.556
σ_l	Invgamma	0.100	2.000	0.087	0.095	0.012	0.072	0.115

Table 3: Parameter estimates

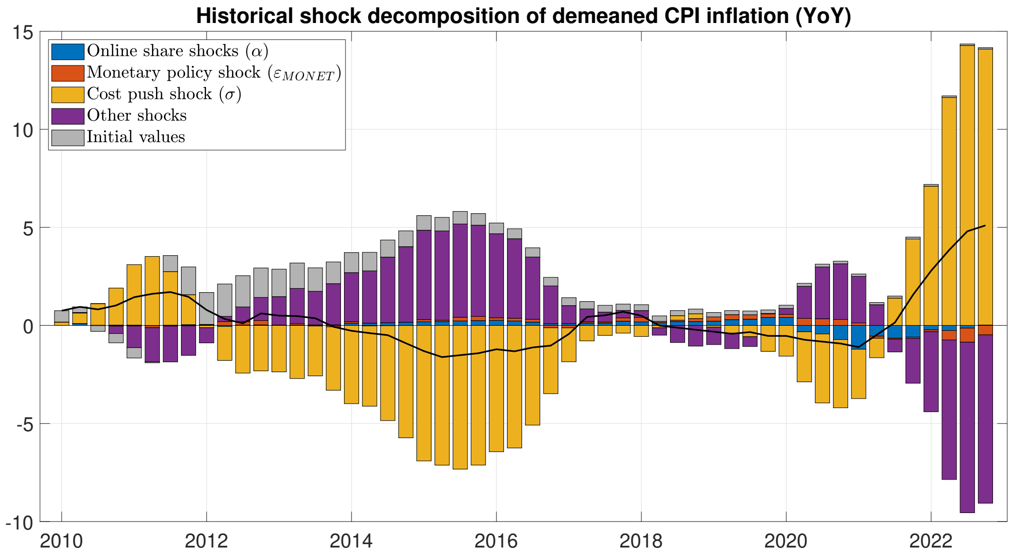


Figure 3: Historical shock decomposition of CPI inflation

4.2 Shock decomposition

Figure 3 illustrates the historical contributions to CPI inflation from various shocks of interest. These include shocks to the share of online retail sales relative to total retail sales (represented by blue bars), monetary policy shocks (orange bars), and cost-push shocks (yellow bars). Contributions from other shocks, including aggregate productivity shock and labour supply shock, are aggregated and illustrated by the purple bars. The analysis spans the period of the COVID-19 pandemic in 2020-2021, providing insight into the predominant factors influencing CPI fluctuations during this time frame.

Notably, the shifts towards online retail sales exerted a disinflationary impact on CPI inflation from the second quarter of 2020 (2020:II) to the fourth quarter of 2021 (2021:IV), aligning with the onset of the COVID-19 health and economic crisis. The magnitude of this shock's contribution increased from 2020:II to 2020:IV, subsequently declining and transitioning to an inflationary impact by the first quarter of 2022. Our finding underscores the role of search and matching friction in good markets in determining CPI inflation.

4.3 Impulse responses

Figure 4 and 5 exhibit the impulse responses of the key variables to a 1 percent increase in the share of online retail sales to total retail sales, α_t, and nominal interest rates, i_t, respectively. The entries display the response of CPI inflation, π_t^C, individual producer price, ρ_t, number of producers, N_t, consumption, C_t, real wage, w_t, market tightness faced by online retailers, $\mathcal{T}_{O,t}$, search cost faced by online retailers, $\mathcal{M}_{O,t}$, market tightness faced by brick-and-mortar retailers, $\mathcal{T}_{B,t}$, and search cost faced by online retailers, $\mathcal{M}_{B,t}$.

4.3.1 A positive shock to the share of online retail sales

Figure 4 exhibits the impulse responses of the key variables to a 1 percent increase in the share of online retail sales to total retail sales, α_t. In line with the analytical expression in subsection 3, the increase in the share of online retail sales to total retail sales mainly lowers the CPI inflation, π_t^C, and stimulates consumption C_t. An increase in the share of online retail sales instigates a demand shift from brick-and-mortar to online retailers who possess higher search efficiency, $\zeta_O > \zeta_B$, charge lower search cost, $\mathcal{M}_{B,t} > \mathcal{M}_{O,t}$, and thus lower retail prices, $\rho_{O,t} < \rho_{B,t}$. Lower price stimulates consumer demand, leading to higher consumption, C_t. Furthermore, higher consumer demand boosts producer's expected profits and share prices, incentivising firm entry and boosting varieties, N_t.

To add decomposition of effects to composition and arbitrage effects.

4.3.2 Monetary policy shock

Figure 5 exhibits the impulse responses of the key variables to contractionary monetary policy, a 1 percent increase in interest rates, i_t. On impact, monetary tightening reduces inflation, π_t^C, and consumption, C_t. Monetary tightening also deters new firms from entering the market (N_t), aligning with empirical findings of Bergin and Corsetti (2005) and Lewis (2006). Contrary to other structural New Keynesian models, the adverse effects of contractionary monetary policy on CPI inflation, π_t^C, are relatively transient. As the number of

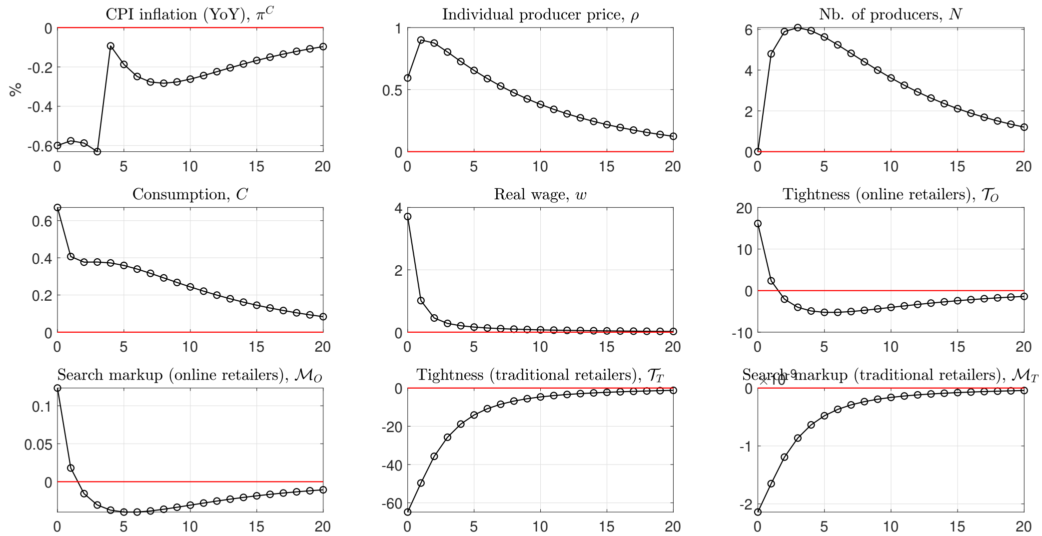


Figure 4: Response to 1% increase in the share of online retail sales

producers, N_t dwindles, frictional good markets tighten ($\mathcal{T}_B$ and $\mathcal{T}_O$ increase), leading to a decrease in the likelihood of retailers successfully sourcing products ($\mathcal{Q}_B$ and $\mathcal{Q}_O$ decrease). Consequently, retailers exert greater search efforts and transfer search costs to higher retail prices ($\mathcal{M}_B$ and $\mathcal{M}_O$ increase) and consumer prices.

5 Conclusion

This study investigates the role of frictional goods markets in shaping Consumer Price Index (CPI) inflation dynamics through the development and estimation of a New Keynesian dynamic stochastic general equilibrium (DSGE) model. This model integrates frictional goods markets and endogenous product entry, employing a search-and-matching framework between retailers and monopolistic producers, each offering a unique product assortment. Distinctively, it differentiates between online and brick-and-mortar retailers based on their matching efficiencies, leveraging shifts in consumer demand towards online sales during the COVID-19 pandemic as a natural experiment. By applying the New Keynesian Phillips Curve, we reveal how online retail sales modulate inflation dynamics. Our analysis sug-

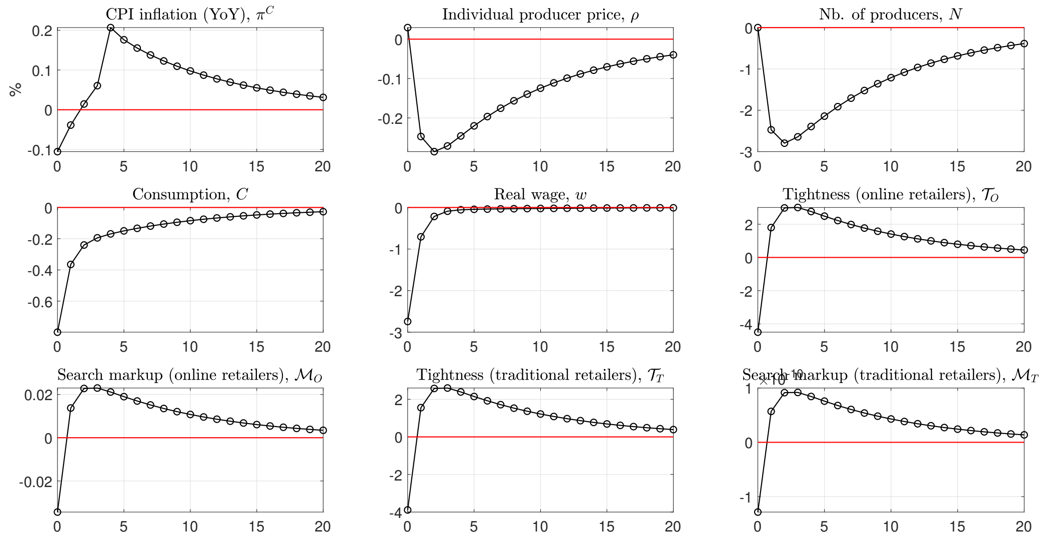


Figure 5: Response to contractionary monetary policy shock

gests that a consumer shift towards online retailers, characterized by lower search costs and enhanced search efficiency, leads to a decrease in CPI inflation.

References

BENASSY, J.-P. (1996): "Taste for variety and optimum production patterns in monopolistic competition," *Economics Letters*, 52, 41–47.

BERGIN, P. R. AND G. CORSETTI (2005): "Towards a Theory of Firm Entry and Stabilization Policy," NBER Working Papers 11821, National Bureau of Economic Research, Inc.

BILBIIE, F. O., I. FUJIWARA, AND F. GHIRONI (2014): "Optimal monetary policy with endogenous entry and product variety," *Journal of Monetary Economics*, 64, 1–20.

BILBIIE, F. O., F. GHIRONI, AND M. J. MELITZ (2008): "Monetary Policy and Business Cycles with Endogenous Entry and Product Variety," in *NBER Macroeconomics Annual 2007, Volume 22*, National Bureau of Economic Research, Inc, NBER Chapters, 299–353.

CAVALLO, A. (2017): "Are Online and Offline Prices Similar? Evidence from Large Multichannel Retailers," *American Economic Review*, 107, 283–303.

DIXIT, A. K. AND J. E. STIGLITZ (1977): "Monopolistic competition and optimum product diversity," *The American economic review*, 67, 297–308.

DONG, M., T. SHOJI, AND Y. TERANISHI (2021): "Product Cycles and Prices: a Search Foundation," Working Papers e161, Tokyo Center for Economic Research.

FRADKIN, A. (2017): *Digital Marketplaces*, London: Palgrave Macmillan UK, 1–14.

GOURIO, F. AND L. RUDANKO (2014): "Customer Capital," *Review of Economic Studies*, 81, 1102–1136.

HARRISON, R. AND OOMEN (2010): "Evaluating and estimating a DSGE model for the United Kingdom," Bank of England working papers 380, Bank of England.

Hottman, C. J., S. J. Redding, and D. E. Weinstein (2016): "Quantifying the Sources of Firm Heterogeneity," *The Quarterly Journal of Economics*, 131, 1291–1364.

Jordà, Ò. (2005): "Estimation and inference of impulse responses by local projections," *American economic review*, 95, 161–182.

Lewis, V. (2006): "Macroeconomic fluctuations and firm entry : theory and evidence," Working Paper Research 103, National Bank of Belgium.

Michaillat, P. and E. Saez (2015): "Aggregate Demand, Idle Time, and Unemployment," *The Quarterly Journal of Economics*, 130, 507–569.

Nakamura, E. (2008): "Pass-Through in Retail and Wholesale," *American Economic Review*, 98, 430–437.

Smets, F. and R. Wouters (2003): "An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area," *Journal of the European Economic Association*, 1, 1123–1175.

——— (2007): "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, 97, 586–606.

Appendix for "Search frictions in Producer markets - Secular trends in CPI, PPI, and pass through?"

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June 12, 2024

Contents

A Data

Table 4 and 5 list the data series for local projection estimation and Bayesian estimation, respectively.

B Model

B.1 Steady state calculations

Total gross operating surplus of non-financial corporations

In this section, we provide the detailed calculations of the steady state. Firstly, the free entry condition in Equation (**??**) provides the steady state of the post-entry firm value,

$$v = f_E.$$

while the Euler equation for shares provides the steady-state value of the real profits of the producers,

$$d = \frac{1 - \beta (1 - \delta)}{\beta (1 - \delta)} v.$$

Variable	Description	Source
Consumer Price	Consumer Price Index excluding energy, food, alcoholic beverages & tobacco	Office for National Statistics
Producer Price	Producer output prices - Domestic manufactured products excluding Duty	Office for National Statistics
Share of Online Retail Sales	Internet sales as a percentage of total retail sales (ratio)	Office for National Statistics

Table 4: Data series for local projections

Variable	Description	Source
Consumer Price	Consumer Price Index excluding energy, food, alcoholic beverages & tobacco	Office for National Statistics
Producer Price	Producer output prices - Domestic manufactured products excluding Duty	Office for National Statistics
Share of Online Retail Sales	Internet sales as a percentage of total retail sales (ratio)	Office for National Statistics
Profit	Total gross operating surplus of non-financial corporations	Office for National Statistics
Productivity	Labour productivity	Office for National Statistics

Table 5: Data series for Bayesian estimation

From the motion of firms, we can write the number of entrants at the steady state

$$H = \frac{\delta}{1-\delta} N$$

To simplify the steady-state calculation, χ is set to the value such that the steady-state labour supply, L, is equal to one.[1] It simplifies intratemporal optimality,

$$\chi = (1 + \tau_L) \frac{w}{C}.$$

Then, from labour market clearing condition,

$$1 = N \left[(\sigma - 1) \frac{d}{w} \right],$$

and the aggregate accounting,

$$C + Hv = w + Nd + d_T + d_O,$$

putting these three steady-state equations together yields

$$N = \left(\frac{Z}{\sigma \mathcal{M} d} \right)^{\frac{1}{1-\psi}}.$$

The steady-state value of other variables can be computed straightforwardly.

[1] *chi* that satisfies such condition is

$$\chi = (1 + \tau_L) \frac{(\sigma_t - 1) d}{\sigma_t d - \frac{\delta}{1-\delta} v}$$