Bank Runs When Liquidity and Solvency Risks Interact

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Abstract

In the context of the modern banking economy, where central banks act as lenders of last resort, I examine the risk of bank runs when liquidity and solvency risks interact. Plenty of studies have demonstrated that central banks can prevent bank runs if they provide sufficient liquidity during a panic. In this model, I showed that equilibrium with bank runs exists when solvency and liquidity shocks interact. Central banks cannot prevent bank runs. The analytical solution and the Monte Carlo experiment demonstrate that the risk of bank runs increases when there is a high degree of risk aversion. With the same level of solvency risk, the chance of bank runs will increase for banks with higher numbers of depositors, which can be interpreted as the sunspots.

JEL classification: E42, E58, G21

Keywords: bank run, liquidity, solvency risk, central bank

1 Introduction

From the Bardi and Peruzzi family bank collapse in 1343 (Hunt 1990) to the fall of Silicon Valley Bank (SVB) in March 2023 (Cookson, Fox, Gil-Bazo, Imbet, and Schiller 2023), bank runs are the phenomena that have occurred throughout the history of banking. Several of these runs were followed by or associated with financial crises. The economic destruction of bank runs has put them in the scope of interest of economists for more than 50 years.

The work of Diamond and Dybvig (1983) is one of the most influential models of bank runs. Consumers face idiosyncratic uncertainty in preferences as they are unsure whether they will be early or late consumers. Banks in this model provide the demand-deposit contract as the risk-sharing mechanism. Depositors deposit their endowments with the banks. Diamond and Dybvig (1983) has proved that the demand-deposit system can bring about the social optimum. Nonetheless, it also has an undesired outcome, which is the bank runs.

The concept of bank runs, as described in Diamond and Dybvig (1983), involves late consumers, namely depositors withdrawing their money from banks, thereby imposing a high liquidity shock on their banks. They believe that their banks may not have enough assets left for them to withdraw, and it is self-fulfilling. In the modern banking world, commercial banks face more complex problems. Nominal loans generate nominal deposits instead of consumers depositing their real endowments. When a customer uses her deposit to purchase goods from a customer of another bank, her deposits will be transferred to the seller's bank, as well as the reserves, which are used as the means of interbank transactions. These types of transactions happen every day, and banks that have negative net flows of deposits will have to borrow reserves from the central bank. Hence, in reality, banks also face the cost of managing deposits. When banks face large liquidity shocks, they may not have enough reserves to satisfy the liquidity demand. In addition, the reserve borrowing costs will also increase.

Another risk that banks are exposed to is solvency risk. Banks' operations to create loans leave them with capital lower than their level of debt. Solvency risks happen when banks may not be able to collect their loans. This can happen due to the low production as observed in Gorton (1988) or their bad monitoring and screening systems (Gupta, Lu, Simaan, and Zaki 2022).

Disasters, such as COVID-19, are examples of events that pose solvency risk to banks. During the COVID-19 pandemic, the GDP of each country plummeted as businesses were unable to operate. Workers lost their jobs or were restricted from working. The loss of income for both businesses and households decreased the ability to pay debts, imposing the solvency risk on commercial banks. However, the solvency risk was not the only risk that banks faced during the COVID-19 pandemic. There was high demand for liquidity. Central banks around the world had to roll out emergency liquidity plans (Cantº, Cavallino, De Fiore, and Yetman 2021). While solvency shocks can cause banks to collapse, large liquidity shocks can also increase the cost of managing liquidity and exacerbate the solvency shocks.

This study aims to investigate bank runs when bank-specific solvency risk interacts with liquidity risk, using the construction of Rivero and Rodriguez (2024) to capture the environment of modern banking.

The analytical solution illustrated that the loss from low production could decrease the deposit rate, making deposits less valuable. However, the magnitude of the loss depends on the degree of liquidity shock. Different combinations of liquidity and solvency risks determine the deposit rates paid to the depositors. Due to the interaction between the two risks, an exact competitive equilibrium cannot be solved analytically. For these reasons, I introduce a Monte Carlo experiment, a simulation in which the production shock is drawn multiple times, making the realized frequency of the shock approach the actual probability from the real data-generating process.

The Monte Carlo experiment shows that bank runs exist when two shocks interact. The chance of bank runs increases with a higher degree of risk aversion. For a certain level of solvency shock, bank runs tend to happen more to banks with a higher number of depositors, indicating the sunspot of the economy. There are some explanations for these counterintuitive results. Firstly, when there is a large number of patient depositors, there are also more people who care about the value of their own deposits, and there are more people to run the banks. Secondly, the patient depositors run the banks because they are risk-averse. The patient depositors have two options to manage solvency risk. Firstly, they can continue saving in terms of deposits, but face the risk that their deposits may lose their real value and their banks may become insolvent. Secondly, they can use their deposits to store the consumption goods for one period. This model assumes that the banks will pay locational deposit rates. The spread between deposit rates in high- and low-production areas increases with the number of impatient depositors, namely, a larger liquidity shock. In locations with larger liquidity shocks, the utility gains from high production can compensate for the utility losses. This is not the same for locations with more patient depositors. The lower spread implies greater stability, but a larger number of depositors means a larger number of people to whom the banks are indebted. The gain in the deposit rates is not high enough to compensate for the utility loss for so many depositors.

Plenty of literature such as Allen, Carletti, and Gale (2014) and Rivero and Rodriguez (2024) has emphasized the role of central banks as the lender of the last resort. As long as the central banks are willing to provide enough liquidity, there will be no bank run. This paper has demonstrated that when a solvency shock interacts with a liquidity shock, central banks are unable to prevent bank runs. The results of this study raise concerns regarding the challenges that central banks face in a world of rising uncertainties.(Jefferson 2023)

2 Literature review

Economists have thrived to understand bank runs, and the research in this field is still ongoing. Most studies on bank runs are also related to liquidity, the central bank, and deposit insurance. Bryant (1980) studied the role of reserves and deposit insurance on bank runs. This study defined a bank run as a situation in which agents learned bad news and came to withdraw their deposits. Deposit insurance could cause complex risk redistribution and did not always have an impact on preventing the runs.

One research that is influential on the subsequent studies is Diamond and Dybvig (1983). In this model, agents face idiosyncratic uncertainty regarding their preferred time for consumption. It introduced deposit contracts as the risk-sharing mechanism. Banks act as intermediaries, gathering all resources from the consumers and offering demand deposit contracts. They have shown that demand deposits can facilitate the social optimum outcome, which is not possible with the market or autarky systems. However, demand deposits can result in an undesirable equilibrium, which is a bank run. One tool to prevent bank runs is to offer deposit insurance.

A number of theoretical studies have been built upon the structure of Diamond and Dybvig (1983). Postlewaite and Vives (1987) incorporated the Prisoner's Dilemma into the model and revealed that there was a unique equilibrium with a positive probability of bank runs, even with the deposit insurance. Allen and Gale (2004a) constructed a model with an aggregate preferences shock. In other words, the banks in their model also face the uncertainty that the whole economy will have high or low impatient agents. They discovered that the aggregate preference shock could lead to either high price volatility, bank runs, or both. Allen and Gale (2004b) explored the model that there were both banks (financial intermediaries) and the financial market that faced aggregate shocks in asset returns and preferences. Banks may not be able to offer contingent deposit contracts, while there may be no contingent bonds for all states of shock in the financial markets. Their results have shown that an economy with both complete contracts and complete markets is incentive-efficient.

In Diamond and Dybvig (1983), Bryant (1980), and several studies, bank runs happen when depositors panic. Banks with more risk factors are more prone to runs. Having a high number of uninsured deposits is also one of the factors. The empirical model from Iyer and Puri (2012) studied micro data on depositors in the banks with runs. Though there are other factors defining the chance of a run, uninsured depositors are the most likely to run the banks. Additionally, the rise of social media has the potential to amplify these risks, as highlighted

Cookson et al. (2023). Nevertheless, there are also some downsides regarding having deposits fully insured. Miller and Luangaram (1998) examined the 1997 Asian financial crisis. They found that the moral hazard caused by deposit insurance was one of the factors leading to the collapse of the financial sectors in several Asian countries.

Other risk factors that can cause the runs are the business cycles and banks' insolvency. An empirical study of the National Bank Era from Gorton (1988) has shown that bank runs occurred when the perception of risks changed. Several economic downturns happened along with bank panics since banks are expected to fail more during recessions. Iyer, Puri, and Ryan (2016) used microdata from a bank in India to show that there were chances of banks' runs when the auditing authority exposed the information about banks' insolvency. Viviani and LE Hanh (2018) used the text-analysis tools to study Federal Deposit Insurance Corporation (FDIC)'s bank failure reports. They discovered that most bank failures were a result of poor management and concentrated lending, which deteriorate banks' solvency.

These models were built on real deposit contracts and real banks. In Diamond and Dybvig (1983), consumers deposit their assets in banks, then banks invest in long and short technologies. In the modern economy, loans create deposits, not the other way round (McLeay, Radia, and Thomas 2014). There exists empirical evidence supporting the notion that banks have the ability to create money from nothing (Werner 2014). Additionally, all contracts made with banks are expressed in nominal terms, not in real terms. Central banks provide reserves as a means of transaction in the interbank markets, and they use the reserves to control interest rates.

Though the banks' activities mostly happen in financial markets, and in nominal terms. The process of money creation has a real impact on real output (Berger and Sedunov 2017). The business downturns associated with financial busts are more severe and take a longer time to recover (Drehmann, Borio, and Tsatsaronis 2012). Several later works incorporate interbank markets and central banks to study bank runs. Freixas, Parigi, and Rochet (2000) created the model in which some consumers have to travel. The interbank market increases welfare by lowering the necessity to hold liquidity. However, there can be a case where the economy becomes stuck in a gridlock equilibrium. They defined "systemic risk" as the risk that spreads from banks to banks by the interbank market and identified the banks that are "too big to fail". Following the global financial crisis, the role of central banks in preventing systemic risks came into focus. Freixas, Martin, and Skeie (2011) built a model in which there was a disparity in liquidity holding, and lowering the monetary policy rate could improve the situation. Hence, monetary policy can also have a macroprudential role. Some research also explored the role of central banks as the lender of last resort. There are some arguments that solvent banks cannot be illiquid. Rochet and Vives (2004) showed that in some circumstances, solvent banks might not be able to acquire liquidity.

Allen, Carletti, et al. (2014) replaced the real deposit contracts in Diamond and Dybvig (1983) with the nominal ones. Entrepreneurs borrow from banks to purchase capital from consumers, which they then invest in both long and short assets. Hence, deposits are created after the loans are made. Consumers use these deposits in their accounts to buy goods, and reserves are the means of transactions in the interbank market. The result illustrated that nominal deposit contracts could lead to the social planner's solution, just as real deposit contracts. Rivero and Rodriguez (2024) revisited Diamond and Dybvig (1983) with the model that incorporates the nominal deposit and debt contracts and interbank market similar to Allen, Carletti, et al. (2014). However, entrepreneurs in this economy own production technology. They borrow from banks to pay the wages to workers. Unlike entrepreneurs in other models that have to choose how much short assets to hold, they can liquidate their technology early without restrictions, except for the total amount of technology that they have. The results from Rivero and Rodriguez (2024) contradict with Allen, Carletti, et al. (2014) as the nominal deposit contracts do not lead to the welfare maximized solution.

This paper contributes to the literature by introducing a solvency shock on top of the liquidity shock in a nominal economy with an interbank market and a central bank.

3 The Economy with the Liquidity and Solvency Risks

The model follows the structure of Rivero and Rodriguez (2024), which incorporates a nominal economy into Diamond and Dybvig (1983). There are three time periods which are $t = 0, 1, 2$. A circular economy with unit 1 consists of continuous location $j \in [0, 1]$. A location comprises one unit of entrepreneurs, one unit of workers, and one unit of representative commercial banks. There is only one central bank for the whole circular economy.

Commercial banks in every location face two types of idiosyncratic risks which are independent from each other; liquidity and solvency risks. Banks face liquidity risks as there is a chance that some agents in a certain location will be the early consumers. The type of preferences is denoted as $\tau \in \{1, 2\}$ according to the time of consumption. For any representative entrepreneur and worker in location j, there is a probability $\lambda(j)$, she will be impatient, preferring to consume in period 1 ($\tau = 1$). And with a probability $1 - \lambda(j)$, she will be patient, preferring to consume in period 2 ($\tau = 2$). $\lambda(j)$ is drawn from the distribution $\Phi(\lambda(j))$ with the support lying within the range $[0, 1]$ and the mean λ. The agent will get 0 utility for consuming in the period that which is not her type. To be specific, the utility $U(c_1, c_2)$ is defined as,

$$U(c_1, c_2) = \begin{cases} u(c_1) \text{ with probability } \lambda(j) \\ u(c_2) \text{ with probability } 1 - \lambda(j) \end{cases}$$

$u(c)$ takes the form of a standard utility function, which is twice differentiable, strictly increasing, and strictly concave. It also satisfies the Inada conditions with the coefficient of relative risk aversion higher than 1.

$$\lim_{z \to 0} u'(z) \to \infty \quad , \quad \lim_{z \to \infty} u'(z) \to 0 \quad , \quad -\frac{zu''(z)}{u'(z)} > 1.$$

I assume that the preference shocks happen equally to both workers and entrepreneurs.

Another risk introduced to this model is the solvency risk, stemming from the probability that some locations will face low production in period 2. At the beginning of the economy $t = 0$, an entrepreneur in location j holds one unit of divisible production technology while a worker in the same location holds indivisible labor input of unit 1. A combination of a unit of technology and a unit of labor yields the output of 2 consumption goods if the production happens at $t = 1$ and 2ρ if the production happens at $t = 2$ where ρ is a random variable. In period 2, all entrepreneurs in location j will be able to deliver the output per capita ρ as,

$$\rho = \begin{cases} R \text{ with probability } \pi \\ r \text{ with probability } 1 - \pi. \end{cases}$$

with $0 < r < 1 < R$. I define the type $\rho = R$ as the high-production and $\rho = r$ as the low-production. Let $\mathbb{E}\,\rho$ be the expected output per capita across locations, I presume that $\mathbb{E}\,\rho = \pi R + (1 - \pi)r > 1$.

3.1 The efficient allocation

Before exploring further banks' and agents' actions in the nominal economy, it is worth mentioning the planner's solution. The planner gathers all resources from both entrepreneurs and workers, then chooses how much technology should be liquidated in period 1, how much technology should be left to produce in period 2, and how much each worker and entrepreneur should consume.

By the law of large numbers, a fraction π of locations will draw the high return R, and a fraction of $1-\pi$ will draw the low return r. Since there is one unit of entrepreneurs in each location j, the gross output per capita will equal the expected output, which is $\mathbb{E}\,\rho = \pi R + (1-\pi)r$. Again, by the law of large numbers, the share of impatient agents for the whole economy is equal to the expectation, λ.

The planner chooses how much technology to liquidate, x, how much technology should be left for the production on period 2, and how much each agent consume to maximize the aggregate utility. Hence, the planner's problem is defined as,

$$\max_{x} \quad \lambda u(c_1) + (1-\lambda)u(c_2),$$

subject to the resource constraints

$$\begin{gathered} 2\lambda c_1 \leq 2x, \quad [t=1] \\ 2(1-\lambda)c_2 \leq 2\,\mathbb{E}\,\rho(1-x) + 2x - 2\lambda c_1, \quad [t=2] \\ 0 \leq x \leq 1. \end{gathered}$$

By the assumption that $\mathbb{E}\,\rho > 1$, it is not efficient for the planner to liquidate and store goods from period 1 to period 2. Hence, the storage, $2x - 2\lambda c_1$, equals zero. Solving the maximization problem, I have the optimal condition such that,

$$u'(c_1^*) = \mathbb{E}\,\rho u'(c_2^*) \tag{1}$$

3.2 Nominal Economy

I explain the nominal economy in this section. Figure 1 summarizes the agents in the model and their actions in each period. The main assumption for this nominal economy is that entrepreneurs in location j can only borrow from a bank in the same location while the workers can buy goods from any entrepreneurs. Banks can do transactions with other banks from other locations in the interbank market.

To produce consumption goods, entrepreneurs need to combine both technology and labor input. Money is used as the mean of payment, and entrepreneurs only own the production technology. With this demand to purchase labor input, the process of money creation begins.

Agents	t=0		t=1	t=2
			• Liquidity shock realized • Entrepreneurs and workers get the same probability of shock • The share of patient agents is λ(j) while the share of impatient agents is 1- λ(j)	• Solvency shock realized. • Get the same ρ for the whole location
commercial banks	• Lend to entrepreneurs $D_0(j)$		• Provide interest rate i_0^d to depositors • Receive the gains of deposits and reserves for the transactions that the entrepreneur in location *j* sell goods • Transfer deposits and reserves for transactions the workers buy food • The reserve position is $M_1(j)$	• Provide interest rate $i_1^d(j;\rho)$ to depositors • Collect the debt from entrepreneurs $(1+i^b)D_0(j)$ • Pay back the reserves with addition rate $(1+i^o)M_1(j)$ to the central bank if it is a borrower or receive $(1+i^o)M_1(j)$ if it is a net lender. • Settle the interbank transaction as in *t=1*
Entre-preneurs	• Borrow $D_0(j)$ from the bank in the same location j • Pay $D_0(j)$ to workers	Patient	• Liquidate *x(j;2)* to produce • Sell $y_1(j;2)$ into the market at the price P_1 • Save deposits $D_1(j;2)$ • Meet the bank's expected asset constraint	• Produce $2\rho(1-x(j;2))$ • Consume $z_2(j;2;\rho)$ • Sell unconsumed $y_2(j;2;\rho)$ into the market at the price P_2 • Have the debt $(1+i^b)D_0(j)$ to pay back to the bank.
		Impatient	• Liquidate *x(j;1)* to produce • Consume $z_1(j;1)$ • Sell unconsumed $y_1(j;1)$ into the market at the price P_1 • Save deposits $D_1(j;1)$ • Meet the bank's expected asset constraint	• Produce $2\rho(1-x(j;2))$ • Sell $y_2(j;1;\rho)$ into the market at the price P_2 • Have the debt $(1+i^b)D_0(j)$ to pay back to the bank.
workers	• Receive $D_0(j)$ as wage • Keep $D_0(j)$ in the bank accounts	Patient	• Withdraw $(1+i_0^d)D_0(j)$ to buy goods at the price P_1 • Consume $c_1(j;1)$	
		Impatient	• Continue saving $(1+i_0^d)D_0(j)$ in the banks	• Withdraw $(1+i_0^d)(1+i_1^d(j;\rho))D_0(j)$ to buy goods at the price P_2 • Consume $c_2(j;2;\rho)$
the central bank			• Provides reserves for the banks' settlement	• Collect the reserves debt with the policy rate $(1+i^o)M_1(j)$ if the commercial bank is the net reserve borrower and pay the interest rate $(1+i^o)M_1(j)$ if the commercial bank is the net lender.

Figure 1: Summary of the model

An entrepreneur asks for loans from a bank in the certain location. When the bank grants the nominal loan defined as $D_0(j)$, the deposit with the same number is generated in the entrepreneur's bank account, as illustrated in Figure 2. After the entrepreneur buys labor input from a worker in the same location, the deposit is transferred from the entrepreneur's account to the worker's. Deposits are used as the mean of payments between entrepreneurs and workers. Banks also accept deposits as the mean of paying debts.

Workers and entrepreneurs realize their types of preferences at the beginning of period 1. Workers who have carried deposits in their accounts from period 0 to 1 gain the interest rate i_0^d.

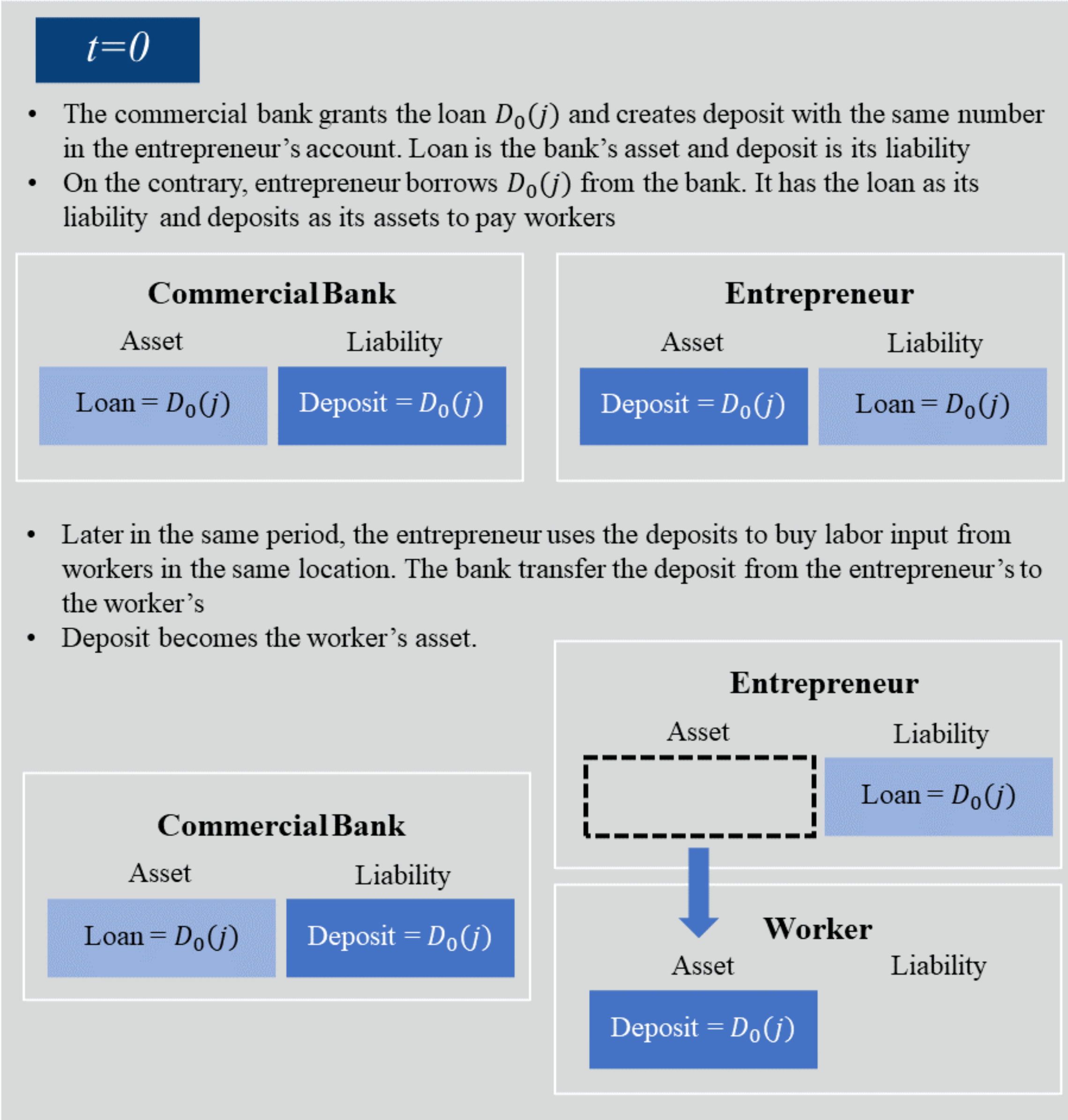

Figure 2: Balance sheets and settlements at $t = 0$

Hence, their total deposit equals $[1 + i_0^d]D_0(j)$. The deposit rate i_0^d is the same across locations as all banks face the same uncertainty at $t = 0$. The Impatient workers need to consume $c_1(j; 1)$. They withdraw their deposits to buy the consumption goods with the price P_1. The patient workers can continue saving their deposits in the bank accounts or buy goods and store them for the next period.

Entrepreneurs choose to liquidate their technology $x(j; \tau)$ according to their types. The goods from liquidation are either consumed or sold in the market. Impatient entrepreneurs consume $z_1(j; 1)$ while patient entrepreneurs do not.

Entrepreneurs accept deposits from workers as the mean of payment and save their income as the deposits $D_1(j;\tau)$. Suppose a worker from location k buys a good from an entrepreneur in location j; her bank in location k must transfer her deposit into the firm's account in location j, as shown in Figure 3.[1] The bank in location k will borrow the reserves from the central bank to use it as the mean of the interbank market's settlement. When it transfers the deposit of the worker to the firm's account, reserves are also transferred to the bank in location j. The bank of the worker is in the borrowing position to the central bank after the settlement. In practice, there is no need to borrow reserves every time a bank makes a payment to another bank. All payments are made, and it is the net interbank position that the central bank settles at the end of period 1.

The production shock is realized at the beginning of period 2. There are risks that banks in the low production area may not be able to collect all debts. I presume that the loss from bad loans will transfer to the deposit rates. Hence, there are two deposit rates provided to the depositors; $i_1^d(j;R)$ if the location gains high production and $i_1^d(j;r)$ if the location gains low production. The deposit rates $i_1^d(j;\rho)$ are location-specific as the number of impatient agents will affect how much the banks can pay. Both entrepreneurs face the market price P_2. Patient entrepreneurs sell the goods which are left from their consumption, while the impatient ones sell all of their goods. All entrepreneurs use the deposits from $t = 1$ and the deposits from selling goods at $t = 2$ to pay the debt and interest rate $(1+i^b)D_0(j)$ where i^b is the lending rate that is not location-specific. Banks gather their loans from the entrepreneurs, pay the debt to the central bank with the rate i^o if they are net reserve borrowers, and receive the gain i^o if they are the net lenders. i^o here is the monetary policy rate set by the central bank, namely, the rate that is used for reserve management.

3.3 Banks' risk management

Debts are fully paid in the models with no uncertainties in production or asset return. In this model, with the uncertainty in production, there is no guarantee that entrepreneurs will pay the full amount of debt. If the representative bank constructs the condition such that banks will take everything from entrepreneurs in case entrepreneurs are not able to pay the debt, only patient entrepreneurs will try to pay in order to prevent zero consumption. Without any further restrictions, impatient entrepreneurs will liquidate everything in period 1 and consume because banks' measures only affect consumption in period 2. For these reasons, the bank needs to set

1. Since there is an infinite continuum of locations, there is zero probability that a worker will buy goods from her location.

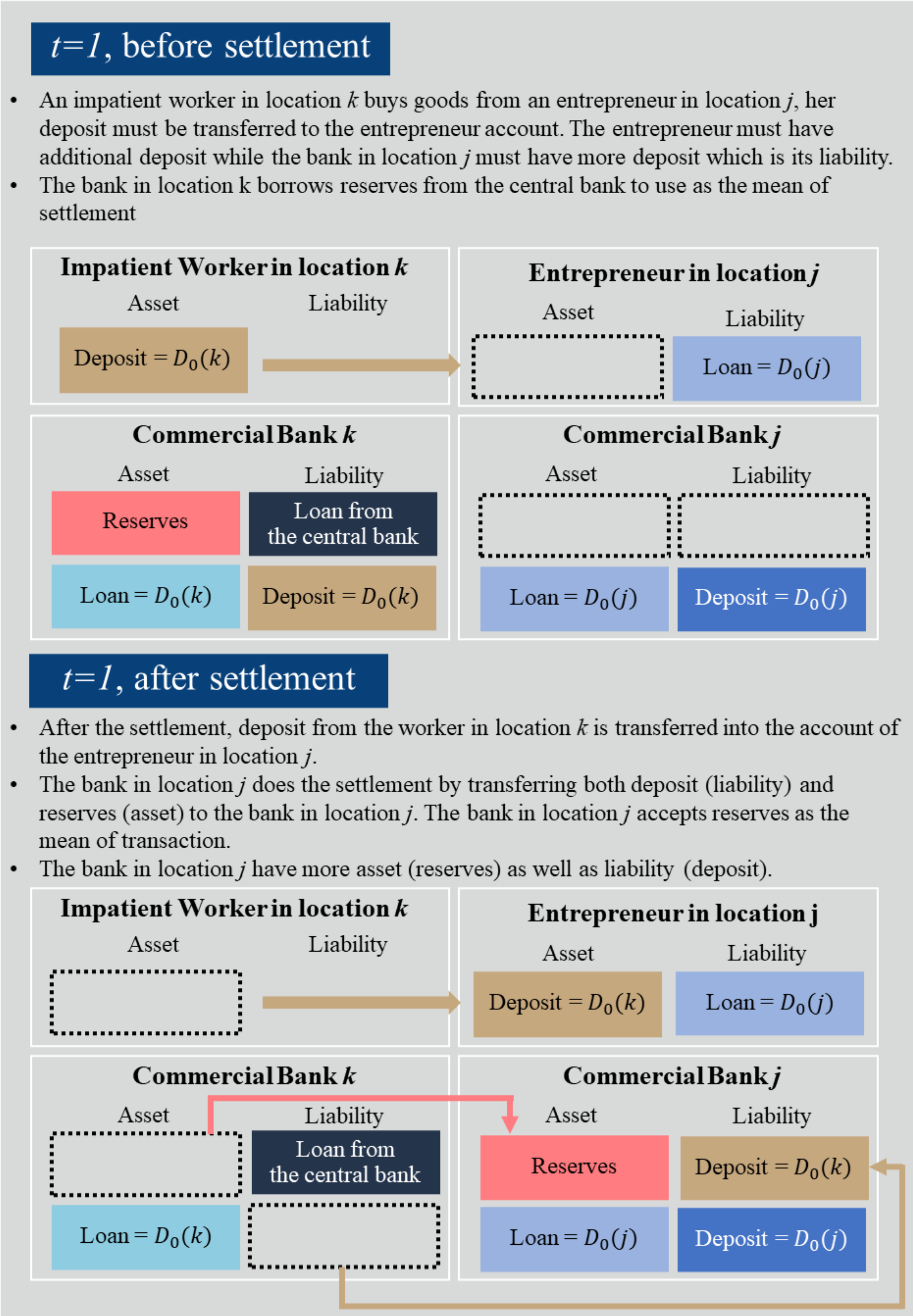

Figure 3: Balance sheets and settlements at $t = 1$

the requirement conditions from banks in both periods 1 and 2.

At $t = 1$, entrepreneurs gain deposits from selling goods.

$$D_1(j;\tau) = P_1\big[2x(j;\tau) - (2-\tau)z_1(j;\tau)\big] \tag{2}$$

Banks require every entrepreneur to meet the nominal expected-asset constraint at $t = 1$.

$$(1+i^b)D_0(j) \leq [1 + \mathbb{E}\, i_1^d(j;\rho)]D_1(j;\tau) + 2P_2\,\mathbb{E}\,\rho(1 - x(j;\tau)) \tag{3}$$

$\mathbb{E}\, i_1^d(j;\rho) = \pi i_1^d(j;R) + (1-\pi)i_1^d(j;r)$ is the expected interest rate paid at $t = 1$. Hence, $[1 + \mathbb{E}\, i_1^d(j;\rho)]D_1(j;\tau)$ is the expected value from saving in deposits. $2P_2\,\mathbb{E}\,\rho(1 - x(j;\tau))$ is the expected gain from saving in terms of real technology. Combining both nominal and real (technology) assets, the expected value should cover the debt that entrepreneurs have pay in at $t = 2$. If the entrepreneurs fail to meet the condition (3), the banks will take everything from them, leading the consumption to be zero.

In period 2, banks will take everything from the entrepreneur if they cannot pay the debt. I assume banks can track all assets of the entrepreneurs, so there is no entrepreneur trying to hide their asset in terms of technology. Entrepreneurs can negotiate with banks. Banks will take $(1-n)$ of the entrepreneur's total assets in the nominal value and let the entrepreneurs have n for consumption. n can be treated as the minimum consumption for entrepreneurs. It is assumed that n is small enough that the asset after banks take $(1-n)$ part is always smaller than the asset left after paying the full debt.

Suppose the nominal asset of an entrepreneur is 100 euros, consisting of 50 euros deposits and 50 euros of the priced value of the goods that have not yet been sold. In practice, the bank will not equally take $(1-n)50$ from the deposits and $(1-n)50$ of the goods as it operates only with nominal assets. Instead, it will take all the deposits and force the entrepreneur to sell goods until the deposits in the account and the inflow deposits from selling goods add up to $(1-n)100$ euros. The goods that are not forced to sell are for the entrepreneur's consumption.

I define $(1-m)$ as the fraction of goods forced to sell and m as what is left for entrepreneurs' consumption.

$$2(1-n)\Big[P_1[1 + 1_1^d(j;r)]x(j;\tau) + P_2 r(1 - x(j;\tau))\Big] = \\ 2P_1[1 + 1_1^d(j;r)]x(j;2) + 2P_2 r(1-m)(1 - x(j;\tau)) \tag{4}$$

The left-hand side (LHS) is $(1-n)$ of the total assets, and the right-hand side (RHS) is the total deposits, including the gains from selling goods.

Solving the equation yields,

$$(1-m) = (1-n) - \frac{(n)P_1[1+1_1^d(j;r)]x(j;\tau)}{P_2 r(1-x(j;\tau))} \tag{5}$$

This method of collecting debt is crucial for the reserves and interest rates calculations, which I will mention in the later part. Notice that impatient entrepreneurs will not negotiate since the consumption in period 2 does not matter to them. They will only try to meet the nominal expected-asset constraint (3). Additionally, only the impatient entrepreneurs will negotiate with the banks if they are not able to pay the dept.

3.4 Impatient entrepreneurs

An impatient entrepreneur in location j liquidates $x(j;1)$. She consumes $z_1(j;1)$ before selling the rest of the goods in the market. The output $y_1(j;1)$ sold to the market takes the value,

$$0 \leq y_1(j;1) \leq 2x(j;1) - z_1(j;1) \tag{6}$$

conditionally on its limited technology resources,

$$0 \leq x(j;1) \leq 1 \tag{7}$$

The deposits gained from selling goods is,

$$0 \leq D_1(j;1) \leq P_1 y_1(j;1) \tag{8}$$

Without any monitoring or restriction, she has incentives to liquidate all technology and consume all the goods in $t=1$. In this mode, impatient entrepreneurs are subject to meeting the banks' expected-asset constraint. Hence, the impatient entrepreneurs' problem is to choose how much to liquidate and maximize her consumption in period 1,

$$\max_{z_1(j;1),x(j;1)} u(z_1(j;1)) \tag{9}$$

subject to equations (6), (8), (7) , and the expected-asset constraint (3).

3.5 Patient entrepreneurs

A patient entrepreneur faces uncertainty in her consumption in period 2. In period 1, she can liquidate some of the technology and save in the form of deposits. Hence, she also faces similar constraints as the impatient entrepreneurs.

$$0 \leq y_1(j;2) \leq 2x(j;2) \tag{10}$$

$$0 \le x(j;1) \le 1 \tag{11}$$

$$0 \le D_1(j;2) \le P_1 y_1(j;2) \tag{12}$$

The production shock is realized at $t = 2$. The consumption equations of the patient entrepreneur with different types of production levels $z_2(j;2;\rho)$ are as the followings.

If the entrepreneur is in the high production area, she can consume what is left after paying debt $(1+i^b)D_0$

$$z_2(j;2;R) = \frac{P_1[1+i_1^d(j;R)]D_1(j;2) + 2P_2R(1-x(j;2)) - (1+i^b)D_0}{P_2} \tag{13}$$

And the output sold to the market is,

$$0 \le y_2(j;2;R) \le 2R(1-x(j;2)) - z_2(j;2;R) \tag{14}$$

If she is in the low production area and fails to pay the debt, she can consume what is left after the negotiation. And by equation (5)

$$\begin{aligned} z_2(j;2;r) &= \frac{n\Big(2P_1[1+i_1^d(j;r)]x(j;2) + 2P_2r(1-x(j;2))\Big)}{P_2} \\ &= 2mr(1-x(j;2)) \end{aligned} \tag{15}$$

And the output that is forced to sell by bank to the market is,

$$0 \le y_2(j;2;r) = 2(1-m)r(1-x(j;2)) \tag{16}$$

The only choice that the impatient entrepreneur can make is how much she should liquidate at $t = 1$ to maximize her expected utility.

$$\max_{x(j;2)} \quad \pi u(z_2(j;2;R)) + (1-\pi)u(z_2(j;2;r)) \tag{17}$$

subject to equations (10), (12), (11), (13) , (15), and the expected-asset constraint (3).

There are two main points to be noted. Firstly, the total assets in both deposits and in technology, in period 1 of patient entrepreneurs will be weakly higher than the impatient entrepreneurs' since they do not have to consume the liquidated good. Secondly, the preference and production uncertainties that are realized in different time periods make two types of entrepreneurs act differently and have different preferences even though they share the same utility function. The patient entrepreneurs maximize the utility at $t = 1$ and had to follow one restriction by the bank which is the expected-asset constraint (3). In contrast, patient entrepreneurs face uncertainty in production. They need to choose how much to liquidate at $t = 1$ to maximize their expected consumption.

3.6 Workers

At $t = 1$, an impatient worker in location j withdraws her deposits to buy consumption goods.

$$c_1(j;1) = \frac{[1+i_0^d]D_0(j)}{P_1} \tag{18}$$

A patient worker continues saving for another period. The return of deposits she receives depends on the production type of her location.

$$c_2(j;2;\rho) = \frac{[1+i_0^d][1+i_1^d(j;\rho)]D_0(j)}{P_2} \tag{19}$$

If patient workers expect a high number of loan losses in period 2, it is possible for patient workers to consider storing the goods from period 1 to period 2. This is because the high loan losses will lower the deposit rates, making the patient workers able to consume less. I will discuss storing and bank runs in the competitive equilibrium part.

3.7 Balance Sheets of Commercial banks

The net worth of the representative bank in location j is,

$$NW_t^B(j) = L_t(j) + M_t(j) - D_t(j) \tag{20}$$

where $L_t(j)$ is the loan provided and $D_t(j)$ is the deposits. $M_t(j)$ is the net reserve position which is negative when the bank is a net reserve borrower and positive when the bank is the a net lender.

3.7.1 Net Worth in Periods 0 and 1

All agents in every location are the same in period 0. Therefore, all banks provide the same amount of loans $D_0(j) = D_0$. With loans D_0 generated, the deposits of the same amount D_0 is also generated. There is no reserve borrowed or lent in this period. By construction, the net worth of period 0 is,

$$\begin{aligned} NW_0^B(j) &= D_0(j) + 0 - D_0(j) \\ &= 0 \end{aligned} \tag{21}$$

In period 1, the amount of loan is still the same, which is D_0 the bank provides interest rates i_0^d for deposits that were carried across periods. Entrepreneurs liquidate, sell goods in the market, and receive income in the form of deposits. The net deposits for the bank is,

$$D_1(j) = (1+i_0^d)D_0 - \lambda(j)P_1c_1(j;1) + \lambda(j)D_1(j;1) + [1-\lambda(j)]D_1(j;2) \tag{22}$$

$(1+i_0^d)D_0$ is the old deposits of the workers plus the interest rates. $\lambda(j)P_1c_1(j;1)$ is the outflow of the impatient workers who withdraw to buy goods. $\lambda(j)D_1(j;1)$ and $[1-\lambda(j)]D_1(j;2)$ are the deposit inflows from the impatient and patient entrepreneurs, respectively. In contrast to the balance sheet in period 0, the idiosyncratic preference shock $\lambda(j)$ starts to affect banks in different locations differently.

The reserves are the net settlement to other banks. If a worker withdraws and pays to an entrepreneur in a different location, the bank has to transfer the deposit using reserves as the mean of the transaction. On the contrary, if a worker from another location pays to the entrepreneur in location j, the bank will gain the flows of deposits and reserves. Hence, the reserves is,

$$M_1(j) = -\lambda(j)P1c_1(j;1) + \lambda(j)D_1(j;1) + [1-\lambda(j)]D_1(j;2) \tag{23}$$

If the reserve position is negative, it means there are more outflows than inflows. The bank is the net borrower to the central bank and vice versa.

The net worth of the bank in this period is,

$$\begin{aligned} NW_1^B(j) &= L_1(j) + M_1(j) - D_1(j) \\ &= D_0 - (1+i_0^d)D_0 \end{aligned} \tag{24}$$

I further assume that there is perfect competition and banks cannot make the rents from the market. Therefore, the net worth in every period must be 0. From equation (24), $i_0^d = 0$.

3.7.2 Loss realization and extra production

Before exploring net worth at $t = 2$, it is necessary to discuss how banks take losses into their balance sheets when the location reaches low production. On the asset side, it lowers the value it can recover from the loan, while on the liability side, the flow of deposits decreases to the same amount. Define $l(j;\tau)$ as the accounting losses banks take from the entrepreneurs of type τ. $(1+i^b-l(j;\tau))D_0$ should equal to the deposits that it can collect. For example, suppose that an impatient entrepreneur i is indebted for 120 euros, but all of her assets can pay only 100 euros. Her bank will take all the deposits in the account and the new deposits from selling goods. However, in the bank account, it will decrease the loan income to 100 euros and the loss $l(j;\tau)$ is 20 euros.

Impatient entrepreneurs do no not need to negotiate with the bank. The bank can force them to sell all of the production goods and take all the entrepreneurs' deposits. Equation (25) shows that when the location reaches low production, the low deposit rate also decreases the value of deposits that the entrepreneurs hold.

$$\begin{aligned}(1+i^b-l(j;1))D_0 &= [1+i_1^d(j;r)]D_1(j;1)+2P_2r(1-x(j;1)) \\ &= [1+i_1^d(j;r)]D_1(j;1)+P_2y_2(j;1;r)\end{aligned} \tag{25}$$

where $y_2(j;1;r) = 2r(1-x(j;1))$ is the goods that the impatient entrepreneurs sell to the market in period 2 when they have low production.

Patient entrepreneurs negotiate, and banks only force them to sell some amount of consumption goods.

$$\begin{aligned}(1+i^b-l(j;2))D_0 &= [1+i_1^d(j;r)]D_1(j;2)+2P_2r(1-m)(1-x(j;2)) \\ &= [1+i_1^d(j;r)]D_1(j;2)+P_2y_2(j;2;r)\end{aligned} \tag{26}$$

where $(1-m)$ takes the value as in equation (5).

In high-production locations, banks can collect their debts. However, there is extra production from the impatient entrepreneurs. This is because, at $t=1$, they are required to hold enough expected assets to pay the debt. The expected constraint (3) binds with equality since there is no additional gain from decreasing the consumption at $t=1$ to have more assets in $t=2$.

Suppose there exists the optimal liquidation $x^*(j;1)$, the optimal consumption at $t=1$, $z_1^*(j;1)$, and the optimal deposits $D_1^*(j;1)$ satisfying the impatient entrepreneur problem; from the expected-asset constraint (3), the value of loans in period 2 $(1+i^b)D_0$ equals to the expected value of the expected assets, as shown in equation (27).

$$(1+i^b)D_0 = P_1[1+\mathbb{E}\, i_1^d(j;\rho)]\big[2x^*(j;1)-z_1^*(j;1)\big]+2P_2\,\mathbb{E}\,\rho(1-x^*(j;1)) \tag{27}$$

When the location achieves high production, the deposit rate paid will be higher than the expected rate, $i_1^d(j;R) > \mathbb{E}\, i_1^d(j;\rho)$ and the production per capital will be higher than the expected value, the $R > \mathbb{E}\,\rho$ From equation (27) with the same value of $x^*(j;1)$, $z_1^*(j;1)$, and $D_1^*(j;1)$, the ex-post assets of the impatient entrepreneurs will be larger than the debts that they need to pay.

$$\begin{aligned}(1+i^b)D_0 &= P_1[1+\mathbb{E}\, i_1^d(j;\rho)]\big[2x^*(j;1)-z_1^*(j;1)\big]+2P_2\,\mathbb{E}\,\rho(1-x^*(j;1)) \\ &< P_1[1+i_1^d(j;1)]D_1^*(j;1)+2P_2R(1-x^*(j;1))\end{aligned} \tag{28}$$

In this setup, the bank will take deposits from the impatient entrepreneurs up to $(1+i^b)D_0$ and leave the rest $W(j;1;R)$ in the entrepreneurs' accounts. This money is left for nothing as I assume that the banks cannot collect more than $(1+i^b)D_0$.

$$\begin{aligned}(1+i^b)D_0 &= P_1[1+i_1^d(j;1)]D_1^*(j;1) + 2P_2R(1-x^*(j;1)) - W(j;1;R)\\ &= P_1[1+i_1^d(j;1)]D_1^*(j;1) + P_2y_2(j;1;R) - W(j;1;R)\end{aligned} \tag{29}$$

3.7.3 Net Worth in period 2 and the Deposit Rates

At $t=2$, loans take different values with different states of production. For the banks in high-production locations, they can collect the loans as the following.

$$L_2(j;R) = (1+i^b)D_0 \tag{30}$$

In the low-production locations, the banks take the losses into their account.

$$L_2(j;r) = \lambda(j)(1+i^b-l(j;1))D_0 + (1-\lambda(j)((1+i^b-l(j;2))D_0 \tag{31}$$

$$= (1+i^b)D_0 - \Big(\lambda(j)l(j;1) + (1-\lambda(j)l(j;2)\Big)D_0 \tag{32}$$

Regarding the deposits, the bank has to pay deposit rate $i_1^d(j;\rho)$ to all deposit holders. Patient entrepreneurs will withdraw their deposits to buy goods with the value $P_2c_2(j;2;\rho)$. All entrepreneurs in the high-production areas use deposits to pay all their debts. With low production, the entrepreneurs cannot pay all the debts. They will pay as in equations (25) and (26). Hence, the deposits with different levels of production are,

$$\begin{aligned}D_2(j;R) = (1-\lambda(j))&\Big[(1+i_0^d)(1+i_1^d(j;R))D_0 - P_2c_2(j;2;R)\Big]\\ &+\lambda(j)\Big[(1+i_1^d(j;R))D_1(j;1) + P_2y_2(j;1;R) - W(j;1;R)\Big]\\ &+(1-\lambda(j))\Big[(1+i_1^d(j;R))D_1(j;2) + P_2y_2(j;2;R)\Big]\end{aligned} \tag{33}$$

$$\begin{aligned}D_2(j;r) = (1-\lambda(j))&\Big[(1+i_0^d)(1+i_1^d(j;r))D_0 - P_2c_2(j;2;r)\Big]\\ &+\lambda(j)\Big[(1+i_1^d(j;r))D_1(j;1) + P_2y_2(j;1;r)\Big]\\ &+(1-\lambda(j))\Big[(1+i_1^d(j;r))D_1(j;2) + P_2y_2(j;2;r)\Big]\end{aligned} \tag{34}$$

In both equations (33) and (34), the first term refers to the value of deposits that patient workers have carried from period 1 to 2 deducted by how much they withdraw for consumption.

The second and the third terms represent the deposit of impatient and patient entrepreneurs, respectively.

Banks hold the reserves $M_1(j)$ from $t = 1$ with the interest rate charged or paid by the central bank $(1 + i^o)$. Hence, the reserves that the banks in the high-production locations are,

$$\begin{aligned} M_2(j;R) = &(1+i^o)M_1(j) - (1-\lambda(j))P_2c_2(j;2;R) \\ &+ \lambda(j)\Big[P_2y_2(j;1;R) - W(j;1;R)\Big] \\ &+ (1-\lambda(j))\Big[P_2y_2(j;2;R)\Big] \end{aligned} \tag{35}$$

And in the low-production locations,

$$\begin{aligned} M_2(j;r) = &(1+i^o)M_1(j) - (1-\lambda(j))P_2c_2(j;2;r) \\ &+ \lambda(j)\Big[P_2y_2(j;1;r)\Big] \\ &+ (1-\lambda(j))\Big[P_2y_2(j;2;r)\Big] \end{aligned} \tag{36}$$

Hence, the bank's net worth at $t = 2$ for the high production is,

$$\begin{aligned} NW_2^B(j;R) &= L_2(j) + M_2(j) - D_2(j;R) \\ &= \Big[1 + i^b - (1+i_0^d)(1+i_1^d(j;R))\Big]D_0 \\ &\quad - \lambda(j)\Big[1 + i^o - (1+i_0^d)(1+i_1^d(j;R))\Big]D_0 \\ &\quad + \lambda(j)[i^o - i_1^d(j;R)]D_1(j;1) + (1-\lambda(j))[i^o - i_1^d(j;R)]D_1(j;2) \end{aligned} \tag{37}$$

which is the same as in Rivero and Rodriguez (2024). The first term is the revenues from generating loans. The other terms are the costs and the revenues associated with the banks' transactions with the central bank.

Net worth for the low production are almost similar, but with the losses from entrepreneurs that are not able to pay the debts.

$$\begin{aligned} NW_2^B(j;R) = &L_2(j) + M_2(j) - D_2(j;r) \\ = &\Big[1 + i^b - (1+i_0^d)(1+i_1^d(j;r))\Big]D_0 - \Big[\lambda(j)l(j;1) + (1-\lambda(j)l(j;2)\Big]D_0 \\ &- \lambda(j)\Big[1 + i^o - (1+i_0^d)(1+i_1^d(j;R))\Big]D_0 \\ &+ \lambda(j)[i^o - i_1^d(j;r)]D_1(j;1) + (1-\lambda(j))[i^o - i_1^d(j;r)]D_1(j;2) \end{aligned} \tag{38}$$

With perfect competition and equating the net worth to 0, the deposit rates for the high and low-production locations are,

$$i_1^d(j;R) = i^o + \frac{(i^b - i^o)D_0}{[1-\lambda(j)]D_0 + \lambda(j)D_1(j;1)) + [1-\lambda(j)]D_1(j;2)} \tag{39}$$

$$i_1^d(j;r) = i^o + \frac{\left[i^b - i^o - \left(\lambda(j)l(j;1) + (1-\lambda(j)l(j;2)\right)\right]D_0}{[1-\lambda(j)]D_0 + \lambda(j)D_1(j;1)) + [1-\lambda(j)]D_1(j;2)} \tag{40}$$

The losses make the interest rate weakly lower if the location reaches low production. Allen, Carletti, et al. (2014) studied the idiosyncratic preference shock and the production (in their paper, asset returns) shock separately. In this model, I combine both shocks and find that the idiosyncratic shock also has an impact on the losses from production shock. How much the loss from low production will also depend on the number of impatient agents in that location.

3.8 The Central Bank

The central bank provides reserves for banks to use as the mean of transaction in the interbank market. It also charges the monetary policy rate to those who are the net reserve borrowers. At $t = 1$, banks are still not subject to high and low production. Therefore, banks are only different to each other by the number of impatient agents in their location. The central bank balance sheet at $t = 1$ equals,

$$NW_1^{CB} = \int_0^1 M_1(j)d\Phi(\lambda(j)) \tag{41}$$

And in the second period, it gains the interest rate.

$$NW_2^{CB} = (1+i^b)\int_0^1 M_1(j)d\Phi(\lambda(j)) \tag{42}$$

4 Competitive Equilibrium and the Monte Carlo Experiment

Equations (39) and (40) are the most important equations in this study as they illustrate that the production losses deteriorate the interest rate and the gain from holding deposits. In the literature without uncertainty in period 1 (Diamond and Dybvig 1983; Allen and Gale 2004a; Rivero and Rodriguez 2024), a bank run happens when patient workers expect that $c_1 > c_2$. In this model with production uncertainty, bank runs occur when the value of withdrawing at

$t = 1$ and storing the good for one period is higher than the expected utility of continuing saving in deposits and accepting the risk of receiving low interest rates.

$$u\left(\frac{D_0}{P_1}\right) \geq \pi u\left(\frac{[1+i_1^d(j;R)]D_0}{P_2}\right) + (1-\pi)u\left(\frac{[1+i_1^d(j;r)]D_0}{P_2}\right) \tag{43}$$

The LHS is the utility from buying good in period 1 and storing, while the RHS is the expected utility from saving in terms of deposits. When a patient worker realizes $\lambda(j)$ and see that her expected utility is lower than the utility from storing, she will withdraw from the bank store the goods.

To make the reference easier, I define $\phi(j)$ as the difference between the expected utility from saving in the deposits and the utility from withdrawing at $t = 1$ and storing the good. Locations with $\phi(j) < 0$ are facing bank runs.

$$\phi(j) = \pi u\left(\frac{[1+i_1^d(j;R)]D_0}{P_2}\right) + (1-\pi)u\left(\frac{[1+i_1^d(j;r)]D_0}{P_2}\right) - u\left(\frac{D_0}{P_1}\right) \tag{44}$$

The value of $\phi(j)$ depends on the prices in the competitive equilibrium as well as other parameters, such as the CRRA coefficients. The prices are also subject to which locations received the low production shock. It is almost impossible to find analytical solutions or inferences. One solution is to simulate the economy and draw the shock for each location. Nevertheless, one round of shock drawing still makes the equilibrium subject to the specific locations that have low production. For these reasons, I construct a Monte Carlo experiment by simulating the economy and drawing the shock for many rounds. By the law of large numbers, the realized frequencies of shock for each location should converge to the probability. Hence, I should be able to make some inferences. The algorithm to solve for the competitive equilibrium is discussed in the appendix.

The competitive equilibrium used in the algorithm is defined as the followings,

Definition 4.1 (Competitive Equilibrium)**.** The competitive equilibrium is, $\forall j$ and $\forall \rho$

- The workers' consumption allocation $\boldsymbol{w}^w = \{\boldsymbol{c_1(j;\tau)}, \boldsymbol{c_2(j;\tau;\rho)}\}$
- The entrepreneurs' allocation $\boldsymbol{w}^f = \{\boldsymbol{z_1(j;\tau)}, \boldsymbol{z_2(j;\tau;\rho)}, \boldsymbol{y_1(j;\tau)}, \boldsymbol{y_2(j;\tau;\rho)}, \boldsymbol{x(j;\tau)}\}$
- The nominal allocation $\boldsymbol{w}^n = \{\boldsymbol{D_0(j)}, \boldsymbol{D_1(j;\tau)}, \boldsymbol{M_1(j)}, \boldsymbol{M_2(j)}\}$
- The price $\boldsymbol{w}^p = \{P_1, P_2\}$
- The market interest rates $\boldsymbol{w}^i = \{\boldsymbol{i_0^d}, \boldsymbol{i_1^d(j;\rho)}, i^b\}$

- The monetary policy i^o

such that

- Given $\boldsymbol{w^p}$, $\boldsymbol{i_0^d}$, $i_1^d(j;\rho)$, and $\boldsymbol{D_0(j)}$; $\boldsymbol{w^w}$ satisfies workers' problem.
- Given $\boldsymbol{w^p}$ and $\boldsymbol{w^i}$; $\boldsymbol{w^f}$ satisfies entrepreneurs' problem.
- Given $\boldsymbol{w^p}$, $\boldsymbol{w^n}$, $\boldsymbol{w^i}$, and i^o ; The banks' net worth is zero $\forall t = 0, 1, 2$.
- Let $\mathbb{1}$ be the index function, all good markets clear

$$\int_0^1 \big[y_1(j;1) + y_1(j;2) - c_1(j;1)\big] d\Phi(\lambda(j)) = 0 \tag{45}$$

$$\int_0^1 \left(\mathbb{1}\{\rho = R\} \sum_{\tau=\{1,2\}} y_2(j;\tau;R) + \mathbb{1}\{\rho = r\} \sum_{\tau=\{1,2\}} y_2(j;\tau;r)\right) d\Phi(\lambda(j))$$
$$= \int_0^1 \left(\mathbb{1}\{\rho = R\}c_2(j;2;R) + \mathbb{1}\{\rho = r\}c_2(j;2;r)\right) d\Phi(\lambda(j)) \tag{46}$$

The utility function in the Monte Carlo experiment takes the form as,

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma} \tag{47}$$

where γ is the constant relative risk aversion (CRRA) coefficients

4.1 Results from the Monte Carlo Experiment

In this session, I discuss the results of the Monte Carlo experiment, which satisfy the definition of the competitive equilibrium (4.1). I focus on changing two parameters which are CRRA coefficient and the π, probability of receiving low production. The details about parameters that remain the same across the experiment are described in Table (1). It is worth noting that all of the cases illustrated have $\mathbb{E}\,\rho > 1$, implying that there is no storing the planner's solutions and it is optimal to produce at $t = 2$.

Figure (4) shows the results from the different CRRA coefficients, namely, the economies with different levels of risk aversions. One variable that I focus on is $\phi(j)$, which is on the top-left corner. The x-axis represents locations and $\lambda(j)$. From equation (44), a location is facing a bank run if $\phi(j)$ is below zero.

It can be counterintuitive that $\phi(j)$ is increasing in $\lambda(j)$, across different CRRA coefficients. I can draw the interpretation from two reasons.

Parameter	Description	Value
R	high production level	1.4
r	low production level	0.3
$\mathbb{E}\,\rho$	expected per capita output at $t = 2$	1.29
λ	expectation of $\lambda(j)$	0.5
n	share of assets the entrepreneurs can negotiate with banks	0.05
i^o	the monetary policy rate	0.05

Table 1: Details about parameters

Firstly, there are more patient workers to run the banks. Unlike impatient workers who have already consume in period 1, these workers have to monitor how much the interest rate their banks can pay in period 2. The second reason is the degree of risk aversion. A location with a higher number of impatient agents has a higher deposit rate in period 1 when there is high production and a lower deposit rate where there is low production. In locations with larger liquidity shocks, the utility gains from high production can compensate for the utility. This is not the same for locations with more patient consumers.

I also examine the chance of bank runs when the economy has a lower probability of high production, namely, when there is a higher widespread of low production. Figure 5 shows the result when I lower π from 0.85 to 0.9. It revealed that the economy is sensitive to higher probability of production losses. With more locations facing low production, the banks can provide lower deposit rates to the depositors. The low return on deposits also makes patient entrepreneurs save lower in terms of deposits. When more entrepreneurs save on technology, the banks are more prone to production losses. This is feed-backed to the deposit rates, making the rates lower.

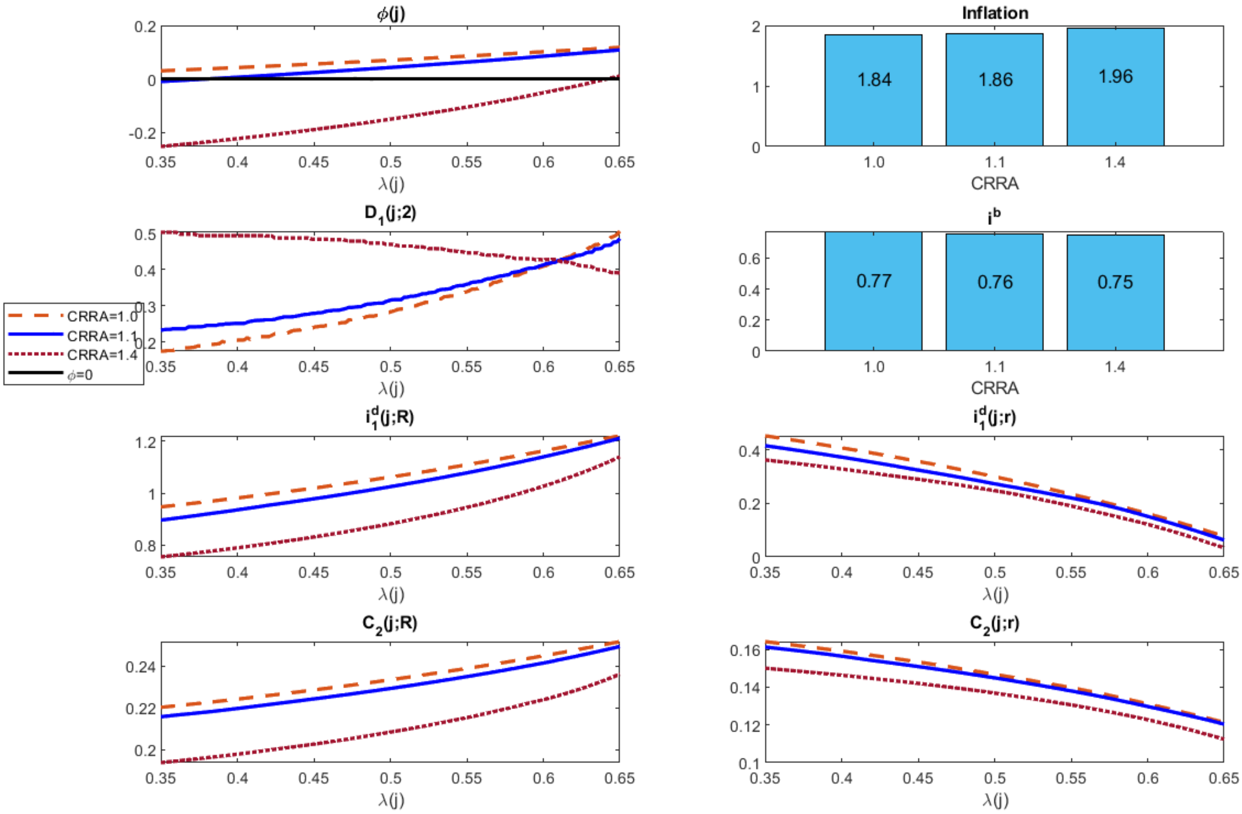

Figure 4: Allocations with different CRRA coeficients

Note: All lines and graphs are constructed from the median results of 50 draws

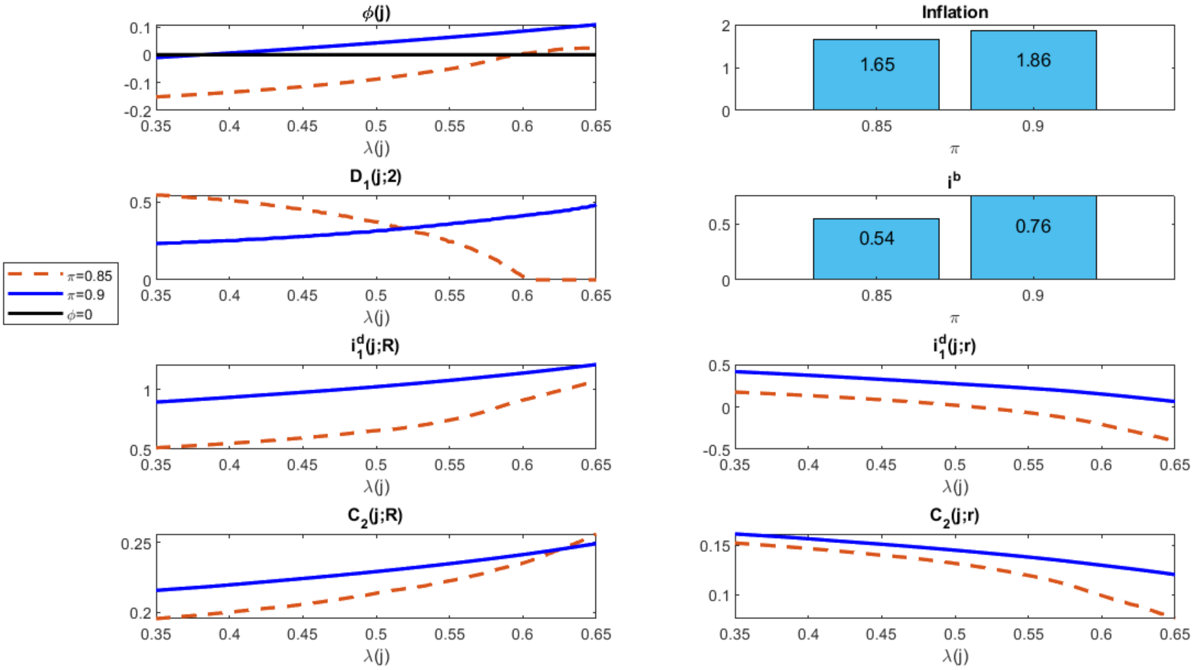

Figure 5: Allocations with different probability of high production

Note: All lines and graphs are constructed from the median results of 50 draws

5 Conclusions

In this model, I examine the risks of bank run when the solvency and liquidity shocks interact in the context of the modern banking system. When the production shock is absent, leaving only the preference shock, the reserve system managed by the central bank can maintain economic stability by supplying liquidity (Allen, Carletti, et al. 2014; Rivero and Rodriguez 2024). There is no bank run, and the prices will absorb the shock. The nominal variables, such as the number of loans provided to each bank, the monetary policy rate, or the lending rate, do not have a real impact on the economy. When the economy experiences a combination of locational shock and idiosyncratic preference shocks, the number of impatient workers affects the deposit rates paid by the bank of that particular location. There is a probability of bank runs, especially when highly risk-averse agents populate the economy or when many locations suffer from widespread low production.

Though the conclusions of this model are crucial, this study has some limitations. Firstly, it assumes that workers can only hold deposits which are nominal assets. In reality, a worker can also invest in technology or other types of assets. Incorporating the models of a family consisting of workers and managers can provide more insight (Lucas 1978; Guner, Ventura, and Xu 2008). Secondly, it has yet tested the cases in which banks are more optimistic or pessimistic about the shocks. Having more prudent banks that require entrepreneurs to hold enough assets even during the low-production time can stabilize the economy too. Such aspects are open to future studies.

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Appendices

A Aggregate Preference Shock

In this session, I briefly discuss the aggregate preference shock from Allen and Gale (2004a). Suppose now there is no uncertainty in production, every location can obtain output per capita R. There is an aggregate uncertainty in period 0 that total number of impatient in the economy is,

$$\mathbb{E}\,\lambda(j) = \sum_j \lambda(j) = \begin{cases} \lambda^H \text{ with probability } \gamma \\ \lambda^L \text{ with probability } 1-\gamma \end{cases}$$

At $t = 1$, every location faces the high and low states all the same. The only decision thank banks make at $t = 0$ is how much to provide $D_0(j)$. Since every agent starts similarly, $D_0(j) = D_0$.

The goal of this exercise is to explore whether states of preference shock will lead to different interest payments. Notice that the only decision thank banks make at $t = 0$ is how much to provide $D_0(j)$. Since every agent starts similarly, $D_0(j) = D_0$. I assume that with the same number of D_0, the states of the economy λ^s for $s \in \{H, L\}$ will affect the deposit rate paid to agents in $t = 2$.

I mainly focus on banks' balance sheets as banks have to deal with the different numbers of impatient agents.

By construction as in equation (24), the net worth of bank j at $t = 0$ is zero.

At $t = 1$, the deposit in state s is,

$$D_1(j;s) = (1 + i_0^d(j;s))D_0 - \lambda(j)P^s 1 c_1(j;1;s) + \lambda^s(j)D_1(j;1;s) + [1 - \lambda^s(j)]D_1(j;2;s) \quad (48)$$

The reserves position is,

$$M_1(j;s) = -\lambda(j)P^s 1 c_1(j;1;s) + \lambda^s(j)D_1(j;1;s) + [1 - \lambda^s(j)]D_1(j;2;s) \quad (49)$$

With the net worth equation (20) and perfect competition, I have that $i_0^d(j;s) = 0 \ \forall j$ and $\forall s$. Hence, the different states of preferences do not cause discrepancies in the interest rate at $t = 1$.

Next, I consider the interest rate provided at $t = 0$. The deposit that the bank has is,

$$\begin{aligned} D_2(j;s) = (1 - \lambda^s(j)) & \Big[(1 + i_0^d(j;s))(1 + i_1^d(j;s))D_0 - P_2^s c_2(j;2;s)\Big] \\ & + \lambda^s(j)\Big[(1 + i_1^d(j;s))D_1(j;1;s) + P^s y_2(j;1;s)\Big] \\ & + (1 - \lambda^s(j))\Big[(1 + i_1^d(j;s))D_1(j;2;s) + P^s y_2(j;2;s)\Big] \end{aligned} \quad (50)$$

And the loan and the reserve positions are,

$$L_2(j;s) = (1 + i^b)D_0 \quad (51)$$

$$\begin{aligned} M_2(j;s) = (1 + i^o)M_1(j;s) & - (1 - \lambda^s(j))P_2^s c_2(j;2;s) \\ & + \lambda^s(j)P^s y_2(j;1;s) \\ & + (1 - \lambda^s(j))(1 + i_1^d(j;s))D_1(j;2;s) + P^s y_2(j;2;s) \end{aligned} \quad (52)$$

$$i_j^d(j;s) = i^o + \frac{(i^b - i^o)D_0}{[1-\lambda^s(j)]D_0 + \lambda(j)D_1(j;1;s)) + [1-\lambda^s(j)]D_1(j;2;s)} \tag{53}$$

With the same reasoning in Rivero and Rodriguez (2024), the market-clearing price is

$$(1 + i_j^d(j;s))\frac{P_1}{P_2} = (1 + i^o)\frac{P_1}{P_2} = (1 + i^b)\frac{P_1}{P_2} = R \tag{54}$$

If $(1+i_j^d(j;s))\frac{P_1}{P_2} > R$, the return on the deposits wil be higher than the return from saving in technology. Hence, no entrepreneur will produce in the second period. On the other hand, no entrepreneur will liquidate in period 1 if $(1+i_j^d(j;s))\frac{P_1}{P_2} < R$. Since i^o is a parameter, I have that all deposit rates are equal across locations and stats.

The aggregate demand shock in the nominal economy does not cause price fluctuations or bank runs, coninciding with Allen, Carletti, et al. (2014). In this model, a central bank acts as the lender of the last resort and entrepreneurs with no limitation in liquidation. This also emphasizes how the monetary policy can tackle demand shocks and create stability.

B The Algorithm for the Mone Carlo Experiment

I used Matlab for computation. All the codes to recalibrate every result can be found in Code-for-Proud-Thesis

At first, I generate

1. A circular economy, making a linear space of 150 locations (J). More numbers would make the results accurate but will take a longer time to iterate the deposit rates
2. I set up the $\lambda(j)$ for each location. Theoretically, it should be drawn. But since two shocks are independent, I can fix the $\lambda(j)$ to be able to plot good graphs.
3. Set seed, and draw 50xJ matrix from a uniform distribution. J is for location and 50 is the number of rounds of the Monte Carlo experiment. If the value is less that π, then the location achieve the high production. The first row refers to the first draw for 50 locations in the economy.

Then for each round of the Monte Carlo experiment, denoted as $p = 1, 2, ..., 50$ (In the code I use t)

1. The outer loop, guess parameter $\theta_0 = [P_1, P_2, i^b]$

2. The inner loop 1, for each guess of θ_0,
   (a) Guess $i_1^d(j;R)$ and $i_1^d(j;r)$ for every location.
   (b) The inner loop 2, for every location $j = 1, 2, .., 150$
      i. Solve the entrepreneurs problems, define the deposits from the entrepreneurs $D_1(j;1)$ and $D_1(j;2)$.
      ii. Calculate the loss if there is low production.
      iii. Use equations (39) and (40) to update the interest rate of locatiion j.
   (c) Gather the vector of new $i_1^d(j;R)$ and $i_1^d(j;r)$, using the smoothing method.
   (d) Calculate the norms between the old and the new guess
   (e) Update the guess until the norm is lower than the tolerance.
3. Calculate output and demand for goods in period 1 and 2.
4. Generate the quadratic loss-function from excess or shortage demand using market-clearing equations (45) and (46). Namely, the loss is the excess or shortage demand that makes the market-clearing conditions do not hold.
5. Use Matlab's finmincon to find $\theta = [P_1^*, P_2^*, i^{b*}]$ that minimize the loss function, namely minimize excess or shortage demand for both markets.

From the market clearing conditions (45) and (46), I define the excess/shortage demand for good markets in period 1 and 2 as,

$$XD_1 = \int_0^1 \big[y_1(j;1) + y_1(j;2) - c_1(j;1)\big] d\Phi(\lambda(j)) = 0 \tag{55}$$

$$\begin{aligned} XD_2 = \int_0^1 \left(\mathbb{1}\{\rho = R\} \sum_{\tau=\{1,2\}} y_2(j;\tau;R) + \mathbb{1}\{\rho = r\} \sum_{\tau=\{1,2\}} y_2(j;\tau;r) \right) d\Phi(\lambda(j)) \\ - \int_0^1 \left(\mathbb{1}\{\rho = R\} c_2(j;2;R) + \mathbb{1}\{\rho = r\} c_2(j;2;r) \right) d\Phi(\lambda(j)) \end{aligned} \tag{56}$$

The lost function takes the quadratic form,

$$L = 100(XD_1^2 + XD_2^2) \tag{57}$$

I multiply the number by 100 as Matlab is not good with a very small number.