Intertwining Inequality and Labor Market under the New Normal

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September 4, 2015*

Abstract

This paper builds on a life cycle model of occupational choices and financial frictions to understand the main channel through which demography and inequality influence the economy. Based on household data from Thailand, younger cohorts are likely to be workers and older cohorts are likely to be entrepreneurs due to age-dependent skills and asset accumulation. Under the new normal faced by the Thai economy as well as others, aging population can lower overall total factor productivity and increase inequality. An increase in equilibrium wage due to shortage of labor supply drives mediocre entrepreneurs to become self-employed – a low-income and low-productivity occupation – and worsens total factor productivity and hence inequality. Moreover, a decline in world interest rates associated with global aging population will exacerbate this negative effect. Reducing financial frictions or alleviating a borrowing constraint of talented entrepreneurs can mitigate this effect while extending retirement age will only improve output per capita while total factor productivity and inequality worsen.

Keywords: Demographic Trends, Occupational Choice, Inequality, Life Cycle Models, Financial Frictions

JEL classification: J11, J24, E24, D91, O15

*Acknowledgements: We thank for helpful comments from participants at the PIER Research Exchange, Piti Disyatat, Surach Tanboon, Nakarin Amarase, Krislert Samphantharak, and Talublugkhana Thanadhidsuwanna. All the remaining imperfections are the exclusive responsibility of the authors.
1 Introduction

Aging population is inevitably set to impact labor market outcomes. One widely discussed implication from aging population is falling labor force participation and consequentially slower economic growth. However, the impacts of changing age structures on occupational compositions have been understudied. Occupations are likely to differ by age.\(^1\) Most of younger cohorts are wage workers whereas most older ones are likely to be entrepreneurs. Given this pattern of occupational choice over a life cycle, labor market will consist of more entrepreneurs as older cohorts are growing. Coupled with a labor shortage of young workers, this will result in heightened competition for workers.

This paper investigates the effects of aging population on labor market structures and its implications on output and inequality through both direct and indirect channels. Firstly, demographic changes directly alter the labor force participation rate and age compositions in the labor market. The labor force will consist of more older cohorts who are mostly entrepreneurs. Secondly, changes in wage due to the direct effect may alter occupational compositions within each age cohort. This indirect effect of aging population comes from endogenous occupational choices made by individuals over the life cycle. Due to falling labor participation of young cohorts, the shortage for labor will put pressures on employers with higher wages. Some employers then switch to become self-employed or workers. Therefore, the paper considers three occupational choices: workers, employers and self-employed. Entrepreneurs refer to being either employers or self-employment.

In fact, the three occupational choices are well reflected by labor market structures in many developing countries (Gollin, 2008; Allub and Erosa, 2014). In Thailand, self-employment accounts for almost 35% of total labor force in 2013 and is mostly found in the agricultural and trade sectors.\(^2\) Among entrepreneurs, self-employed are distinguished from employers by not having employees. Self-employed are unpaid family workers and own-account workers.

\(^1\)For instance, Barrientos et al. (2003) show that self-employment rises with age whereas formal employment opportunities decline with age.

\(^2\)As shown in Figure (19) in Appendix A
This distinction between employers and self-employed highlights the fact that entrepreneurs are vastly heterogenous in their skills, asset holdings and ultimately income levels (Poschke, 2013). The Thai data observe a lower level of average income for self-employed and a high income inequality.\(^3\) As occupational choices and their respective income differ over the life cycle, the changing occupational composition will likely affect overall income and inequality.\(^4\)

In answering the question on how labor market and inequality are intertwined, this paper considers a life cycle model of occupational choices and financial frictions based on that of Allub and Erosa (2014). The model aims to understand life cycle patterns of occupational choices and quantitatively evaluate the impact of aging population on occupational decisions, output, productivity, and inequality. To capture the diverse entrepreneurs especially the self-employed, individuals are heterogeneous in their skills and asset holdings according to which they choose to be wage workers or entrepreneurs. Entrepreneurs are subject to a collateral constraint as their ability to borrow is limited by their wealth. This type of financial frictions is widely observed in developing countries due to limited contract enforcement and poor credit information systems. The setting is a general equilibrium model in a small open economy where wage is endogenous and the real interest rate is exogenously given.

The model is calibrated on Thailand’s household and macro aggregate data during 2006-2013. We then simulate the effects of changing demographics in 2015-2045 during the transition of the model economy. The model projections aim to see how aging population affects output, productivity, inequality and occupational compositions. With aging population, we find slightly worsening productivity and inequality among the labor force. However, output per capita shows a declining trend as population age. This is due to the fact that less people are entering the labor force to produce output while more are retiring.

Moreover, global aging effects will likely depress the real interest rates which will lower funding costs for the entrepreneurs. Aging population drives up wages and results in a surge

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\(^3\)According to Gollin (2008), lower-skilled individuals remain self-employed in poor countries and constitute a larger portion of the labor force than in rich countries.

\(^4\)Chaiwat and Boonyamanond (2013) reports a significant correlation between aging population and inequality in Thailand.
Figure 1: Global and Thailand’s Aging Population

in savings as more individuals are close to retirement (e.g. Catalan et al., 2008; Brooks, 2003; and Domeij and Floden, 2006). On the other hand, demand for investment falls as global labor force shrinks. The projected long-run declines in world interest rates should also spill over to small open economies. As the interest rate in our model is exogenously given, we also conduct a scenario analysis of declining real interest rates as implied by global aging population. The two scenarios for declining interest rates, as influenced by gradual declines in world interest rates of 1% and 2% over 30 years, project slightly worrying outcomes. Under the new normal characterized by global and domestic aging population, output per capita, productivity and inequality are exacerbated. This is driven by a larger fraction of less productive self-employed who now bear lower costs of borrowing.

We then simulate four policy experiments on the calibrated model economy with aging population. The first policy considers an extension of a retirement age. As people work longer, this will increase output per capita. However, productivity and inequality will worsen due to the labor market structure which has a large share of self-employed among older cohorts. Extending retirement age will have a limited effect on occupational choices made by younger cohorts.

The second policy considers an improvement in the financial systems such as better loan
enforcement and liquidation process. This will relax collateral constraints and make financing more accessible for entrepreneurs. We find that the fraction of self-employment at the new steady state is smaller which improves the overall productivity and output per capita. Nonetheless, there is no effect on inequality.

The third policy focuses on developing skill specialization. This policy will enable the young to be very good at either working or managing. Entrepreneurs with better managerial skills become employers who now hire the more productive workers. Specialization is therefore achieved and there is a smaller fraction of self-employed at the new steady state. As a result, this policy will improve output and inequality. However, productivity along the transitional path suffers from the existing lower levels of skill comparative advance among the older cohorts.

The fourth policy combines the second and third policies to consider both an improvement in the financial system and skill specialization. This policy combination yields the best outcome regarding output, productivity and inequality. At the new steady state, skill specialization allows employers with enhanced managerial skills to expand their business and hire better-skilled workers. At the same time, by removing financial frictions, better credit access also facilitates the transition into the new steady state and increase productivity.

In summary, this paper contributes to the existing literature on aging population, labor market and inequality in three ways. Firstly, in addition to the compositional effect of older cohorts in the labor force suggested by Deaton and Paxton (1997) and Fehr et al.(2010), the model incorporates the indirect effects of aging population through labor market mechanisms of occupational choice. Secondly, the scenario analysis goes beyond domestic aging phenomenon and takes into account the spill-over effects from global aging which tends to depress world interest rates and expectedly the real interest rates in small open economies. Thirdly, policy recommendations are drawn from policy experiments simulated from the scenarios previously considered.

The paper proceeds as follows. Section 2 presents stylized facts of labor market and inequality from the Thai household survey data. Section 3 presents the life cycle model of occupational
choices. Section 4 calibrates the model economy and evaluates the performance of the model. The scenario analyses under the new normal are conducted in Section 5. Section 6 delivers the policy simulations on the model economy. Section 7 concludes.

2 Empirical Evidence

This section presents empirical evidence and stylized facts on inequality and occupational choices based on household survey data from Thailand during 2006-2013. The Socio-Economic Household Survey (SES) is a national cross-sectional survey carried out every other year by the National Statistical Office of Thailand (NSO). The objective of the Survey is to collect household economic and social information, such as income, expenditures, debts, housing characteristics, ownership of selected durable goods and changes in assets and liabilities. One novelty of this data set is its coverage on various sources of income including wages, rents, profits and interests. Unlike other studies on Thai labor market most of which use wage income from the Labor Force Surveys (LFS), our paper captures a wider source of income, especially for non-wage earners and income earned from assets. The SES data also include income of unpaid family workers and own-account workers who account for more than a third of the Thai labor force. As our theoretical framework concerns endogenous occupational choices which are specific to individuals, we opt for individual-level data rather than using household-aggregated income.

Empirical data provide three motivating facts for the model: 1) occupational composition differs by age 2) age-profile income differs by occupations, and 3) skill heterogeneity within and across occupations. Individuals are grouped by their occupations according to self-reported occupations: worker, employer and self-employed. Workers are wage earners who work for either the government, state-owned enterprises or the private sector. Employers and self-employed are entrepreneurs whose main income come from business or farm profits. The data suggest that

\[ \text{Averages during 2006-2013 are taken to calculate all statistics in this paper. Each individual year is also computed which confirms that the averages used are not subject to any time effects.} \]

\[ \text{Wage-based income of workers from the LFS is also used to cross check with total income obtained from the SES. Both age-profile average income and variance of log-income show similar trends. Average total income in the SES is slightly higher as it includes non-wage income such as returns to assets (Figure 18 in Appendix A).} \]
most younger cohorts are workers while older cohorts are self-employed (Figure 2). Such composition of the Thai labor market signifies the role of self-employed who are mostly older cohorts in the informal labor force such as agriculture and trade. In addition, the ratio of self-employed among older cohorts has been increasing over the years. With aging population, the size of older cohorts will quickly expand. Given the occupational composition of the labor force, we will likely observe a higher proportion of self-employed in the labor force in the near future.

Moreover, age profile and distribution of income differ across occupations. Over a life-cycle, different occupations observe distinct patterns of income and inequality (Figure 3). Unlike that of entrepreneurs, income of workers increases with age until retirement. Among entrepreneurs, employers and self-employed are vastly different with regards to their income level across all age groups. Turning to distribution of income, Figure (4) shows that most workers earn the average income and their income is less dispersed than that of entrepreneurs. The thick left tails for both employers and self-employed show that there is a concentration of people who earn less than average income. In fact, overall inequality is driven by within-occupation inequality among entrepreneurs. On the other hand, between-occupation inequality is due to a large difference between income of employers and that of the rest. The average income of employers is five times larger than that of workers and self-employed.

Income differentials between employers and self-employed can be partly explained by higher levels of skills possessed by employers. Using education level to reflect skills, Figure (5) illustrates a larger proportion of college and high school graduates among employers in comparison to the self-employed. For all education levels, employers also have higher income than others. On the other hand, workers have the highest ratio of college graduates. Yet, their average monthly income is much lower than that of employers. This evidence reflects the need to incorporate skill heterogeneity of both working skills, as reflected by differences in educational levels.

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7Similar trends are found in the US (Zissimopoulos and Karoly, 2007) and among OECD countries (Blanchflower, 2000).
8By decomposing Generalized Entropy Indices, within-occupation inequality accounts for around 10% of total inequality. In particular, inequality among entrepreneurs is more than 4 times worse than that among workers.
9Poschke (2013) finds that people with either low or high education become entrepreneurs and entrepreneurial income has a greater variance than wage-based income.
Figure 2: Compositions of Occupations by Age
Figure 3: Age-Profile Income and Inequality by Occupation

Figure 4: Distribution of Log Normalized Income by Occupation
Figure 5: Composition of Education by Occupation

Figure 6: Income by Education and Occupation
and managerial skills which allow employers to earn more than workers with similar educational levels. Working skills which can be developed through education are not necessarily correlated with managerial skills which partly build upon experiences.

Due to the age profile of occupational composition and income by occupations observed in the Thai data, Figure (7) displays the average income over the life cycle. Average income increases with age before falling slightly close to retirement. Income heterogeneity within age-cohorts, as reflected by variance of log-income, also increases in age. Moreover, the initial income inequality observed in young cohorts appears to be larger than that in developed countries. This age-profile of income and inequality can be driven by occupational choices made by individuals over a life cycle.

![Figure 7: Age Profile of Average Income (blue) and Variance of Log-Income (red)](image)

In summary, the empirical data highlight the different compositions of occupations across age cohorts. In addition, the age profile of income and inequality greatly differ by occupations.
Workers are the most equal group with age-increasing income and inequality. Income of entrepreneurs is more dispersed especially between employers and the self-employed. Employers and self-employed can be immensely different and must be distinguished to reflect differences in their skills, asset holdings and size of businesses. However, skills developed through education may not fully explain why college-educated workers on average earn less than employers with a college degree. These stylized facts will motivate a life cycle model with endogenous occupational choices and heterogeneous skills.

3 Model

The framework is a small open economy version of a life cycle model with occupational choices and financial frictions. The model is based on that of Allubble and Erosa (2014). Labor markets are characterized by occupational composition. Although the general equilibrium structure is standard, a small open economy setting implies that individuals and financial intermediaries can borrow funds from or invest funds in international capital markets at world interest rates. Individuals are heterogeneous in their skills and asset holdings. They choose their occupation whether to be wage workers or entrepreneurs. To start operating a business, entrepreneurs are subject to collateral constraints due to the possibility of default. Because their ability to borrow is limited by their wealth, some entrepreneurs become self-employed while others become employers.

3.1 Individuals and Demographics

The economy is populated with a continuum of finitely-live individuals, indexed by $i$. Individuals are born at age 20 ($t = 1$) with initial wealth $a_{0i}$, retire at age 60 ($t = T_{ret}$), and die with certainty at age 85 ($t = T$). The probability of surviving from age $t - 1$ to $t$ is $\kappa_t$. The mass of individual at age $t$ is denoted by $\mu_t$. For simplicity, we abstract from marriage and fertility. Each period a generation is born of the same measure of dead individuals, so that
the total population, which we normalize to 1, is constant.\textsuperscript{10} When an individual dies, he is replaced by a descendant who inherits his assets, but not skills.

Individuals have time-separable expected utility preferences over consumption $c_{it}$ given by

$$E \sum_{t=1}^{T} \beta^t (\Pi_{j=1}^{t} \kappa_j) \left[ \frac{c_{it}^{1-\eta}}{1-\eta} \right],$$

and are endowed with one unit of time at every working age. Individuals differ in their working ($z_w$) and managerial ($z_m$) skills. Each individual is born with

$$\alpha_i = (\alpha_{wi}, \alpha_{mi}) \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2_{w} & \rho \sigma_w \sigma_m \\ \rho \sigma_w \sigma_m & \sigma^2_m \end{bmatrix}\right)$$

The skills then evolve stochastically over the life cycle according to

$$\ln(z_{wit}) = X_t(\beta_w) + \alpha_{wi} + u_{wit}$$

$$\ln(z_{mit}) = X_t(\beta_m) + \alpha_{mi} + u_{mit}$$

The life cycle shocks follow the stochastic process

$$u_{jit} = \rho_j u_{jit-1} + \epsilon_{jit} \quad \text{for } j = w, m$$

$\epsilon_t = (\epsilon_{wt}, \epsilon_{mt})$ is jointly drawn from a bivariate normal distribution with correlation coefficient $\rho$.

### 3.2 Production

Output is produced with a constant returns to scale production technology in managerial, labor, and capital inputs. The output produced by an individual supplying $m$ units of

\textsuperscript{10}This setting implies that the model only considers the age structure or composition of a population.
managerial input and using $k$ units of capital and $n$ efficiency units of labor is:

$$y = m^\gamma k^\nu n^\theta$$

where $\gamma + \nu + \theta = 1$. Managerial input is the product of the individuals’ managerial ability ($z_m$) and the time devoted to managing a business ($l_m$), so $m = z_m l_m$. Labor input is $n = n^d + (1 - l_m) z_w$, where $z_w$ is the working skills of the entrepreneur and $n^d$ is workers outside the family. Working-age individuals can either choose to be workers or entrepreneurs. Workers devote all their time working for wage, so $l_m = 0$ and their earnings are $w z_w$. Entrepreneurs are divided into two groups: employers and the self-employed. Employers hire outside labor, $n^d > 0$, and devote all their time managing business, $l_m = 1$. Self-employed entrepreneurs do not hire outside labor. They devote part of their time managing business, $0 < l_m < 1$, and the other for working, $n = (1 - l_m) z_w$. Unlike the self-employed, employers incur a fixed cost of $c_f$.

### 3.3 Credit and Rental Markets

Individuals and financial intermediaries can borrow funds from or invest funds in international capital markets at the world interest rate, $r$. Financial intermediaries, who operate in a competitive market, receive deposits at the interest rate $r$ and rent capital $k$ at the rental rate $R$ to entrepreneurs. Both borrowing and capital rental are done within a period, so individuals’ asset is nonnegative, $a \geq 0$. The zero-profit condition of the intermediaries implies the rental rate $R = r + \delta$, where $\delta$ is the depreciation rate. Capital rental by entrepreneurs, however, are limited by a collateral constraint due to the possibility of default. The upper bound on capital rental of an entrepreneur with asset $a$ is $\lambda a$. Finally, the asset of dying individuals are inherited by the newly born individuals.

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11This can refer to fixed costs of hiring workers or fixed costs of setting up a large-sized business.
3.4 Individuals’ Problem

If a working-age individual chooses to become an entrepreneur at age $t$, the profit maximization problem of the entrepreneur with $(z_{mt}, z_{wt}, a_t)$ taking as given prices and collateral constraints is given by

$$
\pi(z_{mt}, z_{wt}, a_t) = \max_{n, k \in \{0,1\}} (z_{mt}l_m)^{\delta} k^{\nu} n^{\theta} - wn^d - Rk - c_f l_n d > 0 \quad (7)
$$

subject to

$$
n = (1 - l_m)z_{wt} + n^d \quad (8)
$$

$$
k \leq \lambda a_t \quad (9)
$$

$$
n^d \geq 0 \quad (10)
$$

The income of the entrepreneur can be written as

$$
y^e(z_{mt}, z_{wt}, a_t) = \pi(z_{mt}, z_{wt}, a_t) + (1 + r)a_t \quad (11)
$$

If a working-age individual chooses to become a wage worker at age $t$, the worker’s income is

$$
y^w(z_{mt}, z_{wt}, a_t) = w \tilde{z}_{wt} + (1 + r)a_t, \quad (12)
$$

where $\tilde{z}_{wt} = z_{wt}$ if the individual was a wage worker at age $t - 1$, and $\tilde{z}_{wt} = \chi_t z_{wt}$ if the individual was an entrepreneur at age $t - 1$. This is to capture the possibility that individuals can lose their accumulated experience after switching occupation or it is harder to start working as employees at older ages. The income of the working-age individual at age $t$ is the maximum between the entrepreneur’s and worker’s income:

$$
y(z_{mt}, z_{wt}, a_t) = \max \{y^e(z_{mt}, z_{wt}, a_t), y^w(z_{mt}, z_{wt}, a_t)\} \quad (13)
$$

A newborn individual is characterized by his asset $a_0$ and initial skills $(\alpha_m, \alpha_w)$. The individual maximizes by choosing sequences of consumption, assets, occupations, and capital/labor inputs.
if they choose to be entrepreneurs, subject to a sequence of period budget constraints and rental limits.

\[
\max_{c_t, a_{t+1}} \mathbb{E} \sum_{t=1}^{T} \beta^t (\Pi_j) \kappa_j \left[ \frac{c_{1-\eta}^t}{1-\eta} \right]
\]

s.t.

\[
c_t + a_{t+1} = y(z_{mt}, z_{wt}, a_t) ; t \leq T_{ret}
\]
\[
c_t + a_{t+1} = (1 + r) a_t ; t > T_{ret}
\]
\[
c_t \geq 0, a_{t+1} \geq 0, a_0 \text{ and } (\alpha_m, \alpha_w) \text{ given,}
\]

where \(y(z_{mt}, z_{wt}, a_t)\) is given by equations (7) and (13).

The individual optimization problem can be written recursively. The individual states are asset holding \(a\), skills \((z_m, z_w)\), last period occupation \(I_{w-1}\), and age \(t\). The dynamic problem is

\[
V(z_m, z_w, a, I_{w-1}, t) = \max_{c, a'} c^{1-\eta} + \beta \kappa_t \mathbb{E} V(z'_m, z'_w, a', I_{w}, t+1)
\]

s.t.

\[
c + a' \leq \mathbb{I}_w y^w(z_m, z_w, a, I_{w-1}) + (1 - \mathbb{I}_w) y^e(z_m, z_w, a) ; t \leq T_{ret}
\]
\[
c + a' = (1 + r) a ; t > T_{ret}
\]

where \(y^e(z_m, z_w, a)\) and \(y^w(z_m, z_w, a, I_{w-1})\) are given by equations (11) and (12). We also assume that \(\mathbb{I}_{w-1} = 1\) for individuals at age 1.

### 3.5 Steady State Competitive Equilibrium

Given the world interest rate \(r\) and an initial distribution \(\Phi_0(\alpha_m, \alpha_w, a_0)\), a steady state competitive equilibrium is composed of i) invariant distributions of working-age households \(\Phi^w(z_m, z_w, a, I_{w-1})\) and retired individuals \(\Phi^r(z_m, z_w, a, I_{w-1})\); ii) individual’s policy functions: consumption \(c_t(z_m, z_w, a, I_{w-1})\), savings \(s_t(z_m, z_w, a, I_{w-1})\), and occupational choice \(I_{wt}(z_m, z_w, a, I_{w-1})\); iii) entrepreneur’s policy functions: capital input \(k(z_m, z_w, a)\), managerial input \(l_m(z_m, z_w, a)\), and
outside labor input $n_d(z_m, z_w, a)$; and iv) prices $R$ and $w$ such that

1. Given $R, w$ and collateral constraint, entrepreneur’s policy functions solve (7).

2. Given $r$ and $w$, individual’s policy functions solve (18).

3. The zero-profit condition of financial intermediaries holds.

4. Labor market clears.

\[
\sum_{t=1}^{T^{ret}} \mu_t \int n^d(z_m, z_w, a)(1 - \mathbb{I}_{\omega t}(z_m, z_w, a, \hat{\omega}_{t-1})) d\Phi^w_t = \sum_{t=1}^{T^{ret}} \mu_t \int \tilde{z}_w(z_w, \hat{\omega}_{t-1}) \mathbb{I}_w(z_m, z_w, a, \hat{\omega}_{t-1}) d\Phi^w_t
\]

4 Baseline Economy and Calibration

To perform scenario analysis under the new normal, the life cycle model introduced in Section 3 is calibrated to match salient features of the Thai economy for the period 2006-2013. Non-targeted moments including the age-profile of mean income and inequality as well as the occupational composition across age cohorts are used to test the model performance. Despite its parsimony, the model is able to explain the life-cycle pattern of these non-targeted moments fairly well.

4.1 Calibration

The model is calibrated to the Thai economy. The model period is one year. Parameters are grouped into two categories. The first category includes parameters for which the values are either taken from other literature or directly obtained from the data. The second category includes parameters chosen so that endogenous outcomes from the steady state equilibrium match salient features of the data from the Thailand’s Household Socio-Economic Survey for the period 2006-2013.

The productivity process is discretized. The number of grid points for $\alpha_w$ and $\alpha_m$ is 4, so there are 16 pairs of fixed components. The number of grid points for $u_w$ and $u_m$ is 10,
so there are 100 pairs of stochastic components. At each age, there will be 1,600 possible pairs of \((z_m, z_w)\). The age-dependent components, \(X_t(\beta_w)\) and \(X_t(\beta_m)\), are assumed to follow a polynomial of degree 3, so the coefficient vectors \(\beta_w\) and \(\beta_m\) each have 4 elements. Assume that by switching from entrepreneur to worker at age \(t\), individuals’ underlying working skills is adjusted by \(\chi_t = 1/t^\xi\). Lastly, initial wealth inequality that the asset of dying individuals are randomly distributed to the newly born individuals by maintaining the same distribution.

The interest rate \(r\) is set to 3.5\% to match the average real interest rate. The coefficient of relative risk aversion is set to \(\eta = 1.5\), which is in the standard range in the literature. The production side of the model is characterized by the three parameters \((\gamma, \theta, \nu)\). As in Buera et al, (2011), the profit share in production is set to \(\gamma = 0.2\). How the rest of the income is divided between capital and labor is obtained from Pholphirul (2005).\(^\text{12}\) This yields \(\theta = 0.59\) and \(\nu = 0.21\). The depreciation rate on capital \(\delta\) is 4.4\% calculated from NESDB Capital Stock of Thailand. Data on survival probabilities from the Bureau of Policy and Strategy, Ministry of Public Health for Thailand in 2010 are used to determine the age-dependent survival probabilities, \(\kappa_t\). The size of each age cohort \(\mu_t\) is obtained from World Population Prospects by United Nations.

The rest are internally calibrated parameters. The discount factor \(\beta\) is chosen so that the capital to income ratio in the steady state is equal to the capital to GDP ratio in Thailand during the period. The collateral constraint parameter \(\lambda\) is calibrated to match the bank private credit to GDP ratio of Thailand. The fixed cost of employers \(c_f\) is set to match the fraction of employers among entrepreneurs. Turning to the skill process. The coefficient vectors, \(\beta_w\) and \(\beta_m\), the variance of fixed components, \(\sigma^2_w\) and \(\sigma^2_m\), the variance of stochastic components, \(\sigma^2_{\epsilon_w}\) and \(\sigma^2_{\epsilon_m}\), the persistence of stochastic components, \(\rho_w\) and \(\rho_m\), and the correlation, \(\rho\), are jointly calibrated to match the following moments: i) age profile of mean income for workers

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\(^{12}\)Labor income share is the average of two adjusted labor shares. Adjusted labor income share 1 is the ratio of the compensation of employees to GDP minus IUE and indirect taxes, subsidies, and provision for consumption of fixed capital. Adjusted labor income share 2 is the ratio of the sum of the compensation of employees and IUE to GDP minus indirect taxes, subsidies, and provision for consumption of fixed capital. IUE: Income from farm, professions and other unincorporated enterprises, who receive mixed income of profits of the enterprises and compensation to the owners. Many studies define these as income of self-employed workers, which refers to income for own account workers as well as the profits of unincorporated enterprises.
and entrepreneurs; ii) the ratio of median income between employers and self-employed entre-preneurs; iii) the ratio of median income between entrepreneurs and workers; v) the variance of entrepreneurial log income of entrepreneurs and the variance of log income of worker; and v) the proportion of entrepreneurs and workers in the population. Lastly, the parameter governing skill adjustment when switching to from entrepreneur to worker, $\xi$, is chosen to match the persistence of being an entrepreneur between two consecutive years.

Table (1) summarizes all parameter values. The targeted moments are reported in Table (2) with the age profile of mean income for each occupational group illustrated in Figure (8). The model-based counterpart of the data is based on the steady state distribution of the economy.
The model does a good job of generating the large fraction of self-employed entrepreneurs, which, on average, have low income. In particular, approximately 40% of labor is self-employed, and their average income is less than 20% of the average income of employers. However, the model underestimates the variance of log income of entrepreneurs.\textsuperscript{13}

<table>
<thead>
<tr>
<th>Moments</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private capital to GDP ratio</td>
<td>2.2</td>
<td>2.0</td>
</tr>
<tr>
<td>Private credit to GDP</td>
<td>0.9</td>
<td>0.8</td>
</tr>
<tr>
<td>Fraction of employers among entrepreneurs (%)</td>
<td>12</td>
<td>10</td>
</tr>
<tr>
<td>Persistence of being entrepreneurs(%)</td>
<td>96</td>
<td>98</td>
</tr>
<tr>
<td>Variance of (log-) income: workers</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>Variance of (log-) income: entrepreneur</td>
<td>4.8</td>
<td>1.3</td>
</tr>
<tr>
<td>Ratio of median income (entrepreneur/worker)</td>
<td>0.7</td>
<td>0.7</td>
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<tr>
<td>Ratio of median income (self-employed/entrepreneur)</td>
<td>0.2</td>
<td>0.12</td>
</tr>
<tr>
<td>Fraction of entrepreneurs (%)</td>
<td>42</td>
<td>39</td>
</tr>
</tbody>
</table>

\textsuperscript{13}This is due to the model limitation that all underlying productivities are assumed to follow a log-normal distribution. The data, however, show that the distribution of entrepreneurial (log-) income has skewness and fat tail.
4.2 Baseline Economy and Non-Targeted Moments

Individuals are heterogeneous in their skills and wealth. They choose one of the three occupations depending on their relative skills and wealth. Without collateral constraints, it is the skill ratio $z_w/z_m$ that drives occupational choices. An individual with a high $z_w/z_m$ ratio will become a worker. An individual with a low $z_w/z_m$ ratio will become an employer while one with an intermediate $z_w/z_m$ ratio will become self-employed. The left panel of Figure (9) shows the how individuals with wealth $\bar{a}$ choose their occupations in the economy without collateral constraints ($\lambda = \infty$), compared with the middle panel that illustrates the baseline economy ($\lambda = 2.3$). Financial frictions, i.e. collateral constraints, distort returns to skill. They reduce

\[ a = \bar{a}, \; \lambda = \infty \quad a = \bar{a}, \; \lambda = 2.3 \quad a > \bar{a}, \; \lambda = 2.3 \]

Figure 9: Occupation by Skills: Employer (red), Self-Employed (green), and Worker (blue)

the rate of return to the managerial ability of employers, the return to the composite skill input supplied by self-employed individuals, and the wage rate. The presence of collateral constraints increases a chance of individuals becoming self-employed as they prevent talented entrepreneurs to expand their business. As a result, wealth matters when individuals choose occupations in the baseline economy which has a certain degree of financial frictions. The middle and the right panels of Figure (9) illustrate how wealth affects occupational choices. Talented entrepreneurs with higher wealth are less constrained, so they are able to expand their business and become
employers. Because the model has a clear implication on occupational choices, income and inequality, and because scenario analysis will be conducted in Section 5 for the economy under the new normal, it is crucial to see how well the model can match the age-profile of mean income and inequality as well as the occupational composition across age cohorts. Figure (10) compares the model-implied occupational composition by age cohort, in the left panel, to the actual occupational composition from the data for the period 2006-2013, in the right panel. Note that the model is able to replicate the fact that most younger cohorts are workers while older cohorts are entrepreneurs whose majority are self-employed. One possible explanation is that individuals start their career as workers and later on they switch from employees to entrepreneurs.

Figure (11) shows the model-implied mean income and variance of log income over the life cycle. Just as in the data, average income increases over the life cycle before it slightly falls close to retirement. Also, inequality within age-cohorts, as reflected by variance of log-income, increases in age, the similar pattern observed in the data. Overall, the model can capture the life-cycle patterns of occupational compositions, average income and inequality relatively well. All of them are non-targeted moments. However, because the baseline calibration underestimates the variance of (log-) income among entrepreneurs, the model-implied inequality measure is
lower than that observed in the data.

5 Scenario Analysis under the New Normal

The Thai economy is entering an era of the new normal characterized by population aging, both at home and globally. In a small open economy, population aging affects the labor market while global population aging leads to a decline in world interest rates. This section conducts scenario analysis based on the model projections of output, productivity, and inequality under the new normal. As far as the model is concerned, the main mechanism is how individuals change their occupational choices and saving decisions in response to population aging and declining interest rates.

The scenario analysis conducted in this section is a simulation of the model economy to see its transition to the new steady state economy. As the focus of the analysis is on population aging, both at home and globally, Thailand’s demographic change and the path of the world interest rate are the crucial exogenous changes introduced to the economy. Other exogenous variables and parameters are assumed to remain unchanged.
The analysis focuses on the period of 2010-45. In 2010, the model economy is in the steady state equilibrium characterized by the baseline economy. Then, the demographic changes and the new path of the world interest rate are introduced. Thailand’s demographic changes during this period are obtained from United Nations’ World Population Prospects, which provide the demographic data and assess population projections at the national level. However, to simulate the transition of the model economy, it requires the existence of a well-defined transition path from one steady state to the other. Thus, it is necessary that the model economy will converge to the new steady state. Therefore, the demographic structures and the world interest rates are set to remain unchanged after 2045.\textsuperscript{14}

In addition to Thailand’s demographic change, three paths of the world interest rate, as influenced by global population aging, are considered. In Scenario 1, the exogenous interest rate path is constant at 4%. In Scenario 2, interest rates decline gradually by 100 basis points from 4% to 3% and remain unchanged after 2045. In Scenario 3, interest rates decline gradually by 200 basis points from 4% to 2% and remain unchanged afterwards. In all scenarios, individuals have perfect foresight and thus fully anticipate the demographic change and the future path of interest rates.

The extent to which global population aging will depress the path of world interest rates is difficult to predict. Labor supply shortage caused by global aging population results in a surge in savings as more individuals are close to retirement (e.g. Catalan et al., 2008). On the other hand, demand for investment falls as global labor force shrinks. The international spill over of declining world interest rates will make small open economies Thailand also face declines in interest rates. Figure (12) shows output per capita, total factor productivity, wealth inequality and income inequality as simulated in the three scenarios.\textsuperscript{15} When the world interest rates are constant at 4%, Scenario 1 reflects the effects solely from Thailand’s aging population, both directly and indirectly. For the direct effects, given Thailand’s occupational composition, a larger proportion of older cohorts in the labor force implies that there will be a larger proportion

\textsuperscript{14}Even if there is no exogenous change after 2045, it will take almost 100 years after this for the model to converge to the new steady state. In the new steady state, all cohorts are born after 2045 so that they solve the same life-cycle dynamic problem.

\textsuperscript{15}See Appendix B for the detailed definitions.
of self-employed in the labor market (Figure 13). As a result, the overall output and productivity decline as the proportion of unproductive self-employed increases. In fact, output per capita will fall by 30% over the next 30 years which largely reflects the compositional effect of the demographic change. This decrease reflects a larger number of retirees in the economy who no longer produce any output but are accounted for output per capita.

In addition to these direct effects of population aging, the gradual fall in labor force participation associated with population aging indirectly encourages workers and employers to become self-employed. The fall in labor supply drives up wage and pressures some employers to downsize their business. These employers and workers who lose their jobs therefore switch to become self-employed. The smaller decrease in productivity reflects the indirect effects of population aging through the labor market channel as people reallocate to self-employment. With more self-employed, both income and wealth inequality are also worsened among the labor force aged between 20-60. In particular, income inequality measured by the Gini coefficient will rise by 3% over the next 30 years because income among the self-employed is the most unequal. The increase in wealth inequality is more prominent. The Gini coefficient for wealth distribution increases by 10% over the same period. This reflects the compositional effects of larger older cohorts whose asset holdings are even more unequal than incomes.

In Scenario 2 and 3, the decline in world interest rates influenced by global population aging is added to the Thailand’s domestic demographic changes. Scenario 2 and 3 in Figure (12) show that output, productivity and inequality follow the trends as observed in Scenario 1. The interest rate channel through which labor market mechanisms function make funding costs cheaper, encouraging more individuals to become entrepreneurs. Scenario 2 and 3 differ most from the first scenario by income inequality and productivity. When interest rates are expected to decline to 2% in Scenario 3, the Gini coefficient of income rises from .51 to .56, or a 9% increase over 30 years. Higher income inequality is due to the larger proportions of entrepreneurs. The decline in world interest rates enables the talented employers to face lower costs and make more profits. On the other hand, the less-skilled workers also benefit from lower funding costs to start their businesses as self-employed. Due to global population aging faced
by Thailand as well as others, income inequality is expected to exacerbate.

Scenario 1: Interest rates are constant at 4%.
Scenario 2: Interest rates decline gradually from 4% to 3% and remain unchanged afterwards.
Scenario 3: Interest rates decline gradually from 4% to 2% and remain unchanged afterwards.

Figure 12: Scenario Analysis under the New Normal
Figure 13: Scenario Analysis under the New Normal, Occupational Composition
Lower interest rates exacerbate a fall in outputs and significantly worsen wealth inequality when comparing across new steady states. This is due to the larger proportions of entrepreneurs, especially self-employed. The transition paths, however, show a different story. The declining interest rates expected by individuals, especially the young, encourage them to save more in order to smooth their consumption. This in turn enables them to accumulate more wealth and later become entrepreneurs. This channel is not present in the steady state because the interest rate is assumed to remain constant. The declining interest rates expected by individuals, thus, raises output per capita at the beginning of the transition path. The change in saving or asset-holding decisions also dampen the negative effect of lower interest rates on wealth inequality.

6 Policy Experiment

This section considers four policy experiments: i) retirement age extension, ii) an improvement in the financial system, iii) skill development that promotes job specialization, and iv) a combination of Policy II and Policy III. A one-time and permanent policy implementation in 2015 is incorporated to the simulation of the model economy in Scenario 1 – domestic aging population and a constant interest rate path. As in the previous section, the analysis considers both the transition path and the new steady state of the model economy.

6.1 Policy I: Retirement Age Extension

The first policy experiment is the retirement age extension which is widely considered in most literature on aging population. As this paper addresses aging population with a different mechanism, i.e., occupational choices by age cohorts, it is interesting to see how the policy works through this channel. The retirement age is extended from 60 to 65 years old. The simulation results are shown in Figure (14). As people work longer, output per capita is higher than the two base-line scenarios. Lower average productivity is resulted from a concentration of self-employed among older cohorts who now work until 65. However, extending retirement

\footnote{Policy experiments on Scenario 2 and 3 yield similar pattern of transition and almost the same in magnitude.}
Figure 14: Scenario Analysis under the New Normal, Retirement Age Extension

age does not affect occupational choices of other cohorts and therefore shows a limited impact on inequality. The new steady state with retirement age extension shows similar outcomes to that of the base-line scenarios.

6.2 Policy II: Improvement in the Financial System

The second policy experiment is an improvement in the financial system. Better contract enforcement and credit information systems allow financial intermediaries to require less the amount of collateral. In particular, the collateral constraint parameter $\lambda$ is raised from 2.3 in the baseline economy to 25. The simulation results are shown in Figure (15). First, by removing
financial frictions, the misallocation problem is alleviated. With an improvement in the financial system, individuals formerly faced with credit constraints can now borrow more. This enables the productive but constrained entrepreneurs to expand their business and therefore increase productivity and output. This effect can be observed both along the transition path and in the new steady state.

![Graphs showing changes in output per capita, productivity, wealth Gini, income Gini, percentage of self-employed, and percentage of employer over time.]

Figure 15: Scenario Analysis under the New Normal, Improvement in the Financial System

The fraction of self-employed has an interesting dynamic behavior. Initially, when the business expansion of productive employers drives up wage, unproductive employers switch to
self-employed, especially when switching to wage workers can be costly for old age. This increases the fraction of self-employed at the initial stage of transition. In fact, these unproductive individuals become employers only due to the presence of misallocation, which depresses wages. In the new steady state, when most misallocation is resolved, entrepreneurs are more likely to be employers rather than self-employed as suggested by Figure (9). That is, the fraction of self-employed is around 33% in the economy with less financial frictions compared to 39% in the baseline case. Although the fraction of employers seems to be lower than that observed in the baseline scenario, these employers are all productive and own a large business. Lastly, the effect on inequality is so small that the policy will not help alleviate a rising inequality, which is a result from aging population.

6.3 Policy III: Skill Specialization

The third policy experiment is skill development that promotes job specialization. Individuals develop specific skills in their area of expertise, so they specialize either in managing or working. As a result, greater benefits from job specialization or division of labor can be achieved. The experiment considers skill specialization for both fixed component ($\alpha$) and stochastic component ($u$). This can be associated with promoting skill specialization through an education system for the newborn and through mid- to late- career training for older cohorts. In particular, the correlation coefficient, $\rho$, is set to $-0.6$, compared to $0.1$ in the baseline economy. This policy will gradually change the distribution of the skill ratio $z_w/z_m$ toward a bimodal distribution. There will be a larger mass of individuals with high $z_w/z_m$ ratios, who will become workers, and a larger mass of individuals with low $z_w/z_m$ ratios, who will become employers. Less individuals with intermediate $z_w/z_m$ ratios implies that there will be less self-employed entrepreneurs in the economy. The simulation results are shown in Figure (16).

First, for the newborn or young cohorts, specialization fully takes effect up to the point allowed by collateral constraints. With both fixed component ($\alpha_w, \alpha_m$) and expected stochastic components ($u_w, u_m$) governed by negative correlation, young individuals sort themselves
Figure 16: Scenario Analysis under the New Normal, Skill Specialization according to their skill ratios. For older cohorts, the late-career training that promotes skill specialization results in managing and working skills of self-employed entrepreneurs being developed in a way that their intermediate skill ratios gradually increase or decrease. If their skill ratios $z_w/z_m$ decrease, self-employed entrepreneurs can easily switch to employers given that their collateral constraints are not too tight. However, if their skill ratios $z_w/z_m$ increase, some may remain self-employed because switching to wage workers can be costly for older cohort.

Thus, at the initial stage of transition, only the switch from self-employment to employ-
ers are observed. It was not until later in the transition toward the new steady state when the economy consists of most cohorts born with this policy that the fraction of self-employed entrepreneurs goes down from 34% to 28%. The economy enjoys the benefits of specialization from an increase in output per capita. However, the fact that some self-employed entrepreneurs who develop higher skill ratios $z_w/z_m$ cannot switch to wage workers depresses the overall productivity. As a result, total factor productivity initially falls before increasing toward the new steady state when the fraction of self-employed entrepreneurs starts to fall.

Lastly, it is important to note that, during the initial transition, the policy does not significantly affect the fraction of entrepreneurs but does change its composition. Skill specialization leads to a lower fraction of self-employed entrepreneurs – a group with relatively low income and high inequality. In addition, this policy induces the new distribution of skills which supports more equal income distribution. Skill specialization, thus, alleviates the inequality problem created by aging population. In fact, both wealth and income inequality fall compared to the baseline scenario. Later toward the new steady state when the fraction of entrepreneurs eventually falls, the effect on inequality is even more prominent.

### 6.4 Policy IV: Improvement in the Financial System and Skill Specialization

Policy III, which promotes skill specialization, delivers desirable results in terms of output per capita and inequality. Overall productivity, however, initially falls before gaining its momentum after the current old cohorts exit the labor market. With this policy solely implemented, there is still a certain degree of misallocation caused by financial frictions. Specialization takes effect up to the point allowed by collateral constraints. Policy IV aims to fully realize the benefits of specialization by combining Policy III that promotes skill specialization and Policy II that improves the financial system. In particular, the correlation coefficient $\rho$ is set to $-0.6$, and the collateral constraint coefficient $\lambda$ is set to 25. The simulation results are shown in Figure (17). Along the transition, changes in the fraction of self-employed entrepreneurs and the fraction of employers are the combining results of two policies. Productive employers expand
their businesses, driving up the wage rate. Unproductive employers switch to self employment. Self-employed entrepreneurs who develop their managing skills become employers although they find it more difficult to switch due to a higher wage rate. As a result, there is a slight drop in the fraction of self-employed entrepreneurs while the fraction of employers is roughly similar to that of the baseline scenario. The new steady state sees a significantly lower fraction of self-employed entrepreneur but only a slightly lower fraction of employers. Resolving misallocation adds to

Figure 17: Scenario Analysis under the New Normal, Improvement in the Financial System and Skill Specialization

the benefits of skill specialization, as can be seen from a larger increase in output per capita and total factor productivity in the new steady state. At the initial stage of transition, its impact
on total factor productivity also outweighs the impact of skill specialization, resulting in a moderate increase in productivity. Lastly, during the initial transition, skill specialization leads to a lower fraction of entrepreneurs mainly through a lower fraction of self-employed entrepreneurs—a group with relatively low income and high inequality. A drop in the fraction seems to be small and should not have a significant effect on inequality. However, as observed in Policy III, skill specialization induces the new distribution of skills which supports more equal income distribution. The policy, thus, alleviates the inequality problem even during the initial transition. Later toward the new steady state when the fraction of entrepreneurs and the fraction of self-employed entrepreneurs eventually fall, the effect on inequality becomes more prominent. In fact, both wealth and income inequality fall compared to the baseline scenario.

7 Conclusion

Under the new normal characterized by aging population, the paper investigates the changes on output, productivity and inequality. Not only does aging population reduce labor force participation, its effects on increased wage also influence occupational choices made by individuals at all ages. A life cycle model with occupational choices and financial frictions is employed to investigate these effects. The model reflects the important role of self-employment and interesting features of the Thai labor market which are commonly shared by many developing countries. Heterogeneity of skills and asset holdings enables the model to replicate diverse income levels observed in the Thai household survey data. Two sets of skills—managerial and working skills—are determinants of how individuals choose their occupations between being workers, employers or self-employed.

This paper finds negative effects of local and global population aging on inequality through labor market mechanisms. Given the age profile of occupational compositions, older cohorts are likely to be self-employed entrepreneurs. The aging demographics will enlarge the proportions of old self-employed most of which are the low-income and low-productivity group. Due to increased wage, some employers and workers also switch to become self-employed. Declining
world interest rates associated with global population aging have the potential to exacerbate inequality further. Not only do the lower costs of borrowing allow employers to expand their business, lower interest rates also attract low-skilled workers to become self-employed. As a result, the distribution of income becomes more dispersed.

Macroeconomic challenges resulted from aging population can be mitigated by reducing financial frictions or alleviating a borrowing constraint. Extending retirement age will only improve output per capita while total factor productivity worsens. However, neither policy can significantly minimise the adverse effects on inequality. On the other hand, a policy that promotes job specialization can improve inequality at the expense of lower productivity during transitional stages. Most importantly, combing the two policies on reducing financial frictions and promoting skill specialization yield superior outcomes at both the transitional path and the new steady state. As skill specialization reduces self-employment by gradually reallocating them to either employers or workers, better credit access alleviates credit constraints formerly faced by employers and facilitates the expansion of businesses.

In countering the adverse effects from population aging, the policy experiments highlight the importance of managing the size of self-employed in the economy. Whereas some self-employment is related to high-value added manufactured exports, a large fraction of self-employed in an economy is associated with low levels of economic development (Pietrobelli et al., 2004). A small fraction of self-employed exists to serve as a transition from workers to employers when some also face collateral constraints. Here, self-employment is transitory whereby entrepreneurs can initially try out business ideas before upgrading to be employers. However, a large fraction of self-employed reflects an inefficient allocation of skills as the self-employed are neither good at working nor managerial skills and are often financially constrained. Policies which aim to reduce a fraction of self-employment in a developing context are not limited to reducing financial frictions and promoting skill specialization as suggested in this paper. Other economic and institutional factors as well as aggregate productivity are found to affect the rate of self-employment across countries (Pietrobelli et al., 2004; Gollin, 2008). Although they are not in the scope of our study, attention should also be drawn to these factors in order to deal
with upcoming challenges from aging population and a rising self-employment faced by the Thai economy.
APPENDIX

A Data Appendix

Figure 18: Comparisons of Age-Profile Income for Workers in LFS and SES

Figure 19: Compositions of Industry by Occupation
B Definitions and Formulas

B.1 Output per capita

Output per capita is defined as the total output produced divided by the total number of population.

\[
\text{Total Output} = \sum_{t=1}^{T\text{ret}} \mu_t \int o(z_m, z_w, a) (1 - I_w t(z_m, z_w, a, I_w - 1)) d\Phi^w,
\]

where \( o(z_m, z_w, a) = (z_m l_m(z_m, z_w, a))^\gamma k(z_m, z_w, a) ^\nu n(z_m, z_w, a) ^\theta \) and a total population is normalized to 1.

B.2 Productivity

Total factor productivity is an average productivity with which the composite capital and labor input is used in production across entrepreneurs.

\[
\text{Productivity} = \frac{\sum_{t=1}^{T\text{ret}} \mu_t \int o(z_m, z_w, a) (1 - I_w t(z_m, z_w, a, I_w - 1)) d\Phi^w}{\sum_{t=1}^{T\text{ret}} \mu_t \int i(z_m, z_w, a) (1 - I_w t(z_m, z_w, a, I_w - 1)) d\Phi^w},
\]

where \( o(z_m, z_w, a) = (z_m l_m(z_m, z_w, a))^\gamma k(z_m, z_w, a) ^\nu n(z_m, z_w, a) ^\theta \)

and \( i(z_m, z_w, a) = k(z_m, z_w, a) ^\nu n(z_m, z_w, a) ^\theta \).

B.3 Gini Coefficients

Income Gini shown throughout this paper is measured among those in the labor force. For individuals in the labor force uniform on the incomes \( y_i, i = 1, ..., n_{LF} \), indexed in non-decreasing order \( y_i \leq y_{i+1} \):

\[
\text{Income Gini} = \frac{1}{n_{LF}} \left( n_{LF} + 1 - 2 \left( \frac{\sum_{i=1}^{n_{LF}} (n_{LF} + 1 - i) y_i}{\sum_{i=1}^{n_{LF}} y_i} \right) \right).
\]
Wealth Gini shown throughout this paper is measured for a whole population. For a population uniform on the assets \( a_i, i = 1, ..., n \), indexed in non-decreasing order \((a_i \leq a_{i+1})\):

\[
\text{Wealth Gini} = \frac{1}{n} \left( n + 1 - 2 \left( \frac{\sum_{i=1}^{n} (n + 1 - i)a_i}{\sum_{i=1}^{n} a_i} \right) \right).
\]
References


